Anonymous-Query based Rate Control for Wireless Multicast: Approaching Optimality with Constant Feedback

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ABSTRACT

For a multicast group of n receivers, existing techniques either achieve high throughput at the cost of prohibitively large (e.g., $O(n)$) feedback overhead, or achieve low feedback overhead but without either optimal or near-optimal throughput guarantees. Simultaneously achieving good throughput guarantees and low feedback overhead has been an open problem and could be the key reason why wireless multicast has not been successfully deployed in practice. In this paper, we develop a novel anonymous-query based rate control, which approaches the optimal throughput with a constant feedback overhead independent of the number of receivers. In addition to our theoretical results, through implementation on a software-defined ratio platform, we show that the anonymous-query based algorithm achieves low-overhead and robustness in practice.

CCS Concepts

•**Networks** → *Network protocol design; Network control algorithms;*

Keywords

Wireless multicast, near-optimal throughput, low feedback overhead

1. INTRODUCTION

Mobile video is expected to contribute 70% of all the mobile traffic by the end of 2018 [1]. Wireless multicast, which leverages the shared nature of the wireless medium, is an efficient way to distribute the popular video streams to many clients.

In a wireless communication system, the channel condition fluctuates over time due to factors such as multipath, shadowing, mobility, etc. Adapting the physical layer transmission rate according to the channel conditions of the receiver(s), referred to as rate control, is an essential component in the protocol stack in terms of achieving a high throughput. The rate control problem has been extensively studied for wireless unicast, e.g., [2][3][4][5][6]. The rate control in a wireless multicast scenario faces additional challenges: the channel quality of each receiver may correspond to a different stochastic process. As a result, adapting the rate to one particular receiver as in the unicast case may substantially degrade the throughput of the other receivers. Therefore, an appropriate rate control for multicast may need to carefully balance all the receivers by jointly considering the channel state information (CSI) or the channel distribution information (CDI) of all the receivers.

In practice, both the CSI and CDI are only locally available at each receiver. Conventionally, to select the appropriate transmission rate, the transmitter needs to collect the CSI or CDI from all the receivers. For a multicast group of n receivers, existing techniques either achieve high throughput at the cost of prohibitively large ($O(n)$) feedback overhead [7][8], or provide schemes [9][10][11] with low overhead, but without good throughput guarantees.

Thus, simultaneously achieving good throughput guarantees and low feedback overhead has been a long-standing open problem. In this work, we make significant progress towards developing a scheme that achieves high throughput with constant feedback overhead.

The contributions of this paper are summarized as follows.

- We begin with a static multicast rate control problem. We develop a novel Anonymous-Query mechanism that approaches the ϵ-neighborhood of the optimal static rate with a communication overhead of $O(\log 1/\epsilon)$, which is not only independent of the number of receivers n, but also order optimal in terms of ϵ. Our method works as if the receivers were to collaboratively inform the transmitter the optimal rate bit by bit, but without any prior knowledge of CSI or CDI at the transmitter.

- We then compare the achievable throughput of the optimal static rate control with the genie-aided dynamic rate control solution, where the CSI of all receivers are known non-causally at the transmitter and the rate at each time-slot is selected to maximize the

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long-term average system throughput. We show that under some mild conditions, the throughput gain of the optimal genie-aided control over the optimal static rate control method becomes negligible as the number of receivers increases. In particular, when the number of rates the transmitter can choose from is finite, the throughput gain of the optimal genie-aided control decreases exponentially with the number of receivers. This suggests that, compared with the optimal static rate control method, the benefits of exploiting channel realization information from receivers is negligible when the number of receivers is moderately large. Therefore, in a network with a large number of receivers, the static rate control method we propose is able to achieve a throughput close to the throughput of the optimal genie-aided rate control, while incurring a small constant communication overhead independent of the number of receivers. To the best of our knowledge, *our rate control method is the first which approaches the optimal throughput in multicast with a constant communication overhead independent of the number of receivers.*

- Our theoretical results are validated by numerical simulations. Furthermore, via a software defined radio implementation we show the low-overhead and robustness features of the anonymous-query based algorithm.

The rest of this paper is organized as follows. In Section 2, we introduce some related works. In Section 3, we describe the system model and formulate the static rate control problem. In Section 4, we propose an anonymous-query based algorithm to solve the static rate control problem. In Section 5, we compare the achievable throughput of the optimal static rate control with the genie-aided dynamic rate control solution. In Section 6, we evaluate the performance of our method through both numerical simulations and system-level implementations. Finally, in Section 7, we conclude the paper.

2. RELATED WORK

In wireless unicast, the rate control problem has been extensively studied, e.g., [2][3][4][5][6]. Most existing techniques, such as [2][3], adjust the transmission rate according to either the CSI or CDI from the receiver. Recently, Strider code [4] and Spinal code [5] have been proposed, in which no CSI or CDI from the receiver is required before transmission and the transmission rate is determined by the acknowledgement information from the receiver. However, there are two issues with this approach: (i) As shown in a recent work [6], even for a single receiver, the feedback overhead to determine the rate without CSI or CDI still occupies $5-10\%$ of the total time, and thus is non-negligible. (ii) It is not clear whether these new advancements could be successfully applied to a wireless multicast with a large number of receivers.

In wireless multicast, the channel qualities of receivers often correspond to different stochastic processes. One intuitive way of choosing the transmission rate, e.g., [10], is to guarantee the successful reception of every packet at all the receivers. However, as the number of receivers increases, it becomes more and more likely that there exists a receiver(s) whose instantaneous channel happens to be poor. As a result, the achievable throughput of the intuitive method may degrade substantially as the number of receivers increases. A scalable rate control for multicast should allow receivers to occasionally fail to receive packets when their channel conditions are poor. In light of this, there has been a rich body of literatures that study different ways to reliably disseminate information under the broadcast erasure channel model. For example, rateless codes such as LT codes [12], Raptor codes [13] and more recent network coding schemes [14][15][16] allow one to achieve the capacity of broadcast erasure channel. The required feedback overhead in [15][16] can be quite small even when there are a large number of receivers. Note that the transmission rate, e.g., modulation or coding rate in the physical layer, is given a priori in these works. In this work, we focus on devising an efficient multicast rate adaptation in the physical layer, which, by combining with the techniques described above, can achieve a high throughput.

Existing studies about rate control in wireless multicast are either CDI-based or CSI-based. The static multicast rate control problem was formulated and studied in [7]. It is assumed that the transmitter has perfect knowledge about the CDI of the receivers. In [8], the authors optimized transmission rate for multicasting video streams according to CSI of the receivers. Note that the communication overhead for the transmitter to collect CDI or CSI from all the receivers increases linear with respect to the number of receivers n, which could be prohibitive in a large network. To avoid the scalability problem, the method in [9] partitions the receivers into clusters and only selects a subset of the receivers to report CSI or CDI to the transmitter. Yet, it requires precise location information of all the receivers, making it difficult to apply to a network with a large number of mobile clients. In [10], the duration of a busy tone signal is used to indicate the maximum of the instantaneous supportable rate of all receivers. Although the communication overhead of the busy tone can be small even when there are a large number of receivers, the achievable throughput of this approach degrades with the number of receivers, as explained earlier for the intuitive rate control approach. In [11], a rate adaptation method is proposed which could empirically achieve high throughput and low feedback overhead, but it is not clear whether the method is close to the optimal solution. To the best of our knowledge, this is the first work that approaches the optimal throughput (for a large wireless multicast group) with a constant communication overhead independent of the number of receivers.

3. SYSTEM MODEL AND PROBLEM FORMULATION

3.1 System Model

We consider a multicast setting, where we have one transmitter that needs to send a stream of common information to n receivers.

Typical wireless transceivers have multi-rate capabilities. For example, in 802.11ac, the transmitter has the flexibility of choosing between 10 transmission rates from 32.5 Mbit/s to 433.3 Mbit/s by selecting different combinations of modulation and channel coding configurations. Let $\mathcal{R}$ denote the set of transmission rates available at the transmitter. To capture the physical layer capability, we let $r_{\min} \triangleq \min\{\mathcal{R}\}$ and $r_{\max} \triangleq \max\{\mathcal{R}\}$.

We consider a slotted system, where the channel between the transmitter and each of the receivers is assumed to be block fading, where the channel states remain the same within each slot, and vary from one slot to another. Note that a transmission rate would correspond to an associated SNR threshold. Hence, in a time slot t given the channel gain from the transmitter to receiver i, we can find a maximum transmission rate under which the packets sent by the transmitter in slot t can be successfully received by receiver i. We denote such a rate by $R_i[t]$. Across different time slots, $R_i[t]$ would vary with the fluctuation of the channel quality. Since, for each receiver i, there is a one-to-one correspondence between the channel gain and the maximum achievable transmission rate [17], the channel process of a receiver i can be fully captured by the dynamics of $\{R_i[t]\}_{t\in\mathbb{N}}$. In this paper, we assume that $\{R_i[t]\}_{t\in\mathbb{N}}$ is a stationary stochastic process, and we use the random variable R_i to capture its distribution.

We assume that the transmitter has no knowledge of either the distribution or the realization of $\{R_i[t]\}_{1\le i\le n, t\in\mathbb{N}}$, while each receiver i has perfect knowledge of its own channel, i.e., the distribution of $\{R_i[t]\}_{t\in\mathbb{N}}$ as well as its instantaneous realization. This is a practical assumption, since the CSI or CDI can be easily acquired at each receiver by measuring the probing signals sent by the transmitter.

The objective of the transmitter is to maximize the long-term throughput of the common information flow to all the receivers by controlling its transmission rate.

3.2 Static Multicast Rate Control

From the definition of $R_i[t]$, we know that given the transmission rate $r\in\mathcal{R}$, the packet sent in slot t can be successfully received by receiver i if and only if $R_i[t]\ge r$. Thus, if the transmitter transmits at a constant rate $r\in\mathcal{R}$ and the system has only a single receiver i, the long term average throughput of the system can be expressed as a product of the transmission rate r and probability that receiver i can successfully receive a packet at rate r, which we denote as

$$T_i(r) = r\mathbb{P}\left(R_i \ge r\right). \tag{1}$$

We also call $T_i(r)$ the *throughput function* of receiver i.

Note that $\mathbb{P}\left(R_i \ge r\right)$ is a decreasing function of r. From Equation (1), neither a very high rate r nor a very low rate r is desirable to achieve a large throughput, for a high rate r, $\mathbb{P}\left(R_i \ge r\right)$ could be close to zero leading to a low throughput; a low rate r may correspond to the case of $\mathbb{P}\left(R_i \ge r\right)$ being close to one, but now he throughput is limited by r which is low. In this paper, we make the assumption that $T_i(r)$ is a continuous unimodal[1] function in r. Note that this is a mild assumption. For instance, the assumption is true when $R_i[t]$ is calculated based on the Shannon equation and the SNR distribution conforms to either Rayleigh or Rician fading. Two examples of $T_i(r)$ are shown in Figure 1.

When the transmitter sends common information to more than one receiver using a constant rate r, the long term throughput of the system is limited by the weakest receiver under rate r. More precisely, the long term average system throughput under the constant transmission rate r, which we denote as $T(r)$, can be expressed as

$$T(r) = \min_{1\le i\le n} T_i(r). \tag{2}$$

[1]There exists a unique maximum.

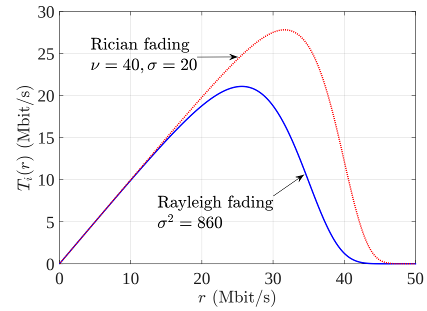


Figure 1: Two examples of $T_i(r)$ with Rayleigh and Rician faded channel models and $R_i[t] = W\log(1+SNR)$ with $W = 5MHz$.

