RANDOMIZED ALGORITHMS FOR CROSS-LAYER NETWORK CONTROL

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Abstract— **We study the problem of joint congestion control and scheduling in wireless networks. We model the wireless network as a directed graph $G = (V, E)$, where V denotes the set of nodes and E denotes the set of wireless links between the nodes. We propose a joint congestion control and scheduling scheme that achieves a fraction $d_I(G)$ of the capacity region, where $d_I(G)$ depends on certain structural properties of graph G as well as the nature of interference constraints. For specific families of graphs, which can represent a wide variety of wireless networks, $d_I(G)$ has been upper bounded by a factor independent of the number of nodes in the network for a wide range of interference models. The scheduling element of our joint congestion control and scheduling scheme is the maximal scheduling policy considered in many of the earlier works. Although, it is widely believed to be amenable to distributed implementation, no algorithms have been proposed for its implementation, except under the node-exclusive interference model which is suitable only for networks in which adjacent nodes can transmit over non-interfering channels. We propose a randomized algorithm for implementing maximal scheduling policy under a 2-hop interference model which is suitable for networks with a limited number of non-interfering channels (e.g., IEEE 802.11 DSSS networks).**

I. INTRODUCTION

Wireless networks have become a ubiquitous part of all modern day communication systems. Unlike wireline networks, where bandwidth and other resources are available in plenty, wireless networks are highly resource constrained. An efficient utilization of the resources is therefore a necessity in case of wireless networts. A seminal contribution in this direction was made in [24], where the authors characterized the *capacity region* of constrained queuing systems, such as a wireless network. They developed a queue length based scheduling scheme that is *throughput-optimal*, i.e., it stabilizes the network provided the user rates fall within the capacity region of the network, where the capacity region is defined to be the set of user arrival rates under which the network is stable (the queue lengths at all the nodes are bounded).

Unlike wireline networks, where all links have fixed capacities, the capacity of a wireless link varies with channel variations due to fading; changes in power allocation, link scheduling, or routing; and changes in network topology etc. This results in the capacity region of a wireless network having a joint dependence on routing, power allocation, link scheduling, and channel variations. In order to maximize the capacity region of the network, one must therefore develop algorithms that can perform jointly optimal routing, link scheduling, and power control under possibly varying channel conditions and network topology. This has spurred recent interest in developing cross-layer optimization algorithms (see, for example, [26], [16], [15], [23], [5]).

Motivated by the works on fair resource allocation in wireline networks [8], [20], [12], [2], [27], researchers have also incorporated congestion control into the cross-layer optimization framework [3], [10], [9], [14], [25], [22], [28], [17]. The congestion control component controls the rate at which users inject data into the network so as to ensure that the user rates fall within the capacity region of the network.

The main component of all the above cross-layer optimization schemes is the optimal scheduler that solves a very difficult global optimization problem of the form:

$$\textbf{maximize} \sum_{l \in \mathcal{L}} p_l r_l \qquad (1)$$
$$\textbf{subject to } \mathbf{r} \in \Delta$$

where $\mathcal{L}$ denotes the set of wireless links; $\mathbf{r}$ is the vector of link rates r_l, $l \in \mathcal{L}$; p_l, $l \in \mathcal{L}$, is the congestion price or possibly some function of the backlog at link l; and Δ is the capacity region of the network. The main difficulty in solving the above optimization problem is that the capacity region Δ depends on the complete network topology and, in general, has no simple representation in terms of the power constraints at the individual links or nodes. The above optimization problem is, in general, NP-Complete and Non-Approximable*.

The above scheduling problem has been considered under several special scenarios of interest, e.g. scenarios with simplified interference models and no power control. The interference models studied in the literature include the node-exclusive interference model [7], [1], [4], [10], [3], [24], [23], [17] and IEEE 802.11 type interference model [25], [21], [6].

