Optimal Energy-Aware Epidemic Routing in DTNs

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ABSTRACT

In this work, we investigate the use of epidemic routing in energy constrained Delay Tolerant Networks (DTNs). In DTNs, connected paths between source and destination rarely occur due to sparse density and mobility of nodes. Epidemic routing is ideally suited in this environment for its simplicity and fully distributed implementation. In epidemic routing, messages are relayed by intermediate nodes through contact opportunities, i.e., when pairs of nodes come within transmission range. Each node needs to decide whether to forward its message upon contact with a new node based on its residual energy level and the age of that message.

We mathematically characterize the fundamental trade-off between energy conservation and forwarding efficacy as a heterogeneous dynamic energy-dependent optimal control problem. We prove, somewhat surprisingly given the complex nature of the problem, that in the mean field regime, the optimal dynamic forwarding decisions follow simple threshold-based structures, in which the forwarding threshold for each node depends on its current remaining energy. We analytically establish this result under generalized classes of utility functions for DTNs. We then characterize the nature of the dependence of these thresholds on the value of the current remaining energy reserves in each node.

1. INTRODUCTION

Motivation

Delay Tolerant Networks (DTNs) are comprised of spatially distributed mobile nodes whose communication range is much smaller than their roaming area; hence, end-to-end connectivity is rare. In such networks, messages are typically relayed by intermediate nodes through random contacts, which are instances of spatial proximity of pairs of nodes. Specifically, a time-stamped message from a source node can flood the network for a chance to contact its destination node [18] within the time frame of relevance of the message.

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Examples of DTNs include disaster response and military/tactical networks where communication devices are carried by disaster relief personnel and soldiers and environmental surveillance, where sensors can be mounted on roaming animals. Opportunistic networking based on DTNs is envisioned to assist ad hoc or infrastructure based communication in next generation networks whenever end-to-end connectivity is hard to achieve. In most of these cases, the intermediate mobile nodes are constrained in their battery reserves. Simple epidemic forwarding schemes that rely on flooding as a method of message propagation may have a detrimental impact on the energy reserves of the intermediate nodes, which can adversely affect the performance of the network. A depleted node, or one left with critically low battery reserve, will not be able to relay new messages in the future. This reduction in the number of relay nodes will in turn undermine the long term throughput of the network, specially for time sensitive messages. On the other hand, an overly conservative strategy would compromise the delivery of the message to the destination in a timely manner. Hence, there is an inherent trade-off between delay-sentive message throughput and energy conservation. In such networks, replacing/recharging the batteries of drained nodes is usually infeasible and/or not cost-efficient. Also, in practice, nodes have distinct levels of remaining energy, hence a forwarding decision that is optimal for a node may be harmful for another. Each node can readily measure its own remaining energy at any given time. The trade-off between energy conservation in each node and message throughput, along with the heterogeneity of the remaining battery reserves of the nodes motivates heterogeneous energy-dependent control, which is the subject of this paper. The state of the art either ignores energy constraints [4, 11, 18], or does not directly utilize current energy-state information in making forwarding decisions [1, 5, 13, 16].

Contributions

We formulate the trade-off between energy conservation and forwarding efficacy as a dynamic energy-dependent optimal control problem: at any given time, each node decides on its forwarding probability based on its current remaining energy. Since the residual energy decreases with transmissions and receptions, the forwarding decisions vary with time. They must, therefore, be determined so as to control the evolution of network states that capture the fraction of nodes holding a copy of the message and the remaining battery reserves of the nodes. We consider two generalized classes of objective functions. These objective functions characterize metrics for end-to-end quality of service and general penalty functions for the final distribution of energies. These functions are defined in the context of an epidemiological model based on mean-field approximation of Markov processes to represent the evolution of the network states (§§2.1, 2.2).

Our main contribution is to prove that dynamic optimal strategies follow simple threshold-based rules (§3, Theorem 1). That is, a node in possession of a copy of the message forwards the message to nodes it encounters that have not yet received it until a certain threshold time that depends on its current remaining energy. As the node forwards the message, it loses energy, and its forwarding time threshold changes accordingly. If the age of the message is past a threshold time corresponding to its current level of energy, the node stops forwarding the message to others, and will only transmit a copy to the destination of the message. The simplicity of the analytically-derived strategies is somewhat surprising given that the system dynamics involve non-linear transitions and a vector of controls, and our objective functions are non-linear and time-dependent (the system is therefore *non-autonomous*). The proofs for optimality of threshold type policies in such cases do not follow from existing optimal control results.

Our third contribution is to characterize the nature of the dependence of the thresholds on the energy levels. Intuitively, the less energy a node has, the more reluctant it should be to transmit the message, as the transmission will drive it closer to critically low battery levels (which in turn will impair timely delivery of future messages). However, surprisingly, our investigations reveal that this intuition can only be confirmed when the penalty associated with low final remaining energies is convex (§3, Theorem 2), and it does not hold in general otherwise. That is, in the former case, higher remaining energy levels lead to longer durations of forwarding, but the monotonicity of the thresholds is not necessarily preserved otherwise.