Since the minimization operation preserves unimodality, $T(r)$ is also a unimodal function.

Note that the throughput in Equation (2) is achievable using many different schemes. For example, rateless codes such as LT codes [12], Raptor codes [13] and more recent network coding schemes [14][16] can be applied in the application layer to achieve the capacity of broadcast erasure channel. In this paper, we limit our attention on the rate adaptation in the physical layer.

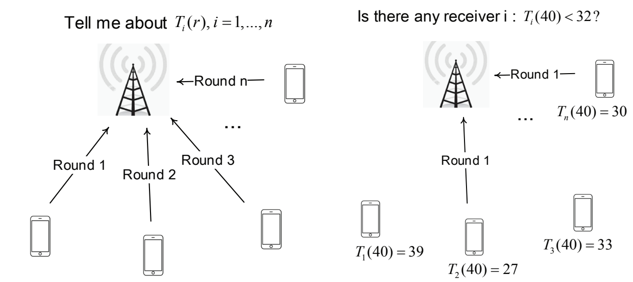
The static multicast rate control problem is to find the transmission rate r that maximizes the throughput of the network, which is formulated in Equation (3).

$$\max_{r\in\mathcal{R}} T(r) = \max_{r\in\mathcal{R}} \min_{1\le i\le n} T_i(r). \tag{3}$$

It can be observed from Equation (3) that the static rate control problem is a max-min optimization problem. Since $T(r)$ is a unimodal function, there is a unique solution $r^*\in\mathcal{R}$ to Equation (3). Solving this problem is straightforward if the complete information about the throughput functions $\{T_i(r)\}_{1\le i\le n}$ is given (e.g., using search techniques such as Golden Section Search or Fibonacci Search), yet the challenge lies in the fact that $T_i(r)$ is known only locally at receiver i. For the transmitter to find the optimal rate r^*, conventional approaches require the transmitter to collect information about $T_i(r)$ from every receiver $1\le i\le n$. In other words, receivers have to feed back information using orthogonal channel resources (either in time or frequency) as shown in Figure 2(a). Consequently, as the number of receivers n increases, the communication overhead of conventional approaches increases proportionally with n. For example, one naive method to approach an ϵ-neighborhood of the optimal rate, i.e., $[r^*-\epsilon/2, r^*+\epsilon/2]$, is to let each receiver i feed back a number of $(r_{\max}-r_{\min})/\epsilon$ samples of $T_i(r)$. The number of bits that needs to be exchanged between the transmitter and all the receivers is in the order[2] of at least $\Theta(n/\epsilon)$, which is prohibitive when there are a large number of receivers in the network or when we desire the solution to be close to the optimal. This is the key motivation of the paper: to find a method to approach r^* with much less communication overhead.

In Section 5, we compare the performance of the static rate control solution with the genie-aided dynamic rate con-

[2]We say $x_n = \Theta(y_n)$ if there exist $z_1, z_2 > 0$ such that $z_1|y_n| \le |x_n| \le z_2|y_n|$ for two real-valued sequences $\{x_n\}$ and $\{y_n\}$.



(a) Traditional information gathering. (b) An example of one round of Anonymous Query

Figure 2: A comparison of traditional information gathering and Anonymous-Query based information gathering.

trol solution, where the channel realizations $\{R_i[t]\}_{1\le i\le n, t\in\mathbb{N}}$ are known non-causally at the transmitter and the rate of transmission at each time-slot is selected to maximize the long-term average system throughput. Surprisingly, we show that under some mild conditions, the throughput gain of the optimal genie-aided control over the static rate control method vanishes as the number of receivers n increases.

4. ANONYMOUS-QUERY BASED MULTICAST RATE CONTROL

In this section, we introduce a novel Anonymous Queries mechanism, based on which an algorithm is developed to solve the static multicast rate control problem. The salient feature of the proposed algorithm is that *the transmitter can approach an ϵ-neighborhood of the optimal solution with a communication overhead $O(\log 1/\epsilon)$, which is not only independent of the number of receivers n, but also order optimal in terms of ϵ.*

4.1 Anonymous Queries-based Signalling

We propose a novel Anonymous Queries mechanism as illustrated in Figure 2(b). In Anonymous Queries, the transmitter and the receivers communicate interactively in rounds. One round of anonymous query is defined as follows.

Definition 1. *(Anonymous Query) One round of anonymous query consists of the following two consecutive stages.*

Stage 1: The transmitter broadcasts a question to a group of n receivers using the lowest data rate[3], where the question follows the following format: "Is there any receiver that satisfies a property $\mathbf{P}$?". Whether or not the property $\mathbf{P}$ is satisfied depends only on information locally available at each receiver.

Stage 2: In response to the question broadcasted by the transmitter, a receiver will send out a beacon signal if the receiver satisfies the property $\mathbf{P}$ specified in the question. If a receiver does not satisfy the property $\mathbf{P}$, it keeps in the receiving mode preparing to receive the next query from the transmitter. At the same time, the transmitter enters the receiving mode and detects the existence of beacon signal from the receivers.

[3]The lowest data rate is used to ensure that all the receivers could receive the question reliably.

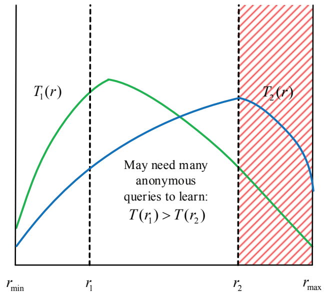
For each round of anonymous query, the transmitter could learn 1 bit information about whether there is any receiver $1 \le i \le n$ satisfying the property $\mathbf{P}$. This learning process is anonymous in the sense that the transmitter cannot learn the actually id of the receiver(s) satisfying the property. The result of the previously sent queries will determine whether more queries are necessary and what query will be sent next.

Since the transmitter can easily detect the existence of beacon signal(s) by energy detection, the receivers satisfying property $\mathbf{P}$ could send the beacon signals in the same channel and do not need to be precisely synchronized or coordinated. This leads to a salient feature of the anonymous queries: *the communication overhead of each round of query is constant for any number of receivers n.* The key challenge is to design an Anonymous-Query based algorithm to learn the optimal rate r^* of Problem (3) using as few queries as possible.

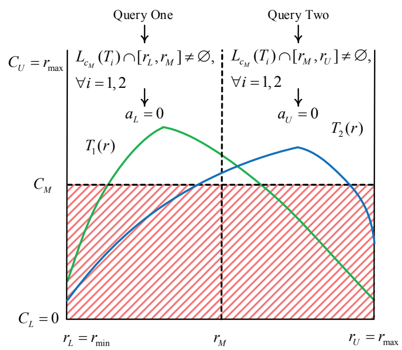
4.2 Anonymous-Query based Rate Control

In this subsection, we present the rate control algorithm based on anonymous queries. For ease of presentation, we first consider the case when the transmitter can choose any rate in the set $\mathcal{R} = [r_{\min}, r_{\max}]$. Later we shall do a minor adjustment to make our algorithm applicable to the case when $\mathcal{R}$ is any finite set.

First, we show that the straightforward incorporations of Anonymous Queries with existing search techniques such as Golden Section Search and Fibonacci Search [18] are not efficient. Given the evaluations of $T(r)$, Golden Section Search or Fibonacci Search can find the maximum of a one dimensional unimodal function $T(r)$. Since $r_{\min} \le r \le r_{\max}$ is bounded, the convergence speed of these methods is known to be order optimal [19], if $T(r)$ is known a priori. We show a toy example in Figure 3(a), in which there are two receivers. The idea of Golden Section Search or Fibonacci Search is that in one iteration, two rates $r_{\min} < r_1 < r_2 < r_{\max}$ are picked. Suppose we know that $T(r_1) > T(r_2)$, then based on the fact $T(r)$ is unimodal, the search region for r^* can be narrowed down to $[r_{\min}, r_2]$. In our problem, however, for a specific r, the evaluation of $T(r) = \min_{1\le i\le n} T_i(r)$ is not readily known by the transmitter, and the value of $T(r)$ can only be approximated by a bisection search using rounds of Anonymous Queries. For the example shown in Figure 2(b), the transmitter queries about "Is there any receiver $1 \le i \le n$ with $T_i(40) < 32$?". Since $T_2(40) = 27, T_n(40) = 30$, the transmitter detects the beacon signal in this round. In the next round, the transmitter may query "Is there any receiver $1 \le i \le n$ with $T_i(40) < 16$?". In this way, the search region for $T(40)$ can be narrowed down. Unfortunately, to learn which one among $T(r_1)$ and $T(r_2)$ is larger, a good number of Anonymous queries may be necessary, especially when $T(r_1)$ and $T(r_2)$ are close. In the worst case, to approach an ϵ-neighborhood of the optimal solution, the number of required queries is $\Theta(\log 1/\epsilon)$ in each iteration of the search algorithm. Since the search algorithm terminates after $\Theta(\log 1/\epsilon)$ iterations, the worst communication overhead of the straightforward incorporation of Anonymous Queries can be $\Theta((\log 1/\epsilon)^2)$ to learn an ϵ-neighborhood of the optimal solution. Note that even though the communication overhead is independent of the number of receivers n com-



(a) Incorporation of Anonymous Queries with existing techniques.



(b) Our Anonymous Queries method.

Figure 3: Comparison of different Anonymous-Query based algorithms in a toy network with two receivers.

pared with a complexity of $O(n)$ using conventional methods, the overhead can still be quite high when ϵ is small.

Next, we propose our Anonymous Query method which will learn the optimal rate much more efficiently.

To begin with, we define the concept of a superlevel set

Definition 2. *(Superlevel set) A set of the form*

$$L_c(T_i) = \{r \in [r_{min}, r_{max}] | T_i(r) \geq c\} \tag{4}$$

is called a superlevel set of function $T_i(r)$ at level c.

The idea to solve Problem (3) using anonymous queries is to search for r^* and $T(r^*)$ at the same time. A lower bound r_L and an upper bound r_U for r^* are maintained. Similarly, a lower bound c_L and an upper bound c_U for $T(r^*)$ are maintained. The anonymous queries are designed such that from the results of queries in each iteration, either the search region $[r_L, r_U]$ for r^* or the search region $[c_L, c_U]$ for $T(r^*)$ can be narrowed down by half. The details of the anonymous-query based rate control scheme are given in Algorithm 1. In lines 2-3, $[r_L, r_U]$ and $[c_L, c_U]$ are initialized. In each iteration of Algorithm 1, r_M and c_M are set to be the middle points of $[r_L, r_U]$ and $[c_L, c_U]$ respectively, as shown in line 7. Then, the transmitter sends two anonymous queries to the receivers on the superlevel sets with the level c_M for $[r_L, r_M]$ and $[r_M, r_U]$ respectively (lines 9-10 and lines 18-19). In response to the anonymous query, a receiver i sends a beacon signal if it satisfies the queried property (lines 11-14, lines 20-23). By detecting the existence of beacon signals for the two queries, the transmitter learns two bits information a_L, and