In our previous work [19], [18], we introduced the notion of a K-hop interference model. An interference model is termed a K-hop interference model if the only constraint imposed on the set of simultaneously transmitting links is that no two links within the set should be within K hops from each other. By varying K, one can capture the interference characteristics of a broad range of wireless networks. The node-exclusive interference model (commonly used model for Bluetooth and FH-CDMA networks [13], [1], [7]) and IEEE 802.11 type interference model (commonly used model for IEEE 802.11 DSSS networks [25], [6]) studied in earlier works correspond to a 1-hop and 2-hop interference model, respectively.

Although, the optimal scheduling problem is polynomial time solvable under the 1-hop interference model, it is NP-Hard and Non-Approximable under all K-hop interference models with $K > 1$ (see [19]). However, it was shown in [18] that the optimal scheduling problem can be approximated within a constant factor under all K-hop interference models for wireless networks whose connectivity graph is geometric. Similar results can be derived for disk graphs and (r, s)-civilized graphs. These results are quite encouraging as wide variety of wireless networks can be well represented using the above families of graphs.

In this paper, we first propose a joint congestion control and scheduling scheme that is guaranteed to achieve a constant fraction of the capacity region for a wide class of wireless networks. The scheduling element of our joint congestion control and scheduling algorithm is the maximal scheduling scheme considered in many of the earlier works [4], [25], [10]. We also provide a randomized distributed algorithm for implementing maximal scheduling policy under the 2-hop interference model.

The rest of the paper is organized as follows. System model and related work are described in Section II. An upper bound on the capacity region is provided in Section III. A novel joint congestion control and scheduling algorithm is proposed and a lower bound on its performance is provided in Section IV. A distributed randomized algorithm for maximal scheduling is proposed in Section V and an upper bound on its running time is provided in case of geometric graphs. Finally, concluding remarks are presented in Section VI.

II. SYSTEM MODEL AND RELATED WORK

We consider a set V of nodes, labeled $1, 2, ..., |V|$, communicating with each other using wireless means. The link (u, v) from node u to node v exists if node u can successfully transmit to node v, provided no other node in the network transmits at the same time. The set of links so formed is denoted by E. Note that the existence of a link between any two nodes depends on many factors (e.g., noise variance at the receiving node, coding and modulation scheme used by the nodes). Although, we do not consider channel variation in this paper, it can easily be incorporated into our model. We refer the interested reader to [16], [15], [11] for related results.

We consider users of K types, labeled $1, 2, ..., K$, sending data over the network. We assume that the arrival process for the type k users is Poisson with rate λ_k. Further, we assume that each user of type k, brings with it a file of size $1/\mu_k$ to be transferred over the network. We assume that users of each type send their data over the same, loop-free route. The extension to the multi-route case is straightforward; we refer the reader to [16], [15], [11], [23] for related results. The user routes are stored in an incidence matrix $[H_k^l]$, where $H_k^l = 1$ if the route of type k users contains link l; and 0 otherwise.

The interference constraints are modeled using a contention matrix $[C_{ij}]_{i,j \in E}$. More precisely, link i is said to interfere with link j if $C_{ij} = 1$; no two links which interfere with each other can be scheduled at the same time. All diagonal entries of C are set to 1. The time is divided into slots of unit duration. Link l can transmit at rate c_l during a slot if no other interfering link is scheduled to transmit during the same slot. Such an interference model has been widely used in the literature (see, for example, [4]), and the interference models used in many other works [19], [18], [10], [9], [11], [4], [25], [3] can be obtained by imposing some additional

*A problem in said to be Non-Approximable if it does not admit any constant factor polynomial time approximation algorithm.

restrictions on the contention matrix. For simplicity of exposition, we assume that C_{ij} is symmetric; i.e., link i interferes with link j if and only if link j interferes with link i.