Finally, our optimal dynamic policy provides a missing *benchmark* for forwarding policies in large networks in which no information about the mobility pattern of the individual nodes is available. This benchmark allows us to identify some even simpler heuristic policies that perform close to the optimal, and also those that substantially compromise performance for simplicity (§4).

Related Literature

Heuristic based routing policies for DTNs involving mobile nodes are proposed in [3,4,11,12,14,17]. In the routing protocol PROPHET introduced in [11], each node maintains a vector of probabilities of delivery and the message is forwarded from lower delivery probability nodes to nodes with higher probability of delivery. Heuristics that take energy consumption into account include NECTAR, proposed in [5], which tries to find a desirable path based on the contact history of the nodes, [3], which proposes the introduction of fixed nodes ("throwboxes") for energy efficient routing, and Spray and Wait [17], which proposes spreading a specific number of copies of the message initially and then waiting for delivery. Encounter-based Routing [14] builds upon the above by having nodes make the decision to spread the limited number of copies of a message contingent on the contact history of an encountered node. Finally, [12] limits transmissions to times when a node has a minimum number of neighbours, which is of limited use when contacts are sparse. Such protocols may lead to large calculations and messaging overhead in the network, while also providing no analytical guarantee on the QoS or energy usage.

In [10], Krifa *et al.* consider another limitation of DTNs: the storage buffers of nodes. Here, if the storage capacities are not constrained, then the best policy is to keep a copy of *all* of the messages (to be delivered to their destinations upon future contact) before they are dropped at expiration of their time-to-live value (TTL). However, if the storage buffer is full when a new message arrives, a decision needs to be made about which message (from the set of the existing messages in the buffer plus the newly arrived one) should be dropped. The decision rule is referred to as a *buffer management* policy. The paper derives policies that approximately optimize for average delay and average throughput.

The problem of finding optimal dynamic forwarding policies in DTNs considering the resource overhead of replications has been investigated in [1,2,13,16,19] among others. These papers either impose an indirect constraint, e.g. restricting the total number of copies of the message in the network to control the energy overhead, or directly consider a cost for the overall energy usage. However, the forwarding rules in these papers do not utilize the current energy levels of the nodes, and are identical for all nodes irrespective of their remaining energy levels. Specifically, the parts of their objective functions that consider the energy overhead only represent the total units of energy used during the process of forwarding copies of a message before its TTL is expired, and the solution is indifferent to the distribution of the residual energy reserves. In the state of the art models, if a node has started with a low energy, or has lost a large portion of its battery reserves during multiple transmissions, it still has to abide by the general rule that is identical for all nodes. However, it is of importance to the network how the aggregate remaining energy is distributed among the nodes, as nodes with critically low remaining energy will compromise the long run performance of the network. Motivated by these two observations, we propose a new framework that yields optimal forwarding policies attaining custom trade-offs between QoS and the desirability of the distribution of the residual energy reserves among the nodes.

2. SYSTEM MODEL

In §2.1, we develop our system dynamics model based on mean-field deterministic ODEs. Subsequently, in §2.2 we consider two general classes of utility functions that cogently combine a measure of QoS with a penalty for the impact of the policy on the residual energy of the nodes. We present the model for a *single-delivery* setting. Particularly, each message is destined for a single destination and it is sufficient for only one copy of the message to be delivered to its destination.[1]

[1]The single-delivery setting can also capture cases where there are multiple destinations that are, unlike the source and the intermediate nodes, inter-connected through a backbone network. An example of such a setting is where the final destination is (a group of) base stations in a cellular network. Note that in the latter case there is no additional benefit in delivering more than one copy of the message to one of the destinations, since once one destination receives the message, it can instantly inform the other destinations of its reception using the high-speed backbone network. So we can assume all destination nodes receive the message virtually simultaneously. The only modification in our modeling would be a re-scaling of the rate of contact between

2.1 System Dynamics

We begin with some definitions: a node that has received a copy of the message and is not its destination is referred to as an *infective*; a (non-destination) node that has not yet received a copy of the message is called a *susceptible*. The maximum energy capacity of a node is B units for each node. Transmission of the message between a pair of nodes consumes τ units of available energy in the transmitter and r units in the receiver. Naturally, $r \leq \tau$. When an infective node contacts a susceptible at time t, the message is transmitted with a certain forwarding probability if the infective (transmitter) and susceptible (receiver) have at least τ and r units of reserve energy, respectively.

Two nodes contact each other at rate $\hat{\beta}$. We assume that inter-contact times are exponentially distributed and uniform among nodes, an assumption common to many mobility models (e.g. Random Walker, Random Waypoint, Random Direction, etc. [7]). Moreover, it is shown in [7] that

$$\hat{\beta} \propto \frac{\text{average relative speed of nodes} \times \text{communication ranges}}{\text{the roaming area}} \tag{1}$$

We define $S_i(t)$ (resp., $I_i(t)$) to be the *fraction* of the susceptible (resp., infective) nodes that have i energy units at time t. Hence, for $t \in [0, T]$: $\sum_{i=0}^{B}(S_i(t) + I_i(t)) = 1$.