Algorithm 1: Anonymous-Query based Rate Control

1 - - - - - - - Initializations - - - - - - - - - - - - - -
2 $r_L := r_{\min}$; $r_U := r_{\max}$;
3 $c_L := 0$; $c_U := r_{\max}$;
4 $r_F := r_{\min}$;
5 - - - - - - Anonymous queries - - - - - - - - - - -
6 **while** $r_U - r_L > \epsilon$ *and* $c_U - c_L > \epsilon$ **do**
7 $r_M := \frac{1}{2}(r_L + r_U)$; $c_M := \frac{1}{2}(c_L + c_U)$
8 - - - - - - Anonymous query for $[r_L, r_M]$ - - - - - - - -
9 Tx broadcasts an anonymous query:
10 $\exists$ Rx i such that $L_{c_M}(T_i) \cap [r_L, r_M] = \emptyset$?
11 **foreach** *Rx* $i \in \{1, ..., n\}$ **do**
12 **if** $T_i(r) < c_M, \forall r \in [r_L, r_M]$ **then**
13 send out a beacon signal;
14 **end**
15 Tx detects the existence of beacon signal;
16 **if** *beacon signal is detected* **then** $a_L := 1$; **else** $a_L := 0$;
17 - - - - - - Anonymous query for $[r_M, r_U]$ - - - - - - - -
18 Tx broadcasts an anonymous query:
19 $\exists$ Rx i such that $L_{c_M}(T_i) \cap [r_M, r_U] = \emptyset$?
20 **foreach** *Rx* $i \in \{1, ..., n\}$ **do**
21 **if** $T_i(r) < c_M, \forall r \in [r_M, r_U]$ **then**
22 send out a beacon signal;
23 **end**
24 Tx detects the existence of beacon signal;
25 **if** *beacon signal is detected* **then** $a_U := 1$; **else** $a_U := 0$;
26 - - - - - - Tx updates for the new queries - - - - - - - -
27 **switch** a_L, a_U **do**
28 **case** $a_L = 1$ *and* $a_U = 1$ $c_U := c_M$;
29 **case** $a_L = 0$ *and* $a_U = 0$ $c_L := c_M$; $r_F := r_M$;
30 **case** $a_L = 1$ *and* $a_U = 0$ $r_L := r_M$;
31 **case** $a_L = 0$ *and* $a_U = 1$ $r_U := r_M$;
32 **end**
33 **end**
34 **if** $r_U - r_L \leq \epsilon$ **then** $\hat{r} := r_L$; **else** $\hat{r} := r_F$;
35 - - - - - Starting data transmission - - - - - - - - - -
36 Transmit packets at rate $\hat{r}$.

a_U, which altogether determine the updates of the search regions for $[r_L, r_U]$ and $[c_L, c_U]$, as shown in lines 27-32. r_F is a rate which always guarantees that $T(r_F) \geq c_L$. Anonymous queries are sent until either $r_U - r_L \leq \epsilon$ or $c_U - c_L \leq \epsilon$. In the end (lines 34-36), depending on whether $r_U - r_L \leq \epsilon$, the transmitter sets up its transmission rate $\hat{r}$ among r_L and r_F. For the toy network shown in Figure 3(b) that consists of two receivers, after two rounds of Anonymous Queries, $a_L = 0$ and $a_U = 0$. As a result, the search region for $T(r^*)$ can be cut by half (For the mathematical reasoning, please refer to Theorem 1 and its proof in Appendix A.).

We define the communication overhead of Algorithm 1 as follows.

Definition 3. *(communication overhead) The communication overhead of Algorithm 1 is defined as the number of required anonymous queries.*

Theorem 1 characterizes the convergence rate of Algorithm 1.

Theorem 1. *To achieve an ϵ-neighborhood of the optimal rate r^* or the throughput $T(r^*)$, that is either we find a rate*

$r_L \in [r_{min}, r_{max}]$ such that $r^* - \epsilon \le r_L \le r^*$ or we find a rate $r_F \in [r_{min}, r_{max}]$ such that $|T(r_F) - T(r^*)| \le \epsilon$, the communication overhead of Algorithm 1 scales as $O(\log \frac{1}{\epsilon})$.

PROOF. See Appendix A. $\square$

REMARK 1.1. *By Theorem 1, the communication overhead of Algorithm 1 is independent of the number of receivers n.*

REMARK 1.2. *Getting to know the ϵ-neighborhood of the optimal rate r^* is equivalent to acquire the first $\Theta(\log \frac{1}{\epsilon})$ bits of r^* in the binary number system. Thus, the communication overhead of any algorithm which could learn the ϵ-neighborhood of r^* is trivially lower bounded by $\Theta(\log \frac{1}{\epsilon})$. By Theorem 1, Algorithm 1 is order optimal in terms of ϵ.*

REMARK 1.3. *The key idea behind Theorem 1 is to show that at the end of every iteration of Algorithm 1, we have $c_L \le T(r_F) \le c_U$ and $r_L \le r^* \le r_U$, $c_L \le T(r^*) \le c_U$. Since in each iteration of Algorithm 1, either the search region $[r_L, r_U]$ for r^* or the search region $[c_L, c_U]$ for $T(r^*)$ can be narrowed down by half, after $\lceil \log_2 \frac{r_{max} - r_{min}}{\epsilon} \rceil + \lceil \log_2 \frac{r_{max}}{\epsilon} \rceil$ iterations of Algorithm 1, either $r_U - r_L \le \epsilon$ or $c_U - c_L \le \epsilon$. Thus, if $r_U - r_L \le \epsilon$, we know that r_L is within the ϵ-neighborhood of an optimal rate r^*, i.e., $|r_L - r^*| \le \epsilon$, otherwise we know a rate r_F, the throughput of which is in an ϵ-neighborhood of the throughput of an optimal rate, i.e., $|T(r_F) - T(r^*)| \le \epsilon$.*

REMARK 1.4. *It can be easily shown that $T(\hat{r}) \ge T(r^*) - \epsilon$ always holds. Hence, the throughput under the rate $\hat{r}$ calculated by Algorithm 1 is close to the throughput under the optimal state rate r^*.*

To see this, by Theorem 1, for the rate $\hat{r}$ calculated by Algorithm 1, we have either $T(r^*) - T(\hat{r}) \le \epsilon$ or $r^* - \epsilon \le \hat{r} \le r^*$. Given that $r^* - \epsilon \le \hat{r} \le r^*$, from Equation (1), we have for any $1 \le i \le n$

$$T_i(r^*) - T_i(\hat{r}) = r^* \mathbb{P}(R_i \ge r^*) - \hat{r} \mathbb{P}(R_i \ge \hat{r}) \stackrel{(a)}{\le} \mathbb{P}(R_i \ge \hat{r})(r^* - \hat{r}) \le \epsilon,$$

where step (a) follows from the fact $\mathbb{P}(R_i \ge r)$ is a decreasing function in r. By the definition in Equation (2), we have

$$T(r^*) - T(\hat{r}) = \min_{1 \le i \le n} T_i(r^*) - \min_{1 \le i \le n} T_i(\hat{r}) \le \min_{1 \le i \le n} (T_i(\hat{r}) + \epsilon) - \min_{1 \le i \le n} T_i(\hat{r}) = \epsilon.$$

Thus, $T(r^*) - T(\hat{r}) \le \epsilon$ holds.

To allow Algorithm 1 to work with a finite $\mathcal{R}$, a minor adjustment on Algorithm 1 is sufficient: Rather than updating r_M to be $\frac{1}{2}(r_L + r_U)$ in line 7, set r_M to be the median of the set $\mathcal{R} \cap [r_L, r_U]$. As a direct corollary of Theorem 1, with a communication overhead of $O(\log \frac{1}{\epsilon} + \log m)$, either we find the optimal rate $r^* \in \mathcal{R}$, or we find a rate $r_F \in \mathcal{R}$ such that $|T(r_F) - T(r^*)| \le \epsilon$.

5. COMPARISON WITH GENIE-AIDED RATE CONTROL

In this section, we compare the achievable throughput of our rate control method with the genie-aided dynamic rate control solution, in which the channel state realizations are known non-causally at the transmitter and the rate at each time-slot is selected to maximize the long-term average system throughput. We show that under some mild conditions, the throughput gain of the optimal genie-aided control over our rate control method vanishes as the number of receivers n increases. *In a network with a large number of receivers, the static rate control method given by Algorithm 1 is able to achieve a throughput close to the throughput of the optimal genie-aided rate control, while incurring a small constant communication overhead independent of the number of receivers.*

5.1 Genie-aided Rate Control

In the genie-aided rate control, we allow the transmitter to be very powerful and even foresee the channel realizations of all the receivers in all the time slots, i.e., $\{R_i[t]\}_{1 \le i \le n, t \in \mathbb{N}}$. As a result, it could then select the transmission rate $\vec{r} = \{r[t]\}_{t \in \mathbb{N}}$ according to the channel realizations. Under a specific vector of rate $\vec{r} = \{r[t]\}_{t \in \mathbb{N}}$, the achievable throughput of any receiver i can be expressed as

$$T_i^G(\vec{r}) = \limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} r[\tau] 1_{R_i[\tau] \ge r[\tau]}, \tag{5}$$

where 1_A is the indicator function for event A.

The optimal genie-aided rate control finds the transmission rate $\vec{r}$ which maximizes the throughput of the network as formulated in Equation (6).

$$T^* \triangleq \max_{\vec{r} = \{r[t]\}, t \in \mathbb{N}} T^G(\vec{r}) = \max_{\vec{r} = \{r[t]\}, t \in \mathbb{N}} \min_{1 \le i \le n} T_i^G(\vec{r}), \tag{6}$$

where T^* is the achievable throughput of the optimal genie-aided rate control.

5.2 Performance Limits of Genie-aided Rate Control

We assume that for any receiver i, its channel states across different time slots are independently distributed. For any pair of receivers $i_1 \ne i_2$, $\{R_{i_1}[t]\}_{t \in \mathbb{N}}$ and $\{R_{i_2}[t]\}_{t \in \mathbb{N}}$ are independent. We assume that there are K classes of receivers, $\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_K$. The distributions of R_i for the receivers in the same class $i \in \mathcal{C}_k$ are the same and we use the random variable $R_{\mathcal{C}_k}$ to denote its distribution, i.e., for any receiver $i \in \mathcal{C}_k$, $\mathbb{P}(R_i \ge r) = \mathbb{P}(R_{\mathcal{C}_k} \ge r), \forall r \in \mathcal{R}$. For any two classes $\mathcal{C}_k$ and $\mathcal{C}_{k'}$, the corresponding channel distributions follow a stochastic order, that is either $\mathbb{P}(R_{\mathcal{C}_k} \ge r) < \mathbb{P}(R_{\mathcal{C}_{k'}} \ge r), \forall r \in \mathcal{R}$ (denoted as $R_{\mathcal{C}_k} \prec R_{\mathcal{C}_{k'}}$) or $\mathbb{P}(R_{\mathcal{C}_k} \ge r) > \mathbb{P}(R_{\mathcal{C}_{k'}} \ge r), \forall r \in \mathcal{R}$ (denoted as $R_{\mathcal{C}_k} \succ R_{\mathcal{C}_{k'}}$). Note that the stochastic order of different classes of channels is a mild condition. For example, if the channel distribution follows the commonly adopted Rayleigh distribution, the stochastic ordering is satisfied, since the Rayleigh fading model is determined by a single parameter. Without loss of generality, we assume $R_{\mathcal{C}_1} \prec R_{\mathcal{C}_2} \prec ... \prec R_{\mathcal{C}_K}$. Thus, $\mathcal{C}_1$ corresponds the bottleneck class of receivers.

Let $n_{\mathcal{C}_k}$ denote the number of receivers in class $\mathcal{C}_k$. Then, $\sum_{k=1}^{K} n_{\mathcal{C}_k} = n$. We assume that $n_{\mathcal{C}_k}$ is non-decreasing with n and $n_{\mathcal{C}_k} \to \infty$ as $n \to \infty$ for any $1 \le k \le K$.