Let $\vec{\lambda} = (\lambda_1, \lambda_2, ..., \lambda_K)$ be the vector of user arrival rates. Let $n_k(t)$ and $Q_l(t)$ denote the number of type k users and queue backlog at link l in the network at time t, respectively. As in [10], [16], we say that the network is stable if

$$\limsup_{t\to\infty} \frac{1}{t} \int_0^t 1_{\left\{\sum_{k=1}^K n_k(t)+\sum_{l\in E} Q_l(t)>N\right\}} dt \to 0 \quad (2)$$

as $N \to \infty$. The capacity region of the network is defined to be set of user arrival rate vectors for which the network can be stabilized under some scheduling policy. The capacity region of constrained queuing systems, such as a wireless network, has been well characterized in [24]. For our model, the capacity region is given by the set

$$\Omega = \left\{ \vec{\lambda} : \left[\sum_{k=1}^K \frac{H_k^l \lambda_k}{\mu_k c_l} \right]_{l\in E} \in Co(\mathcal{S}) \right\}, \quad (3)$$

where $Co(\mathcal{S})$ represents the convex hull of all link schedules $\mathcal{S}$ that satisfy the interference constraints imposed by our model.

A scheduling scheme is said to be *throughput-optimal* if it can stabilize the network for all arrival rate vectors within Ω. Tassiulas and Ephremides [24] proposed a queue length based scheduling scheme and showed that it is throughput-optimal. However, the proposed scheme requires centralized computation and is NP-Hard in most cases of interest. Since it is difficult to do centralized computation in an ad hoc setting, a considerable amount of effort has been put forth in devising simple distributed schemes that can achieve a certain fraction of the capacity region.

A scheduling scheme that has been widely studied in the literature in this context is the so-called *maximal scheduling scheme* [10], [4], [18], [25][†]. In [10] and [25], the performance of maximal scheduling scheme is studied under a joint congestion control and scheduling framework in a multi-hop traffic setting with 1-hop interference model and 2-hop interference model, respectively. In [4] and [18], a more restrictive setting with single-hop traffic and no congestion control is considered with contention matrix based interference model and K-hop interference models, respectively.

Although, maximal scheduling scheme has been studied in the literature from the point of view of being amenable to distributed implementation, no explicit distributed algorithms have been proposed for its implementation.

The main contributions of this work are as follows.

- We propose a joint congestion control and scheduling algorithm based on maximal scheduling scheme under a multi-hop setting and study its performance under the contention matrix based interference model. In doing so, we extend the earlier results of Chaporkar et al. [4] for a single-hop (MAC layer) setting to a multi-hop setting.
- We then restrict our attention to wireless networks whose connectivity graph is geometric and consider K-hop interference models introduced in our earlier work [18]. In this setting, we show that our joint congestion control and scheduling algorithm is within a constant factor of the optimal for all values of K. Similar results can be obtained for wireless networks whose connectivity graph is a disk graph or a (r, s)-civilized graph. These results extend our earlier results in [18].
- We provide a randomized distributed algorithm for implementing the maximal scheduling scheme under the 2-hop interference model. We show that the algorithm runs in $O(\log^2 |V|)$ rounds in case of geometric graphs. Note that the commonly used IEEE 802.11 RTS/CTS communication scheme corresponds to a 2-hop interference model. Moreover, numerical results in [18] indicate that of all the K-hop interference models, the 2-hop interference model yields best performance in IEEE 802.11 DSSS PHY based networks under a wide range of node densities.

III. AN UPPER BOUND ON THE CAPACITY REGION

In this section, we derive an upper bound on the capacity region under the contention matrix based interference model. The upper bound will be in terms of the *interference degree* of the network graph which we define formally next (see also [4] and [18]).

Definition 1. *The interference set $I(e)$ of link e is the set of links that interfere with link e, i.e.,*

$$I(e) = \{l \in E : C_{el} = 1\}.$$

[†]Note that the terminologies used in these works and some minor details of the schemes differ slightly from each other, but the main idea is the same.