It can be expected that the achievable throughput of the network under the genie-aided rate control to be higher than our static rate control policy in Algorithm 1, as the static rate control is just a special case of the genie-aided control. In this subsection, we characterize the throughput gain of

$$I(\mathcal{R}) = -\log \max_{\tilde{r}\neq r^*, \tilde{r}\in\mathcal{R}} \left\{ \left(1-\frac{r^*}{\tilde{r}}\right)^{-(1-\frac{r^*}{\tilde{r}})} \left(\frac{r^*}{\tilde{r}}\right)^{-\frac{r^*}{\tilde{r}}} \mathbb{P}(r^* \le R_{\mathcal{C}_1} < \tilde{r})^{1-\frac{r^*}{\tilde{r}}} \mathbb{P}(R_{\mathcal{C}_1} \ge \tilde{r})^{\frac{r^*}{\tilde{r}}} + \mathbb{P}(R_{\mathcal{C}_1} < r^*), \quad if\ \tilde{r} > r^*; \right.$$
$$\left. \left(1-\frac{\tilde{r}}{r^*}\right)^{-(1-\frac{\tilde{r}}{r^*})} \left(\frac{\tilde{r}}{r^*}\right)^{-\frac{\tilde{r}}{r^*}} \mathbb{P}(\tilde{r} \le R_{\mathcal{C}_1} < r^*)^{1-\frac{\tilde{r}}{r^*}} \mathbb{P}(R_{\mathcal{C}_1} \ge r^*)^{\frac{\tilde{r}}{r^*}} + \mathbb{P}(R_{\mathcal{C}_1} < \tilde{r}), \quad if\ \tilde{r} < r^* \right\}. \quad (7)$$

the optimal genie-aided rate control over our static rate control policy given by Algorithm 1 as the number of receivers increases.

THEOREM 2. *Let $\hat{r} \in \mathcal{R}$ denote the rate calculated by Algorithm 1. Given that the rate set $\mathcal{R}$ that the transmitter can choose from is countable, the throughput gain of the optimal genie-aided rate control over the rate control method (with the rate calculated by Algorithm 1), i.e., $\{r[t] = \hat{r}\}_{t\in\mathbb{N}}$ is no more than ϵ as $n \to \infty$, where ϵ is independent of n and can be made arbitrarily small at the cost of $O(log\frac{1}{\epsilon})$ communication overhead.*

$$\lim_{n\to\infty} T^* \le T(\hat{r}) + \epsilon \qquad w.p.1, \quad (8)$$

in which w.p.1 denotes "with probability 1".

PROOF. See Appendix B. □

REMARK 2.1. *The genie-aided rate control may achieve a higher throughput than the optimal static rate control because of the opportunistic gains. However, Theorem 2 suggests that for wireless multicast, the opportunistic gains of the genie-aided rate control vanishes as n increases. This is in sharp contrast to wireless unicasts, where the opportunistic gains increases as n increases [20].*

REMARK 2.2. *To prove Theorem 2, by Theorem 1 and Remark 1.4, it suffices to prove that the throughput gain of the optimal genie-aided rate control over the optimal static rate control policy, i.e., $\{r[t] = r^*\}_{t\in\mathbb{N}}$ vanishes as $n \to \infty$.*

$$\lim_{n\to\infty} T^* = T(r^*) \qquad w.p.1. \quad (9)$$

THEOREM 3. *Let $\hat{r} \in \mathcal{R}$ denote the rate calculated by Algorithm 1. If the rate set $\mathcal{R}$ the transmitter can choose from is finite, and there exists $0 < \kappa \le 1$ such that $\kappa n \le n_{\mathcal{C}_1} \le \kappa n + 1$, the throughput gain of the optimal genie-aided rate control over the rate control method (with the rate calculated by Algorithm 1), i.e., $\{r[t] = \hat{r}\}_{t\in\mathbb{N}}$, decreases exponentially fast with respect to n.*

$$e^{-I(\mathcal{R})\kappa n + o(n)} \le T^* - T(\hat{r}) \le e^{-I(\mathcal{R})\kappa n + o(n)} + \epsilon. \quad (10)$$

Here[4], ϵ is independent of n and can be made arbitrarily small at the cost of $O(log\frac{1}{\epsilon})$ communication overhead, and the decay rate $I(\mathcal{R})$ is strictly greater than 0, given by Equation (7).

PROOF. See Appendix C. □

REMARK 3.1. *Theorem 3 considers a finite rate set $\mathcal{R}$, which is the case in practice. It implies that, for moderately large n, the throughput gain of the genie-aided rate control has become negligible.*

[4]We say $x_n = o(y_n)$ if $\lim_{n\to\infty} \frac{|x_n|}{|y_n|} = 0$ for two real-valued sequences $\{x_n\}$ and $\{y_n\}$.

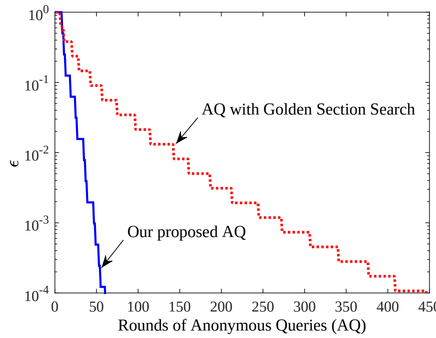


Figure 4: Evolution of the communication overhead to approach the ϵ-neighborhood of r^*.

REMARK 3.2. *To prove Theorem 3, by Theorem 1 and Remark 1.4, it suffices to prove that the throughput gain of the optimal genie-aided rate control over the optimal static rate control policy, i.e., $\{r[t] = r^*\}_{t\in\mathbb{N}}$ decreases exponentially fast with respect to n.*

$$T^* - T(r^*) = e^{-I(\mathcal{R})\kappa n + o(n)}. \quad (11)$$

The proof of Equation (11) is composed of three parts. First, we prove a lemma which characterizes the probability that some rate $\tilde{r} \neq r^*$ will lead to a higher sum throughput of receivers in class $\mathcal{C}_1$ than the static optimal rate r^*. Secondly, based on the lemma, we obtain an asymptotic upper bound of the throughput gain for any genie-aided rate control policy. Lastly, we construct a specific genie-aided rate control policy, whose throughput gain achieves the asymptotic upper bound.

REMARK 3.3. *By Equation (7) in Theorem 3, the decay rate of the throughput gain of the optimal genie-aided rate control depends on the available rate set $\mathcal{R}$ as well as the distribution $\mathbb{P}(R_{\mathcal{C}_1} \ge r), \forall r \in \mathcal{R}$. Since in Equation (7) the maximization is taken over all possible $\tilde{r} \neq r^*, \tilde{r} \in \mathcal{R}$, given the same r^*, the decay rate is non-increasing with the increase of the number of available rates in set $\mathcal{R}$. This suggests that, for the optimal genie-aided rate control, the channel realization information could bring a higher throughput gain over the static rate control method when there are more available rates in $\mathcal{R}$.*

6. EXPERIMENTAL RESULTS

In this section, we first present some simulations results to validate the theoretical results. Then, via a software defined radio implementation we show the low-overhead and robustness features of the anonymous-query based algorithm.

6.1 Numerical Results

In Figure 4, we compare the communication overhead of

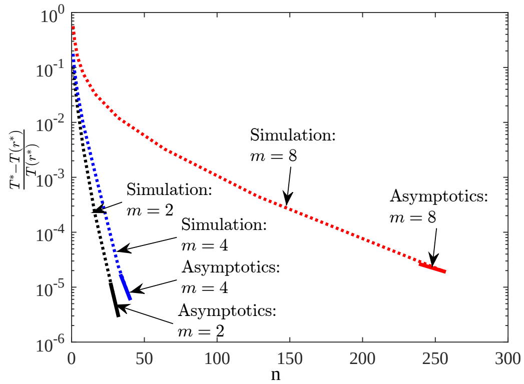


Figure 5: Throughput gain of the optimal genie-aided rate control with respect to the number of receivers n, when there is a single class of receivers. Here, m is the number of rates in set $\mathcal{R}$ that the transmitter may choose from.

our proposed rate control in Algorithm (1) with a straightforward incorporation of Anonymous Query with the Golden Section Search [18] (discussed in Section 4). In the experiment, the network contains 100 receivers, the channel of which is modeled by either Rayleigh fading or Rician fading with random coefficients. Given the SNR at the receiver, $R_i[t]$ is calculated by the Shannon capacity with a bandwidth of 20 MHz. Initially, $r_{\min} = 0Mbps$ and $r_{\max} = 200Mbps$. For fair comparison, we record the number of required anonymous queries to narrow the search region for r^* below $\epsilon \times 200Mbps$. It can be observed that the convergence rate of our method is much faster than the Golden Section Search method. With less than 40 rounds of anonymous queries, our method could achieve a $2Mbps$ neighborhood of the optimal rate r^*. The reason why both methods have a staircase shaped curve in Figure 4 is the following. For our proposed method, the staircase exists since after one iteration of Algorithm 1, either the search region for r^* or the search region for $T(r^*)$ is guaranteed to be narrowed down. For the Golden Section Search, the staircase exists because it may require multiple rounds to learn the relationship between $T(r_1)$ and $T(r_2)$ before the search region (separated by r_1, r_2) can be narrowed down, as explained in Section 4.2.

Figure 5 plots the throughput gain of the optimal genie-aided rate control over the optimal static rate control as a function of the number of receivers n, when the transmitter has m rates to choose from. Note that only when all receivers are in the same class, the optimal genie-aided rate control policy is known explicitly, which selects the rate maximizing the sum throughput of all receivers in each time slot. Thus, to show the achievable throughput of the optimal genie-aided rate control, all the receivers belong to the same class in this experiment. From the Figure 5, we have the following observations. First, the throughput gain of the optimal genie-aided rate control decreases fast with respect to n. Second, for sufficiently large n, the throughput gain decays exponentially and matches the predicted asymptotic decay rate from Equation (7). Third, the decay rate is non-increasing with the increase of the number of available rates in set $\mathcal{R}$, which is consistent with what is indicated in Remark 3.3.

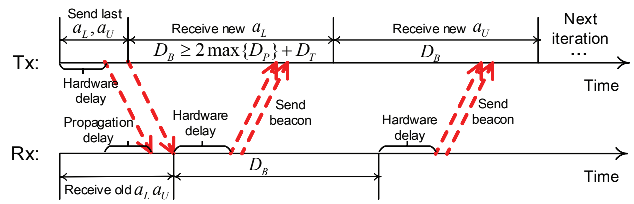


Figure 6: Implementation of anonymous queries in Algorithm 1.

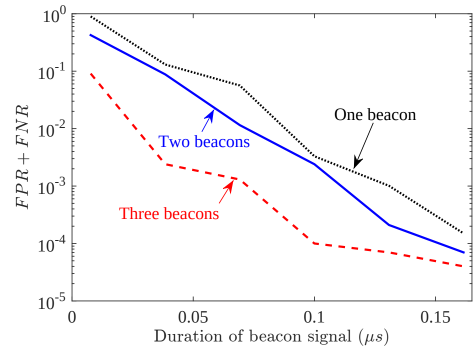


Figure 7: Impact of beacon duration and the number of simultaneous transmitted beacon signals on the probability of beacon detection errors.

6.2 System Implementation

To understand the overhead and the robustness of the anonymous queries-based algorithm in practice, we build a proof-of-concept system on the NI SDR platform [21]. The transceivers send signals on a 20 MHz channel at 2.4 GHz. Figure 6 illustrates how we implemented one iteration of anonymous queries in Algorithm 1. In the beginning of the iteration, the transmitter broadcasts a_L, a_U, which are the 2-bits information that the transmitter has learned in the last iteration. Following the procedure of Algorithm 1, a receiver could then know the two anonymous queries in the new iteration. In response, a receiver decides whether or not to send a beacon signal by checking the property in the anonymous query. At the same time, the transmitter listens on the channel for a time window D_B. D_B is set to be larger than twice the maximum possible propagation delay D_P in the network plus the time D_T required to send a beacon signal. In our experiment, a large proportion of D_T is caused by the hardware. For example, there is a delay to exchange information between the analog front and the digital front. Remember that there are two anonymous queries in one iteration of Algorithm 1. To guarantee that the second beacon won't interfere the first beacon at the transmitter, a receiver could not start sending the second beacon until a delay of D_B. In our implementation, the receivers do not need to be precisely synchronized and only need to follow a local time schedule.

Since the anonymous query mechanism depends on the beacon signal being correctly detected, we study the impact of beacon duration and the number of simultaneous transmitted beacon signals on the probability of beacon detection errors. Detection error rate, measured by the false positive rate (FPR) plus the false negative rate (FNR), is shown in

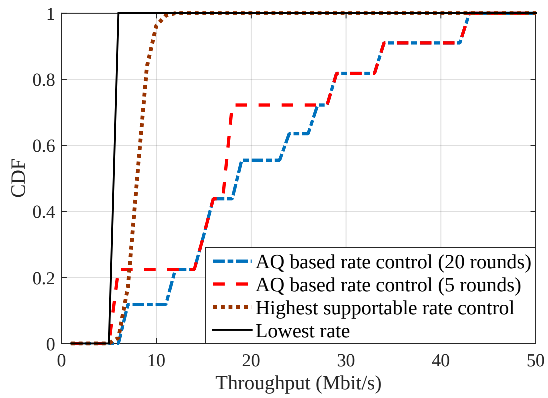


Figure 8: Achievable multicast throughput of random networks with $n = 50$ receivers.

Figure 7. It can be observed that the detection error rate drops with the increase of the number of simultaneous transmitted beacon signals. It makes sense since when multiple receivers send beacons, it is easier for the transmitter to detect an energy impulse. By properly setting the duration of a beacon signal, the detection error probability can be kept under a negligible level. Hence, the anonymous-queries based algorithm can be quite robust in practice.

We implement anonymous queries on a system with one transmitter and 11 receivers. To emulate a Wi-Fi network, the receivers are randomly deployed within 20 meters to the transmitter. With our specific implementation, Algorithm 1 could converge within 0.5 ms. Note that in practice, the channel distribution of a receiver also changes over time. Thus, Algorithm 1 needs to be performed once for every period of time when there might be a noticeable change in the channel distribution of a receiver. The communication overhead, compared with the time period is essentially negligible in a Wi-Fi system.

Finally, we compare the achievable throughput of our rate control method with two existing baseline methods through trace-driven simulations. First, the throughput functions $\{T_i(r)\}_i$ are obtained through experimental measurements of receivers under different channel conditions, e.g., different SNR levels and different mobility status. We implement the 802.11a protocol which supports 8 possible rates, i.e., $\{6, 9, 12, 18, 24, 36, 48, 54\}$ Mbit/s. Then, the traces of $\{T_i(r)\}_i$ are fed into the simulator and 1000 networks with 50 receivers are generated, where each receiver may conform to one of the traces randomly. For each network generated, we compare our rate control method and two baseline methods. The first baseline method is to transmit at the lowest rate, which is 6 Mbps. The second baseline method [10] is to transmit at the highest supportable rate of all receivers at any given time. In Figure 8, we show the CDF of the achievable throughput of different methods. It can be observed that the second baseline only outperforms the first baseline method slightly. This is because the probability that there exists one receiver whose channel condition happens to be poor is high. Compared with the two baseline methods, Anonymous Query based method achieves a much higher throughput. It is worthy to note that the communication overhead of our method is extremely low. With 20 rounds of anonymous queries, the transmitter has been able to find the optimal static rate most of the time. Even after 5 rounds of anonymous queries, our method significantly outperforms the highest supportable rate control method in terms of throughput.

7. CONCLUSION

In this paper, we develop a novel anonymous-query based rate control for wireless multicast, which approaches the optimal throughput with a constant feedback overhead independent of the number of receivers. Through implementation on a software-defined ratio platform, we show that the anonymous-query based algorithm achieves low-overhead and robustness in practice.

8. ACKNOWLEDGMENTS

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References

[1] "Cisco visual networking index: Forecast and methodology, 2014-2019." http://www.cisco.com.

[2] J. Camp and E. Knightly, "Modulation rate adaptation in urban and vehicular environments: cross-layer implementation and experimental evaluation," *IEEE/ACM Trans. Netw.*, vol. 18, no. 6, pp. 1949–1962, 2010.

[3] M. Vutukuru, H. Balakrishnan, and K. Jamieson, "Cross-layer wireless bit rate adaptation," *ACM SIGCOMM*, vol. 39, no. 4, pp. 3–14, 2009.

[4] A. Gudipati and S. Katti, "Strider: Automatic rate adaptation and collision handling," in *ACM SIGCOMM*, vol. 41, pp. 158–169, 2011.

[5] J. Perry, P. A. Iannucci, K. E. Fleming, H. Balakrishnan, and D. Shah, "Spinal codes," in *ACM SIGCOMM*, pp. 49–60, 2012.

[6] P. A. Iannucci, J. Perry, H. Balakrishnan, and D. Shah, "No symbol left behind: a link-layer protocol for rateless codes," in *ACM MOBICOM*, pp. 17–28, 2012.

[7] J. Luo and A. Ephremides, "Wireless multicasting with channel uncertainty," in *IEEE CISS*, pp. 800–805, 2007.

[8] J. Yoon, H. Zhang, S. Banerjee, and S. Rangarajan, "Muvi: a multicast video delivery scheme for 4g cellular networks," in *ACM MOBICOM*, pp. 209–220, 2012.

[9] Y. Bejerano, J. Ferragut, K. Guo, V. Gupta, C. Gutterman, T. Nandagopal, and G. Zussman, "Scalable wifi multicast services for very large groups," in *IEEE ICNP*, pp. 1–12, 2013.

[10] A. Chen, D. Lee, G. Chandrasekaran, and P. Sinha, "Himac: High throughput mac layer multicasting in wireless networks," in *IEEE MASS*, pp. 41–50, 2006.

[11] V. Gupta, C. Gutterman, Y. Bejerano, and G. Zussman, "Experimental evaluation of large scale wifi multicast rate control," in *IEEE INFOCOM 2016*, Apr. 2016.

[12] M. Luby, "LT codes," in *IEEE FOCS 2002*, pp. 271–280, 2002.

[13] A. Shokrollahi, "Raptor codes," *IEEE Trans. Inf. Theory*, vol. 52, pp. 2551–2567, Jun. 2006.

[14] T. Ho, M. Médard, R. Koetter, D. R. Karger, M. Effros, J. Shi, and B. Leong, "A random linear network coding approach to multicast," *IEEE Trans. Inf. Theory*, vol. 52, pp. 4413–4430, Oct. 2006.

[15] A. Rezaee, F. du Pin Calmon, L. Zeger, and M. Medard, "Speeding multicast by acknowledgment reduction technique (SMART) enabling robustness of QoE to the number of users," *IEEE J. Select. Areas Commun.*, vol. 30, pp. 1270–1280, Aug. 2012.

[16] F. Wu, Y. Sun, Y. Yang, K. Srinivasan, and N. Shroff, "Constant-delay and constant-feedback moving window network coding for wireless multicast: Design and asymptotic analysis," *IEEE J. Select. Areas Commun.*, vol. 33, pp. 127–140, Feb 2015.

[17] D. Halperin, W. Hu, A. Sheth, and D. Wetherall, "Predictable 802.11 packet delivery from wireless channel measurements," *ACM SIGCOMM*, vol. 41, no. 4, pp. 159–170, 2011.

[18] J. Kiefer, "Sequential minimax search for a maximum," *Proceedings of the American Mathematical Society*, vol. 4, no. 3, pp. 502–506, 1953.

[19] M. Avriel and D. J. Wilde, "Optimality proof for the symmetric fibonacci search technique," *Fibonacci Quarterly*, vol. 4, no. 3, pp. 265–269, 1966.

[20] X. Liu, E. K. Chong, and N. B. Shroff, "A framework for opportunistic scheduling in wireless networks," *Computer networks*, vol. 41, no. 4, pp. 451–474, 2003.

[21] FlexRIO SDR. http://www.ni.com. National Instruments.

[22] A. Dembo and O. Zeitouni, *Large deviations techniques and applications.* Springer-Verlag New York, Inc., 2 ed., 2010.

APPENDIX

A. PROOF OF THEOREM 1

The key to prove Theorem 1 is to show that at the end of every iteration of Algorithm 1, it is guaranteed that $c_L \le T(r^*) \le c_U$, $c_L \le T(r_F) \le c_U$ and $r_L \le r^* \le r_U$. Before delving into the proof, we provides some preliminary properties regarding the superlevel sets.

By definition of superlevel set and $T(r)$ in Equation (2),

$$L_c(T) = \bigcap_{i=1}^{n} L_c(T_i). \tag{12}$$

A throughput $c \in [0, r_{\max}]$ is achievable by the network, i.e., $c \le T(r^*)$, if and only if $L_c(T) \ne \emptyset$. For any $0 \le c \le T(r^*)$, since by Equations (1) and (3) $T_i(r^*) \ge c$, it follows that $r^* \in L_c(T)$.

Lemma 1. *At the end of every iteration of Algorithm 1,*

$$r_L \le r^* \le r_U. \tag{13}$$

Proof. See Appendix D. □

Lemma 2. *At the end of every iteration of Algorithm 1,*

$$c_L \le T(r^*) \le c_U, \tag{14}$$

$$c_L \le T(r_F) \le c_U. \tag{15}$$

Proof. See Appendix E. □

In each iteration of Algorithm 1, depending on the results of the two Anonymous Queries a_L, a_U, either the search region $[r_L, r_U]$ for r^* or the search region $[c_L, c_U]$ for $T(r^*)$ is narrowed down by half. Therefore, after $\lceil \log_2 \frac{r_{\max}-r_{\min}}{\epsilon} \rceil + \lceil \log_2 \frac{r_{\max}}{\epsilon} \rceil$ iterations of Algorithm 1, we have either $r_U - r_L \le \epsilon$ or $c_U - c_L \le \epsilon$. If $r_U - r_L \le \epsilon$, it follows from Lemma 1 that $r^* - r_L \le r_U - r_L \le \epsilon$ and thus we know that r_L is within the ϵ-neighborhood of an optimal rate r^*, If $r_U - r_L > \epsilon$, we must have $c_U - c_L \le \epsilon$. In this case, it follows from Lemma 2 that $|T(r_F) - T(r^*)| \le c_U - c_L \le \epsilon$ and thus we know a rate r_F, the throughput of which is in an ϵ-neighborhood of the throughput of the optimal rate r^*.

Note that in each iteration of Algorithm 1, there are two rounds of Anonymous Queries, the number of Anonymous Queries needed to guarantee the above precision is $2\lceil \log_2 \frac{r_{\max}-r_{\min}}{\epsilon} \rceil + 2\lceil \log_2 \frac{r_{\max}}{\epsilon} \rceil$, the theorem is proved.

B. PROOF OF THEOREM 2

By Theorem 1 and Remark 1.4, it suffices to prove Equation (9).