Definition 2. *A set of links A is said to be a non-interfering set if it does not contain any interfering links, i.e., for each pair of links $u, v \in A$ with $u \neq v$, we have $C_{uv} = 0$.*

Definition 3. *The interference degree $d_I(e)$ of link e is the maximum number of links belonging to its interference set which do not interfere with each other, i.e.,*

$$d_I(e) = \max_{A \subseteq I(e): A \text{ is a non-interfering set}} |A|.$$

Definition 4. *The interference degree $d_I(G)$ of graph $G = (V, E)$ is the maximum interference degree across its constituent links, i.e., $d_I(G) = \max_{e \in E} d_I(G)$.*

We are now ready to upper bound the capacity region.

Theorem 1. *The capacity region Ω specified by (3) consists of arrival rate vectors $\vec{\lambda}$ that satisfy $\sum_{l \in I(e)} \sum_{k=1}^{K} \frac{H_k^l \lambda_k}{\mu_k c_l} \leq d_I(e)$ for all $e \in E$.*

Proof: Let $\mathcal{S}$ be a link schedule that activates the same set of non-interfering links $A_{\mathcal{S}}$ during every slot. Consider a link $e \in E$. Since the set of links $A_{\mathcal{S}}$ is non-interfering, it contains at most $d_I(e)$ links from $I(e)$. Thus, the link rate vectors $[x_l]_{l \in E}$ under $A_{\mathcal{S}}$ must satisfy

$$\sum_{l \in I(e)} \frac{x_l}{c_l} \leq d_I(e) \text{ for all } e \in E. \tag{4}$$

Since this result holds for all link schedules that satisfy the interference constraints, it follows that all *feasible* link rate vectors under the contention matrix based interference model must satisfy the constraints given in (4). The result now follows by noting that a packet arrival rate vector $\vec{\lambda}$ incduces an average load of $\sum_{k=1}^{K} H_k^l \lambda_k / \mu_k$ on link l.

The following result is a direct consequence of Theorem 1 and the fact that $d_I(G) = \max_{e \in E} d_I(e)$.

Corollary 1. *The capacity region Ω specified by (3) consists of arrival rate vectors $\vec{\lambda}$ that satisfy $\sum_{l \in I(e)} \sum_{k=1}^{K} \frac{H_k^l \lambda_k}{\mu_k c_l} \leq d_I(G)$ for all $e \in E$.*

IV. JOINT CONGESTION CONTROL AND SCHEDULING SCHEME

We now propose a joint congestion control and scheduling scheme that is guaranteed to achieve a fraction $d_I(G)$ of the capacity region. The scheme uses *congestion prices* $q_l(t)$, $l \in E$ to maintain an estimate of the congestion level in the network at time t. The congestion control and scheduling are performed using these congestion prices. T ime is divided into slots of unit duration, and both congestion prices and user rates are updated at the beginning of each slot. The detailed description of the scheme is as follows:

- *Congestion price update*: The congestion prices are updated as follows:

$$q_l(t+1) = \left(q_l(t) + \alpha \Delta q_l(t)\right)^+, \tag{5}$$

  where

$$\Delta q_l(t) = \sum_{j \in I(l)} \left[\sum_{k=1}^{K} H_k^j \int_t^{t+1} \frac{n_k(t) x_k(t)}{c_j} - 1_{j \in \mathcal{S}(t)} \right],$$

  and $\mathcal{S}(t)$ denotes the set of links that are scheduled to transmit during the time slot beginning at time t.
- *User rate update:* The data rates of type k users are updated as follows:

$$x_k(t+1) = \min \left\{ \frac{1}{\sum_{l \in E} q_l(t+1) \sum_{j \in I(l)} \frac{H_k^j}{c_j}}, M_k \right\},$$

  where M_k is the maximum data rate of type k users.
- *Transmission scheduling:* The link transmissions are scheduled in accordance with the *maximal scheduling scheme*, i.e., the subset of edges M chosen for transmission during any slot satisfies that for each edge $e \in E$, either $I(e) \cap M \neq \Phi$ or $q_e \leq 1$.