Let us denote $N^t_{\mathcal{C}}(r)$ as the number of receivers in class $\mathcal{C}$ that can support rate r in time slot t. More precisely, $N^t_{\mathcal{C}}(r) \triangleq \sum_{i\in\mathcal{C}} \mathbf{1}_{R_i[t]\ge r}$. Since $\{R_i[t]\}$ is i.i.d. across receivers in the same class, for any $r \in \mathcal{R}$ and any time slot t, we have, according to the strong law of large numbers,

$$\lim_{n_{\mathcal{C}}\to\infty} \frac{rN^t_{\mathcal{C}}(r)}{n_{\mathcal{C}}} = r\mathbb{P}(R_{\mathcal{C}} \ge r). \text{ w.p.1}$$

Given that $\mathcal{R}$ is a countable set, we further have, for any time slot t,

$$\lim_{n_{\mathcal{C}}\to\infty} \max_{r\in\mathcal{R}} \frac{rN^t_{\mathcal{C}}(r)}{n_{\mathcal{C}}} = \max_{r\in\mathcal{R}} r\mathbb{P}(R_{\mathcal{C}} \ge r). \text{ w.p.1},$$

which, by applying the Lebesgue dominated convergence theorem, yields

$$\begin{aligned} &\lim_{n_{\mathcal{C}}\to\infty} \mathbb{E}\left[\max_{r\in\mathcal{R}} \frac{rN^t_{\mathcal{C}}(r)}{n_{\mathcal{C}}}\right] \\ =&\mathbb{E}\left[\lim_{n_{\mathcal{C}}\to\infty} \max_{r\in\mathcal{R}} \frac{rN^t_{\mathcal{C}}(r)}{n_{\mathcal{C}}}\right] = \max_{r\in\mathcal{R}} r\mathbb{P}(R_{\mathcal{C}} \ge r). \end{aligned} \tag{16}$$

Let us first focus on the genie-aided rate control problem. Based on Equation (6), we have

$$\begin{aligned} T^* =& \max_{\vec{r}=\{r[t]\}, t\in\mathbb{N}} \min_{1\le i\le n} \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=1}^{t} r[\tau] 1_{R_i[\tau]\ge r[\tau]} \\ \le& \max_{\vec{r}=\{r[t]\}, t\in\mathbb{N}} \min_{1\le k\le K} \frac{1}{n_{\mathcal{C}_k}} \sum_{i\in\mathcal{C}_k} \left(\lim_{t\to\infty} \frac{1}{t} \sum_{\tau=1}^{t} r[\tau] 1_{R_i[\tau]\ge r[\tau]} \right) \\ =& \max_{\vec{r}=\{r[t]\}, t\in\mathbb{N}} \min_{1\le k\le K} \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=1}^{t} \frac{r[\tau] N^t_{\mathcal{C}_k}(r[\tau])}{n_{\mathcal{C}_k}} \\ \le& \min_{1\le k\le K} \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=1}^{t} \max_{r[\tau]\in\mathcal{R}} \frac{r[\tau] N^t_{\mathcal{C}_k}(r[\tau])}{n_{\mathcal{C}_k}} \\ =& \min_{1\le k\le K} \mathbb{E}\left[\max_{r\in\mathcal{R}} \frac{rN^0_{\mathcal{C}_k}(r)}{n_{\mathcal{C}_k}}\right] \text{ a.s.} \end{aligned} \tag{17}$$

By combining Equation (16), and the fact that $n_{\mathcal{C}_k} \to \infty$ as $n \to \infty$ for any $1 \le k \le K$, we obtain,

$$\begin{aligned} \lim_{n\to\infty} T^* \le& \min_{1\le k\le K} \max_{r\in\mathcal{R}} r\mathbb{P}(R_{\mathcal{C}_k} \ge r) \\ =& \max_{r\in\mathcal{R}} r\mathbb{P}(R_{\mathcal{C}_1} \ge r). \end{aligned} \tag{18}$$

The last equality in the above equation holds because of the assumption that the random variables $\{R_{\mathcal{C}_k}\}_{1\le k\le K}$ follow the stochastic ordering of $R_{\mathcal{C}_1} \prec R_{\mathcal{C}_2} \prec ... \prec R_{\mathcal{C}_K}$.

Next, let us shift our focus to the static rate control problem. From Equation (3), the throughput using the optimal static rate control can be expressed as

$$\begin{aligned} \max_{r\in\mathcal{R}} T(r) = \max_{r\in\mathcal{R}} \min_{1\le i\le n} T_i(r) =& \max_{r\in\mathcal{R}} \min_{1\le k\le K} \min_{i\in\mathcal{C}_k} T_i(r) \\ =& \max_{r\in\mathcal{R}} \min_{1\le k\le K} r\mathbb{P}(R_{\mathcal{C}_k} \ge r) \\ =& \max_{r\in\mathcal{R}} r\mathbb{P}(R_{\mathcal{C}_1} \ge r). \end{aligned} \tag{19}$$

Again, the last equality is due to the stochastic ordering assumption of $\{R_{\mathcal{C}_k}\}_{1\le k\le K}$.

Remember that $r^* = \arg\max_{r\in\mathcal{R}} T(r)$. From Equation (18) and Equation (19), we know that $\lim_{n\to\infty} T^* \le \max_{r\in\mathcal{R}} T(r)$, which, by combining with the fact that $T^* \ge \max_{r\in\mathcal{R}} T(r)$ for any n, Equation (9) holds and the proof is completed.

C. PROOF OF THEOREM 3

By Theorem 1 and Remark 1.4, it suffices to prove Equation (11).

The proof is composed of three parts. First, we prove a lemma which characterizes the probability that some rate $\tilde{r} \neq r^*$ will lead to a higher sum throughput of receivers in class $\mathcal{C}_1$ than the static optimal rate r^*. Secondly, based on the lemma, we obtain an asymptotic upper bound of the throughput gain for any genie-aided rate control policy. Lastly, we construct a specific genie-aided rate control policy, whose throughput gain achieves the asymptotic upper bound.

Let us focus on the receivers in the class $\mathcal{C}_1$. Lemma 3 characterizes the probability that the sum throughput of receivers $i \in \mathcal{C}_1$ under a rate $\tilde{r}$ is higher the sum throughput under the optimal static rate r^* by $n_{\mathcal{C}_1}\nu$ in an arbitrary time slot t, where $\nu \ge 0$.

LEMMA 3. *In any time slot t, the probability that the sum throughput of all receivers $i \in \mathcal{C}_1$ under a rate $\tilde{r} \neq r^*$ is higher than the sum throughput under the static optimal rate r^* by $n_{\mathcal{C}_1}\nu$ for $\nu \ge 0$ decays exponentially with respect to $n_{\mathcal{C}_1}$,*

$$-\lim_{n_{\mathcal{C}_1}\to\infty} \frac{1}{n_{\mathcal{C}_1}} \log \mathbb{P}\left(\tilde{r}N^t_{\mathcal{C}_1}(\tilde{r}) \ge r^* N^t_{\mathcal{C}_1}(r^*) + n_{\mathcal{C}_1}\nu\right) = I_\nu(\tilde{r}), \tag{20}$$

where the decay rate $I_\nu(\tilde{r})$ is a positive number given by

$$I_\nu(\tilde{r}) = \sup_{\theta\in\mathbb{R}} \left\{-\log \mathbb{E}\left(e^{-\theta\left(\tilde{r}1_{R_i[t]\ge\tilde{r}} - r^* 1_{R_i[t]\ge r^*} - \nu\right)}\right)\right\} \tag{21}$$

At $\nu = 0$, if $\tilde{r} > r^*$,

$$I_0(\tilde{r}) = -\log\Big(\left(1 - r^*/\tilde{r}\right)^{-(1-r^*/\tilde{r})}\left(r^*/\tilde{r}\right)^{-r^*/\tilde{r}} \times$$
$$\mathbb{P}(r^* \le R_{\mathcal{C}_1} < \tilde{r})^{1-\frac{r^*}{\tilde{r}}}\,\mathbb{P}(R_{\mathcal{C}_1} \ge \tilde{r})^{\frac{r^*}{\tilde{r}}} + \mathbb{P}(R_{\mathcal{C}_1} < r^*)\Big).$$

At $\nu = 0$, if $\tilde{r} < r^*$,

$$I_0(\tilde{r}) = -\log\Big(\left(1 - \tilde{r}/r^*\right)^{-(1-\tilde{r}/r^*)}\left(\tilde{r}/r^*\right)^{-\tilde{r}/r^*} \times$$
$$\mathbb{P}(\tilde{r} \le R_{\mathcal{C}_1} < r^*)^{1-\frac{\tilde{r}}{r^*}}\,\mathbb{P}(R_{\mathcal{C}_1} \ge r^*)^{\frac{\tilde{r}}{r^*}} + \mathbb{P}(R_{\mathcal{C}_1} < \tilde{r})\Big). \tag{22}$$

In addition,

$$\lim_{\nu\to 0^+} I_\nu(\tilde{r}) = I_0(r). \tag{23}$$

PROOF. See Appendix F. □

Provided with the above lemma and remember that $r^* = \arg\max_{r\in\mathcal{R}} T(r)$, our next task is to show

$$T^* - \max_{r\in\mathcal{R}} T(r) \le e^{-I(\mathcal{R})\kappa n + o(n)}. \tag{24}$$

From Equation (17) in the proof of Theorem 2 we know that

$$\begin{aligned}
T^* \le& \min_{1\le k\le K} \mathbb{E}\left[\max_{r\in\mathcal{R}} \frac{rN^0_{\mathcal{C}_k}(r)}{n_{\mathcal{C}_k}}\right] \\
\le& \mathbb{E}\left[\max_{r\in\mathcal{R}} \frac{rN^0_{\mathcal{C}_1}(r)}{n_{\mathcal{C}_1}}\right] \\
\le& \mathbb{P}\left(r^* = \arg\max_{r\in\mathcal{R}} \frac{rN^0_{\mathcal{C}_1}(r)}{n_{\mathcal{C}_1}}\right) \mathbb{E}\left[\frac{r^*N^0_{\mathcal{C}_1}(r^*)}{n_{\mathcal{C}_1}}\right] + \\
& \mathbb{P}\left(r^* \neq \arg\max_{r\in\mathcal{R}} \frac{rN^0_{\mathcal{C}_1}(r)}{n_{\mathcal{C}_1}}\right) r_{\max} \\
\le& \mathbb{E}\left[\frac{r^*N^0_{\mathcal{C}_1}(r^*)}{n_{\mathcal{C}_1}}\right] + \mathbb{P}\left(r^* \neq \arg\max_{r\in\mathcal{R}} \frac{rN^0_{\mathcal{C}_1}(r)}{n_{\mathcal{C}_1}}\right) r_{\max} \\
=& T(r^*) + \mathbb{P}\left(r^* \neq \arg\max_{r\in\mathcal{R}} \frac{rN^0_{\mathcal{C}_1}(r)}{n_{\mathcal{C}_1}}\right) r_{\max}.
\end{aligned}$$

Thus, to prove Equation (24), it suffices to show that[5]

$$\mathbb{P}\left(r^* \neq \arg\max_{r\in\mathcal{R}} \frac{rN^0_{\mathcal{C}_1}(r)}{n_{\mathcal{C}_1}}\right) \le e^{-I(\mathcal{R})\kappa n + o(n_{\mathcal{C}_1})},$$

which can be obtained by using Lemma 3, as showed in the equation below.

$$\begin{aligned}
&\mathbb{P}\left(r^* \neq \arg\max_{r\in\mathcal{R}} \frac{rN^0_{\mathcal{C}_1}(r)}{n_{\mathcal{C}_1}}\right) \\
\overset{(a)}{\le}& \sum_{\tilde{r}\neq r^*} \mathbb{P}\left(\tilde{r}N^0_{\mathcal{C}_1}(\tilde{r}) \ge r^*N^0_{\mathcal{C}_1}(r^*)\right) \\
\le& m \max_{\tilde{r}\neq r^*} \left\{\mathbb{P}\left(\tilde{r}N^0_{\mathcal{C}_1}(\tilde{r}) \ge r^*N^0_{\mathcal{C}_1}(r^*)\right)\right\} \\
\overset{(b)}{=}& m \max_{\tilde{r}\neq r^*} e^{-I_0(\tilde{r})n_{\mathcal{C}_1} + o(n_{\mathcal{C}_1})} \\
=& m \cdot e^{-\min_{\tilde{r}\neq r^*}\{I_0(\tilde{r})\}n_{\mathcal{C}_1} + o(n_{\mathcal{C}_1})} \\
\overset{(c)}{=}& e^{-I(\mathcal{R})n_{\mathcal{C}_1} + o(n_{\mathcal{C}_1})} && (25)\\
\overset{(d)}{=}& e^{-I(\mathcal{R})\kappa n + o(n)} && (26)
\end{aligned}$$

where step (a) uses the union bound; in step (b), Equation (20) with $\nu = 0$ is applied; step (c) uses the definition of $I(\mathcal{R})$ in Equation (7), the definition of $I_0(r)$ in Equation (22) and the fact that m is independent of $n_{\mathcal{C}_1}$; step (d) uses the facts that $\kappa n \le n_{\mathcal{C}_1} \le \kappa n + 1$.

To complete the proof, as the last step, we need to show that

$$T^* - \max_{r\in\mathcal{R}} T(r) \ge e^{-I(\mathcal{R})\kappa n + o(n)}.$$

It suffices to show that, for any given $\epsilon_0 > 0$, there exists a specific genie-aided rate control policy $\vec{r} = \{r[t]\}_{t\in\mathbb{N}}$ with

$$T^G(\vec{r}) - T(r^*) \ge e^{-(I(\mathcal{R})+\epsilon_0)\kappa n + o(n)}. \tag{27}$$

Let $\tilde{r} \triangleq \arg\min_{r\in\mathcal{R}, r\neq r^*} I_0(r)$. By Equation (23) in Lemma 3, for any $\epsilon_0 > 0$, we can find $\nu > 0$ such that $I_\nu(\tilde{r}) - I_0(\tilde{r}) < \epsilon_0$. For any such (ϵ_0, ν) pair, we can construct a genie-aided rate control policy which determines the rate $r[t]$ in slot t accord-

[5]Here we abuse notation slightly by representing $-\lim_{x\to\infty} \frac{1}{x}\log f(x) \ge I$ as $f(x) \le e^{-Ix+o(x)}$.

ing to the following rule.

$$r[t] = \begin{cases} \tilde{r} & \text{if } \tilde{r}N^t_{\mathcal{C}_1}(\tilde{r}) \geq r^* N^t_{\mathcal{C}_1}(r^*) + n_{\mathcal{C}_1}\nu \\ r^* & \text{otherwise} \end{cases}$$

Then, for the receivers $i \in \mathcal{C}_1$,

$$\begin{aligned} T_i^G(\vec{r}) - T(r^*) &= \nu \mathbb{P}\left(\tilde{r}N^t_{\mathcal{C}_1}(\tilde{r}) \geq r^* N^t_{\mathcal{C}_1}(r^*) + n_{\mathcal{C}_1}\nu\right) \\ &\overset{(a)}{=} \nu e^{-I_\nu(\tilde{r})n_{\mathcal{C}_1} + o(n_{\mathcal{C}_1})} \overset{(b)}{\geq} \nu e^{-(I(\mathcal{R})+\epsilon_0)n_{\mathcal{C}_1} + o(n_{\mathcal{C}_1})} \\ &\overset{(c)}{=} e^{-(I(\mathcal{R})+\epsilon_0)\kappa n + o(n)}, \end{aligned} \tag{28}$$

where step (a) uses Lemma 3, step (b) uses the facts that $I_\nu(\tilde{r}) - I_0(\tilde{r}) < \epsilon_0$ and $I_0(\tilde{r}) = I(\mathcal{R})$, and step (c) uses the fact ν is independent of $n_{\mathcal{C}_1}$.

For the receivers $i \in \mathcal{C}_k$ with $k \neq 1$,

$$\begin{aligned} &T_i^G(\vec{r}) - T(r^*) \\ &\geq \left(1 - \mathbb{P}\left(\tilde{r}N^t_{\mathcal{C}_1}(\tilde{r}) \geq r^* N^t_{\mathcal{C}_1}(r^*) + n_{\mathcal{C}_1}\nu\right)\right) T_i(r^*) - T(r^*) \\ &\geq \left(1 - e^{-I_\nu(\tilde{r})n_{\mathcal{C}_1} + o(n_{\mathcal{C}_1})}\right)(T_i(r^*) - T(r^*)) \end{aligned}$$

Since $T_i(r^*) - T(r^*) > 0$ for any receiver $i \in \mathcal{C}_k$ with $k \neq 1$, by combining the above equation with Equation (28), we can obtain Equation (27), which completes the whole proof.

D. PROOF OF LEMMA 1

We prove by induction. Initially, $r_L = r_{\min}, r_U = r_{\max}$ and thus Equation (13) trivially holds.

Suppose Equation (13) holds for the previous iteration. According to Algorithm 1, in the cases of $a_L = 1, a_U = 1$ and $a_L = 0, a_U = 0$, r_L, r_U are not updated and thus Equation (13) would still hold for the new iteration. We only need to focus on the other two cases when $a_L = 1, a_U = 0$ and $a_L = 0, a_U = 1$. Since the two cases are symmetric, without loss of generality, we consider one of the cases, $a_L = 1, a_U = 0$. There are two possibilities, when the throughput c_M can be achieved by the network and when the throughput c_M is not achievable for the network.

Case 1: the throughput c_M can be achieved by the network.

When the throughput c_M can be achieved by the network, by the property shown with Equation (12), we have $L_{c_M}(T) \neq \emptyset$ and $r^* \in L_{c_M}(T)$. By the assumption of induction, $r_L \leq r^* \leq r_U$. Thus, it follows that

$$L_{c_M}(T) \cap [r_L, r_U] \neq \emptyset. \tag{29}$$

By the design of Anonymous Queries in Algorithm 1, $a_L = 1$ implies that, there exists some receiver i such that

$$L_{c_M}(T_i) \cap [r_L, r_M] = \emptyset. \tag{30}$$

From Equations (12) and (30), we have

$$L_{c_M}(T) \cap [r_L, r_M] = \emptyset, \tag{31}$$

which implies $r^* \notin [r_L, r_U]$. Together with our assumption that $r^* \in [r_L, r_U]$, it follows that $r^* \in [r_M, r_U]$. Since in the new interation $r_L := r_M$, Equation (13) still holds in this case.

Case 2: the throughput c_M cannot be achieved by the network.

When the throughput c_M is not achievable for the network, by the property shown with Equation (12), it follows that $L_{c_M}(T) = \emptyset$.

By the design of Anonymous Queries in Algorithm 1, $a_U = 0$ equivalently implies that, for any receiver $1 \leq i \leq n$, we have

$$L_{c_M}(T_i) \cap [r_M, r_U] \neq \emptyset, \tag{32}$$

Since $T_i(r)_{1\leq i \leq n}$ is a unimodal function in r, we can express the superlevel set as an closed interval

$$L_c(T_i) \cap [r_L, r_U] \triangleq \left[J_i^{st}(c), J_i^{end}(c)\right], \forall\, 0 \leq c \leq c_M, \tag{33}$$

where $J_i^{st}(c)$ and $J_i^{end}(c)$ correspond to the starting and end points of the closed interval $L_c(T_i) \cap [r_L, r_U]$. Note that $J_i^{st}(c)$ and $J_i^{end}(c)$ are well defined for $0 \leq c \leq c_M$, since $L_c(T_i) \cap [r_L, r_U] \neq \emptyset$ from Equation (32). Besides, $J_i^{st}(c)$ is non-decreasing and $J_i^{end}(c)$ is non-increasing with respect to c. Since for any $1 \leq i \leq n$, $T_i(r)$ is a continuous function of r, by the definition in Equation (33), $J_i^{st}(c)$ and $J_i^{end}(c)$ are continuous functions of c when $0 \leq c \leq c_M$.

For $0 \leq c \leq c_M$, define $J^{st}(c) \triangleq \max_{1\leq i\leq n} J_i^{st}(c)$ and $J^{end}(c) \triangleq \min_{1\leq i\leq n} J_i^{end}(c)$. If $J^{st}(c_M) \leq J^{end}(c_M)$, it follows that $J^{st}(c_M) \in L_{c_M}(T)$ contradicting with the assumption that $L_{c_M}(T) = \emptyset$. Thus, we must have $J^{st}(c_M) > J^{end}(c_M)$. From Equations (32)(33), $J_i^{end}(c_M) \geq r_M$ for any receiver $1 \leq i \leq n$. It follows that

$$J^{end}(c_M) \geq r_M. \tag{34}$$

Since c_M is not an achievable throughput by our assumption, we have $T(r^*) < c_M$. Since $T(r)$ is a unimodal function with the unique r^* which achieves the maximum $T(r^*)$, we have $J^{st}(T(r^*)) = J^{end}(T(r^*)) = r^*$. By the monotonic property of $J^{end}(c)$ and Equation (34), $J^{end}(T(r^*)) \geq J^{end}(c_M) \geq r_M$. Thus, we have $r^* = J^{end}(T(r^*)) \geq r_M$. Combining with the assumption that $r^* \in [r_L, r_U]$, it follows that $r^* \in [r_M, r_U]$. Note that in the new iteration $r_L := r_M$, Equation (13) also holds in this case.

By induction, the proof is complete.

E. PROOF OF LEMMA 2

We prove by induction. Initially, $c_L = 0, c_U = r_{\max}, r_F = r_{\min}$. From Equations (1) and (2), Equations (14) and (15) must hold.

Suppose Equations (14) and (15) holds for the previous iteration. According Algorithm 1, in the cases of $a_L = 1, a_U = 0$ and $a_L = 0, a_U = 1$, c_L, c_U, r_F are not updated and thus Equations (14) and (15) would still hold for the new iteration. We only need to focus on the other two cases when $a_L = 0, a_U = 0$ and $a_L = 1, a_U = 1$.

Case 1: $a_L = 0, a_U = 0$.

Now we discuss the case that $a_L = 0, a_U = 0$. By the design of Anonymous Queries in Algorithm 1, $a_L = 0, a_U = 0$ equivalently implies that for any $1 \leq i \leq n$,

$$L_{c_M}(T_i) \cap [r_L, r_M] \neq \emptyset, \tag{35}$$

$$L_{c_M}(T_i) \cap [r_M, r_U] \neq \emptyset. \tag{36}$$

Since $T_i(r)_{1\le i\le n}$ is a unimodal function in r, a non-empty superlevel set of $T_i(r)$ must be an interval. From Equations (35) and (36), it follows that

$$r_M \in L_{c_M}(T_i), \forall 1 \le i \le n. \tag{37}$$

Thus, $\bigcap_{i=1}^n L_{c_M}(T_i) \neq \emptyset$. By Equation (12), c_M is achievable for the network. Thus, $c_M \le T(r^*) \le c_U$. Since in the new iteration, $c_L := c_M$, Equation (14) still holds. Also, in the new iteration, $r_F := r_M$. By Equation (37), we have $T(r_M) \ge c_M$. Together with the fact that $T(r_M) \le T^* \le c_U$, Equation (15) holds for the new iteration as well.

Case 2: $a_L = 1, a_U = 1$.

Next we discuss the case that $a_L = 1, a_U = 1$. By the design of Anonymous Queries in Algorithm 1, $a_L = 1, a_U = 1$ equivalently means that there exist some $i_1, i_2 \in \{1, ..., n\}$ (i_1 might be equal to i_2.) such that

$$L_{c_M}(T_{i_1}) \cap [r_L, r_M] = \emptyset, \tag{38}$$

$$L_{c_M}T_{i_2}) \cap [r_M, r_U] = \emptyset. \tag{39}$$

It follows that,

$$\left(\bigcap_{i=1}^n L_{c_M}(T_i)\right)\bigcap [r_L, r_U] \subset L_{c_M}(T_{i_1}) \cap L_{c_M}(T_{i_2}) \cap ([r_L, r_M] \cup [r_M, r_U]) = \emptyset \tag{40}$$

By Lemma 1, we have $r_L \le r^* \le r_U$. If c_M is achievable for the network, $r^* \in \bigcap_{i=1}^n L_{c_M}(T_i)$ due to the property shown with Equation (12). However, this contradicts Equation (40). Hence, c_M is not achievable for the network. $c_L \le T(r^*) \le c_M$.