We note that the above scheme is similar in spirit to a scheme proposed in [10] under the node exclusive interference model. However, our scheme is more general and works for all contention matrix based interference models, including the node exclusive interference model. Some salient features of our scheme are worth noting: (i) the congestion price of a link depends not only on its own backlog, but also that of the links belonging to its interference set; (ii) the data rate of type k users depends on the congestion prices of all those links that either belong to the route of type k users or interfere with such a link. It is this proper choice of user rate and congestion price update that allows our scheme to achieve a fraction $d_I(G)$ of the capacity region.

Theorem 2. *If the stepsize α is small enough, the above joint congestion control and scheduling scheme stabilizes the network for all arrival rate vectors that belong to $\Omega^o / d_I(G)$, where Ω^o denotes the interior of set Ω.*

Proof: The main idea of the proof is to construct an appropriate Lyapunov function with a negative drift. We shall use the Lyapunov function $V(\vec{n}, \vec{q}) = V_n(\vec{n}) + V_q(\vec{q})$, where $V_n(\vec{n}) = \sum_{k=1}^{K} \frac{\beta n_k^2}{2\lambda_k}$ and $V_q(\vec{q}) = \sum_{l \in E} \frac{q_l^2}{2\alpha}$.

Let

$$\Delta V_q = E\left[V_q(\vec{q}(t+1)) - V_q(\vec{q}(t)) | \vec{n}(t), \vec{q}(t)\right]$$

and

$$\Delta V_n = E\left[V_n(\vec{n}(t+1)) - V_n(\vec{n}(t)) | \vec{n}(t), \vec{q}(t)\right].$$

Since all scheduled links l must have a congestion price $q_l \geq 1$, the projection operator in (5) is not required provided $\alpha \leq 1$. Using some algebraic manipulations, it can be shown that

$$\begin{aligned} \Delta V_q \leq C_1 \alpha \sum_{k=1}^{K} \int_t^{t+1} E\left[n_k^2(t) x_k^2(t) | \vec{n}(t), \vec{q}(t)\right] dt \\ + C_2 - \sum_{l \in E} q_l(t) 1_{\{q_l(t)>1\}} + \\ \sum_{l \in E} q_l(t) \sum_{j \in I(l)} \sum_{k=1}^{K} \frac{H_k^j}{c_j} \int_t^{t+1} E\left[n_k(t) x_k(t) | \vec{n}(t), \vec{q}(t)\right] dt. \end{aligned} \tag{6}$$

Following the line of analysis in [10] used to prove Theorem 7, it can be shown that

$$\begin{aligned} \Delta V_n \leq \sum_{l \in E} q_l(t) \sum_{j \in I(l)} \sum_{k=1}^{K} \frac{\beta \lambda_k H_k^j}{\mu_k c_j} \\ - \sum_{l \in E} q_l(t) \sum_{j \in I(l)} \sum_{k=1}^{K} \frac{H_k^j}{c_j} \int_t^{t+1} E\left[n_k(t) x_k(t) | \vec{n}(t), \vec{q}(t)\right] dt \\ - (\beta - 1) \sum_{k=1}^{K} \int_t^{t+1} E\left[n_k(t) | \vec{n}(t), \vec{q}(t)\right] dt + C_3 \\ - \sum_{k=1}^{K} \frac{\mu_k}{4 \lambda_k M_k} \int_t^{t+1} E\left[n_k^2(t) x_k^2(t) | \vec{n}(t), \vec{q}(t)\right] dt \end{aligned} \tag{7}$$

Observe that

$$\sum_{l \in E} q_l(t) 1_{\{q_l(t)>1\}} \geq \sum_{l \in E} q_l(t) - |E|. \tag{8}$$

Also, for all $\lambda \in \Omega^o / d_I(G)$, we have

$$\sum_{j \in I(l)} \sum_{k=1}^{K} \frac{\beta \lambda_k H_k^j}{\mu_k c_j} < 1 \text{ for all } l \in E. \tag{9}$$