In the new iteration, $c_U := c_M$, by the conclusion we draw above, Equation (14) still holds. Notice that r_F, C_L are not updated. Together with the fact that $T(r_F) \le T(r^*) \le c_M$, Equation (15) holds for the new iteration as well.

By induction, the proof is complete.

F. PROOF OF LEMMA 3

By the definition of $N_{\mathcal{C}}^t(r)$,

$$\mathbb{P}\left(\tilde{r}N_{\mathcal{C}_1}^t(\tilde{r}) \ge r^* N_{\mathcal{C}_1}^t(r^*) + n_{\mathcal{C}_1}\nu\right) \tag{41}$$

$$=\mathbb{P}\left(\sum_{i\in\mathcal{C}_1}\left(\tilde{r}1_{R_i[t]\ge\tilde{r}} - r^*1_{R_i[t]\ge r^*} - \nu\right) \ge 0\right). \tag{42}$$

Notice that $\{\tilde{r}1_{R_i[t]\ge\tilde{r}} - r^*1_{R_i[t]\ge r^*} - \nu\}_{i\in\mathcal{C}_1}$ are *i.i.d.* random variables and $T_i(r^*) > T_i(\tilde{r})$ for $i \in \mathcal{C}_1$, we have

$$\mathbb{E}(\tilde{r}1_{R_i[t]\ge\tilde{r}} - r^*1_{R_i[t]\ge r^*} - \nu)$$
$$=\tilde{r}\mathbb{P}(R_i[t] \ge \tilde{r}) - r^*\mathbb{P}(R_i[t] \ge r^*) - \nu < -\nu < 0. \tag{43}$$

According to the Cramer's Theorem (see Theorem 2.1.24 in [22]), there exists $I_\nu(\tilde{r}) > 0$ such that

$$\mathbb{P}\left(\sum_{i\in\mathcal{C}_1}\left(\tilde{r}1_{R_i[t]\ge\tilde{r}} - r^*1_{R_i[t]\ge r^*} - \nu\right) \ge 0\right) = e^{-I_\nu(\tilde{r})n_{\mathcal{C}_1} + o(n_{\mathcal{C}_1})}, \tag{44}$$

where $I_\nu(\tilde{r})$ is the rate function defined in Equation (21). Thus, Equation (21) is proved.

When $\nu = 0$, we prove the case when $\tilde{r} > r^*$, and the case when $\tilde{r} < r^*$ can be proved in the same way. Combining $\tilde{r} > r^*$ with Equation (21), we have

$$I_0(\tilde{r}) = \sup_{\theta\in\mathbb{R}^1}\Big\{-\log\Big(\mathbb{P}(R_i[t] \ge \tilde{r})e^{-\theta(\tilde{r}-r^*)} + \mathbb{P}(r^* \le R_i[t] < \tilde{r})e^{\theta r^*} + \mathbb{P}(R_i[t] < r^*)\Big)\Big\}. \tag{46}$$

To find the supreme of Equation (46), we let its first derivative to be zero. There is a unique θ_0^* where the supreme in Equation (46) is achieved, given by

$$\theta_0^* = -\frac{1}{\tilde{r}}\log\frac{\mathbb{P}(r^* \le R_i[t] < \tilde{r})r^*}{\mathbb{P}(R_i[t] \ge \tilde{r})(\tilde{r} - r^*)}. \tag{47}$$

Inserting Equation (47) back to Equation (46), when $\tilde{r} > r^*$, Equation (22) is proved.

It is easy to verify that

$$\mathbb{P}(r^* \le R_{\mathcal{C}_1} < \tilde{r})^{1-\frac{r^*}{\tilde{r}}}\mathbb{P}(R_{\mathcal{C}_1} \ge \tilde{r})^{\frac{r^*}{\tilde{r}}}$$
$$\le\left(1 - \frac{r^*}{\tilde{r}}\right)^{1-\frac{r^*}{\tilde{r}}}\left(\frac{r^*}{\tilde{r}}\right)^{\frac{r^*}{\tilde{r}}}\left(\mathbb{P}(r^* \le R_{\mathcal{C}_1} < \tilde{r}) + \mathbb{P}(R_{\mathcal{C}_1} \ge \tilde{r})\right), \tag{48}$$

where the equality holds when $\tilde{r}\mathbb{P}(R_{\mathcal{C}_1} \ge \tilde{r}) = r^*\mathbb{P}(R_{\mathcal{C}_1} \ge r^*)$. Since $T(r^*) > T(\tilde{r})$, the equality in Equation (48) does not hold.

Combining Equations (48) and (22), it follows that the decay rate is strictly positive, $I_0(\tilde{r}) > 0$.

Lastly, we prove Equation (23). Again, we prove the case when $\tilde{r} > r^*$, and the case when $\tilde{r} < r^*$ can be proved in the same way. Combining $\tilde{r} > r^*$ with Equation (21), we have

$$I_\nu(\tilde{r}) = \sup_{\theta\in\mathbb{R}^1}\Big\{-\log\Big(\mathbb{P}(R_i[t] \ge \tilde{r})e^{-\theta(\tilde{r}-r^*-\nu)} + \mathbb{P}(r^* \le R_i[t] < \tilde{r})e^{\theta(r^*+\nu)} + \mathbb{P}(R_i[t] < r^*)e^{\theta\nu}\Big)\Big\}. \tag{49}$$

Let θ_ν^* be the θ where the supreme of Equation (49) is achieved, denoted as $\theta_\nu^* = \Delta\theta_\nu + \theta_0^*$. The first derivative of Equation (49) on θ_ν^* is zero, i.e.,

$$-(\tilde{r} - r^* - \nu)\mathbb{P}(R_i[t] \ge \tilde{r})e^{-\theta_\nu^*(\tilde{r}-r^*-\nu)} + (r^* + \nu)\mathbb{P}(r^* \le R_i[t] < \tilde{r})e^{\theta_\nu^*(r^*+\nu)} - \nu\mathbb{P}(R_i[t] < r^*)e^{\theta_\nu^*\nu} = 0. \tag{50}$$

Noting Equation (50) holds for θ_0^* when $\nu = 0$. From Equation (50), we could derive Equation (45).

To simplify Equation (45), in the following, we use H to denote

$$H \triangleq r^{*\,1-\frac{r^*}{\tilde{r}}}(\tilde{r} - r^*)^{\frac{r^*}{\tilde{r}}}\mathbb{P}(R_{\mathcal{C}_1} \ge \tilde{r})^{\frac{r^*}{\tilde{r}}}\mathbb{P}(r^* \le R_{\mathcal{C}_1} < \tilde{r})^{1-\frac{r^*}{\tilde{r}}}. \tag{51}$$

Clearly, H is independent of ν.

On the other hand, by Equation (50), if $\Delta\theta_\nu^* \ge 0$, for any $0 \le \nu < \tilde{r} - r^*$, we have

$$e^{\theta_\nu^* r^*} = \frac{(\tilde{r} - r^* - \nu)\mathbb{P}(R_i[t] \ge \tilde{r})e^{-\theta_\nu^*(\tilde{r}-r^*)} + \nu\mathbb{P}(R_i[t] < r^*)}{(r^* + \nu)\mathbb{P}(r^* \le R_i[t] < \tilde{r})}$$
$$\overset{(a)}{\le} \frac{(\tilde{r} - r^*)\mathbb{P}(R_i[t] \ge \tilde{r})e^{-\theta_0^*(\tilde{r}-r^*)} + (\tilde{r} - r^*)\mathbb{P}(R_i[t] < r^*)}{r^*\mathbb{P}(r^* \le R_i[t] < \tilde{r})}$$
$$\triangleq A_+, \tag{52}$$

where step (a) uses the assumption that $\theta_\nu^* = \Delta\theta_\nu + \theta_0^* \ge \theta_0^*$.

$$r^{*1-\frac{r^*}{\tilde{r}}}(\tilde{r}-r^*)^{\frac{r^*}{\tilde{r}}}\mathbb{P}(R_{\mathcal{C}_1}\geq\tilde{r})^{\frac{r^*}{\tilde{r}}}\mathbb{P}(r^*\leq R_{\mathcal{C}_1}<\tilde{r})^{1-\frac{r^*}{\tilde{r}}}\left(e^{\Delta\theta_\nu r^*}-e^{-\Delta\theta_\nu(\tilde{r}-r^*)}\right)$$
$$=\nu\left(\mathbb{P}(R_i[t]<r^*)-\mathbb{P}(r^*\leq R_i[t]<\tilde{r})e^{\theta_\nu^* r^*}-\mathbb{P}(R_i[t]\geq\tilde{r})e^{-\theta_\nu^*(\tilde{r}-r^*)}\right)\leq\nu\mathbb{P}(R_i[t]<r^*)\triangleq\nu B_0. \tag{45}$$

It follows from Equations (45) and (52) that

$$\begin{aligned}&H\left(e^{\Delta\theta_\nu r^*}-e^{-\Delta\theta_\nu(\tilde{r}-r^*)}\right)\\ \geq&\nu\left(-\mathbb{P}(r^*\leq R_i[t]<\tilde{r})A_+-\mathbb{P}(R_i[t]\geq\tilde{r})e^{-\theta_0^*(\tilde{r}-r^*)}\right)\\ \triangleq&-\nu B_1.\end{aligned} \tag{53}$$

By Equation (50), if $\Delta\theta_\nu^*<0$, for any $0\leq\nu<\frac{1}{2}(\tilde{r}-r^*)$, we have

$$\begin{aligned}e^{-\theta_\nu^*(\tilde{r}-r^*)}=&\frac{(r^*+\nu)\mathbb{P}(r^*\leq R_i[t]<\tilde{r})e^{\theta_\nu^* r^*}-\nu\mathbb{P}(R_i[t]<r^*)}{(\tilde{r}-r^*-\nu)\mathbb{P}(R_i[t]\geq\tilde{r})}\\ \stackrel{(a)}{\leq}&\frac{(r^*+\tilde{r})\mathbb{P}(r^*\leq R_i[t]<\tilde{r})e^{\theta_0^* r^*}}{(\tilde{r}-r^*)\mathbb{P}(R_i[t]\geq\tilde{r})}\triangleq A_-,\end{aligned} \tag{54}$$

where step (a) uses the assumption that $\theta_\nu^*=\Delta\theta_\nu+\theta_0^*<\theta_0^*$. It follows from Equations (45) and (54) that

$$\begin{aligned}&H\left(e^{\Delta\theta_\nu r^*}-e^{-\Delta\theta_\nu(\tilde{r}-r^*)}\right)\\ \geq&\nu\left(-\mathbb{P}(r^*\leq R_i[t]<\tilde{r})e^{\theta_0^* r^*}-\mathbb{P}(R_i[t]\geq\tilde{r})A_-\right)\triangleq-\nu B_2.\end{aligned} \tag{55}$$

From Equations (45), (53) and (55), for small enough $\nu<\frac{1}{2}(\tilde{r}-r^*)$, there exists $A=\max\{B_0,B_1,B_2\}>0$ independent of ν such that

$$\left|H\left(e^{\Delta\theta_\nu r^*}-e^{-\Delta\theta_\nu(\tilde{r}-r^*)}\right)\right|\leq\nu A, \tag{56}$$

Notice that the left side of Equation (56) is a continuous function of $\Delta\theta_\nu$ which is equal to 0 when and only when $\Delta\theta_\nu=0$. By Equation (56), as $\nu\to0^+$, $\Delta\theta_\nu\to0$ and $\theta_\nu^*\to\theta_0^*$.

Together with Equation (49), Equation (23) is proved.