Using (6)-(9), it follows that given any $\epsilon > 0$ we can choose $\beta > 1$ and $\alpha > 0$ such that

$$\Delta V_n + \Delta V_q \leq C_4 - \epsilon \left(\sum_{l \in E} q_l(t) + \int_t^{t+1} E\left[n_k(t) | \vec{n}(t), \vec{q}(t)\right] dt \right).$$

The result now follows using Theorem 2 in [16] and observing that $Q_l(t) = q_l(t)/\alpha$ for $l \in E$.

Theorem 3 provides a lower bound on the performance of our joint congestion control and scheduling scheme in general wireless networks. We now consider its performance in case of wireless networks whose connectivity graph is a geometric graph. Note that if same power level is used for all transmissions and the noise variance is same at all the nodes in the network, then the connectivity graph is indeed a geometric graph. Such conditions are often assumed in the literature. Further, we shall restrict our attention to K-hop interference models. We have the following result:

Theorem 3. *If the underlying connectivity graph of the network is a geometric graph and the stepsize α is small enough, the above joint congestion control and scheduling scheme stabilizes the network for all arrival rate vectors that belong to $\Omega^o/49$ under all K-hop interference models.*

Similar performance bounds can be shown to hold for disk graphs and (r, s)-civilized graphs; the details are omitted for want of space.

V. RANDOMIZED DISTRIBUTED MAXIMAL SCHEDULING UNDER 2-HOP INTERFERENCE MODEL

Although the performance of maximal scheduling has been analyzed under various interference models [10], [4], [18], an explicit distributed implementation has not been provided in the literature. Moreover, the overhead in implementing such an algorithm has been ignored in most of previous works. We now propose a randomized distributed algorithm for implementing maximal scheduling under the 2-hop interference model (see [18]). We next describe our distributed computing model.

As in [6] and other related works, we assume a *synchronous message passing distributed computing model*, which is a variation of the standard models used in the distributed computing literature. The main difference is the broadcast feature in the model which is typical of wireless networks. More precisely, the distributed computing architecture is modeled as a graph with undirected edges (we assume bidirectional links between the nodes as required by the 2-hop interference model). Each node has a unique ID. The clocks at all the nodes are synchronized and the communication takes place in rounds, each occupying a slot. Each slot is further divided into six subslots. A node can transmit a control packet of length $O(1)$ during each subslot. A packet transmission from node u is heard by all nodes w in its neighborhood, henceforth denoted by $N(u)$, unless the node w itself transmits or some node in $N(w)$ other than u transmits.

We now define some terminology to be used in the sequel. A directed link from u to v will be denoted by

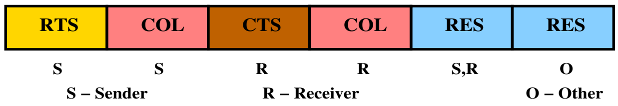

Fig. 1. The sequence of control packets exchanged during a slot.

(u, v); and a bidirectional link will be denoted by uv or vu. The congestion price of link (u, v) will be denoted by $q_{u,v}$. The set of nodes $v \in N(u)$ such that $q_{u,v} > 1$ will be denoted by $N^Q(v)$. We are now ready to describe our algorithm.

Distributed Maximal Scheduling($G = (V, E), q : E \to \mathbb{R}^+, M$)

1) $M := \phi$ and $b(u) = -1, u \in V$.
2) **while** $b(u) \neq -1 \forall u$ **do**
3) Each node u with $b(u) = 0$ decides to transmit with probability $\frac{1}{|N^Q(u)|+1}$. Upon deciding to transmit, it chooses a node at random from $N^Q(u)$ and sends a RTS message to that node during the first subslot.
4) If a sender node detects any other transmission then it sends a COL packet during the second subslot.
5) If a receiver node successfully receives the RTS packet and does not hear a COL packet (or a collision due to multiple such packets), then it sends a CTS message during the third subslot.
6) If a receiver node detects any other transmission then it sends a COL packet during the fourth subslot.
7) If no COL packet is sent during the fourth subslot, the sender u and receiver v both send a MC packet during the fifth subslot and set $b(u) = b(v) = 1$. $M := M \cup (u, v)$.
8) All nodes w hearing a MC packet set $b(w) = 0$ and further transmit a copy of it during the sixth subslot.
9) A node x with $w \in N^Q(x)$ where $b(w) = 0$ deletes node w from $N^Q(x)$.

The algorithm requires that each node should know the network topology within its two hop neighborhood. The main feature of our algorithm is the careful exchange of control packets that ensures that no two links within two hops of each other decide to transmit at the same time (see Figure 1). A node that wishes to transmit, first sends a *ready to send* (RTS) packet. If it detects an ongoing transmission while sending the RTS packet, it sends a *collision* (COL) packet during the next subslot. Successful reception of an RTS packet by the receiver guarantees that no other transmitter can be within one hop of the receiver. Further, no collision packet being sent guarantees that no two nodes within one hop can decide to transmit at the same time.

If the receiver does not hear a COL packet or a collision due to multiple such packets, it sends a *clear to send* (CTS) packet during the third subslot. If it detects an ongoing transmission while sending the CTS packet, it sends a COL packet during the fourth subslot. Thus no collision packet being sent guarantees that no two nodes within one hop can decide to receive at the same time. If no collision packet is transmitted, both sender and receiver transmit a *reservation* (RES) packet during the next subslot. All nodes which hear the RES packet, further transmit a copy of it during the next subslot. The RES packet contains the IDs of sender and receiver. Each two hop neighbor v of the sender and the receiver uses its knowledge of its two hop topology to determine which of its one hop neighbors are also one hop neighbors of the senser or the receiver, and removes them from its neighborhood $N^Q(v)$.

It is clear from the above discussion that when the algorithm terminates it returns a maximal matching satisfying the 2-hop interference constraints. We can show the following bound on the running time of the above algorithm in case of geometric graphs; the details of the proof will appear in a companion paper.

Theorem 4. *The Distributed Maximal Scheduling terminates in $O(\log^2 |V|)$-rounds with high probability in case of geometric graphs.*

The same performance bound holds for disk graphs as well as (r, s)-civilized graphs. We further believe that a similar performance bound may hold in case of general graphs.

VI. CONCLUDING REMARKS

We considered the problem of throughput-optimal cross-layer design of wireless networks. We provided an upper bound on the capacity region of wireless networks under all contention matrix based interference models. We then proposed a joint congestion control and scheduling algorithm and showed a lower bound of $d_I(G)$ on its performance. The lower bound $d_I(G)$ depends on certain structural properties of the connectivity graph of the wireless network. We have previously shown that $d_I(G)$

can be upper bounded by 49 in case of all geometric graphs and under all K-hop interference models. Thus, our joint congestion control and scheduling algorithm performs within a constant factor of the optimal in case of geometric graphs under all K-hop interference models.

We also proposed a distributed randomized algorithm for maximal scheduling under the 2-hop interference model. We showed that the algorithm runs in $O(\log^2 |V|)$ time in case of geometric graphs. We beleive that a similar performance bound may hold in case of general graphs. We plan to investigate this issue further in our future work. It is for the first time in the literature that an explicit distributed algorithm for implementing the maximal scheduling has been proposed. The overhead of implementing the maximal scheduling has so far been ignored in the literature. However, the overhead might be significant in case of large networks and must therefore be properly accounted for in the analysis.

An interesting direction for future research would be to develop distributed algorithms for maximal scheduling under some other interference models. Also, it would be interesting to determine the minimal overhead required for implementing maximal scheduling under various network settings and interference models. Further, we would like to investigate if one can develop cross-layer algorithms that offer better performance than the algorithm proposed in this paper.

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