At any given time, each node can directly observe its own level of available energy, and its forwarding decision should, in general, utilize such information. Hence, upon an instance of contact between a susceptible node with i units of energy and an infective node with j units of energy at time t, as long as $i \geq r$ **and** $j \geq \tau$, the message is passed with probability $u_i(t)$ ($0 \leq u_i \leq 1$). Consequently, the susceptible node transforms to an infective node with $i - r$ units of energy, and the infective node to an infective node with $j - \tau$ units of energy. We assume that upon contact between an infective and another node, the infective can identify (through a low-load exchange of control messages) whether the other node has a copy of the message (i.e., is infective), or does not (i.e., is susceptible), and also whether the contacted node is a destination. We assume that each instance of such exchanges consumes an insignificant amount of energy.

Let N be the total number of nodes and define $\beta := N\hat{\beta}$. Following (1), $\hat{\beta}$ is inversely proportional to the roaming area, that scales with N. Hence, if we can define a density of nodes, β has a nontrivial value. The system dynamics in the mean-field regime (i.e., for large N) over any finite interval can be approximated as follows ([6, Theorem 1]):

$$\dot{S}_i = -\beta S_i \sum_{j=\tau}^{B} u_j I_j \qquad r \leq i \leq B \tag{2a}$$

$$\dot{I}_i = -\beta u_i I_i \sum_{j=r}^{B} S_j \qquad B - r < i \leq B \tag{2b}$$

$$\dot{I}_i = \beta S_{i+r} \sum_{j=\tau}^{B} u_j I_j - \beta u_i I_i \sum_{j=r}^{B} S_j \quad B - \tau < i \leq B - r \tag{2c}$$

$$\dot{I}_i = \beta S_{i+r} \sum_{j=\tau}^{B} u_j I_j + \beta u_{i+\tau} I_{i+\tau} \sum_{j=r}^{B} S_j - \beta u_i I_i \sum_{j=r}^{B} S_j \qquad \tau \leq i \leq B - \tau \tag{2d}$$

a mobile node and the destination by the number of these inter-connected sinks (Base Stations).

$$\dot{S}_i = 0 \qquad i < r, i = 0 \tag{2e}$$

$$\dot{I}_i = \beta S_{i+r} \sum_{j=\tau}^{B} u_j I_j + \beta u_{j+\tau} I_{i+\tau} \sum_{j=r}^{B} S_j \qquad i < \tau \tag{2f}$$

Note that in the above differential equations and in the rest of the paper, whenever not ambiguous, the dependence on t is made implicit. Each differential equation of the above set is explained in the following:[2]

(2a): The rate of decrease in the fraction of susceptible nodes with energy level i is proportional to the rate of contacts between those nodes and the infective nodes with energy levels equal to or higher than τ.

(2b): The rate of decrease in the fraction of infective nodes with energy level i such that $i > B - r$ is proportional to their rate of contact with any susceptible with more than r units of energy. No susceptible or infective can be transformed to an infective with such a high level of energy.

(2c): Similarly, the rate of change in the fraction of infectives with energy level i such that $B - \tau < i \leq B - r$ is due to the transformation of susceptibles with energy level $i + r$ upon contact with infectives that have τ units of energy, along with the mechanism in (2b). No infective can be transformed to an infective of such a high energy level.

(2d): This is the non-marginal equation for the evolution of the infectives. Here, three mechanisms are in place: (2b),(2c) and one more: infectives of energy level $i + \tau$ convert to infectives with energy level i upon contact with susceptibles that have sufficient energy for message exchange.

(2e): Susceptibles with less than r units of energy cannot convert to infectives.

(2f): Infectives with less than τ units of energy cannot convert to any other type.

The initial conditions are

$$\mathbf{S}(0) = \mathbf{S}_0 := (S_{01}, \ldots, S_{0B}) \ \& \ \mathbf{I}(0) = \mathbf{I}_0 := (I_{01}, \ldots, I_{0B}) \tag{3}$$

and the state constraints are

$$\mathbf{S} \succeq 0, \quad \mathbf{I} \succeq 0, \quad \sum_{i=0}^{B}(S_i(t) + I_i(t)) = 1, \quad \forall t \in [0, T]. \tag{4}$$

2.2 Objective functions

The objective function of the network represents both a measure of the efficacy of the policy in ensuring the timely delivery of the message and the effect of the policy on the residual energy reserves of the nodes. In what follows, we first develop each of the components of the objective function separately, and then we combine them to yield the overall objective function of the network.

Measure of timely delivery. One plausible measure of QoS in the context of the DTN is to maximize the probability of the delivery of a message to the destination before a deadline time T. At time T, which is the time after which the message is irrelevant, all infectives will drop the message.

[2]The system dynamics for the single-delivery case can ignore the single instance of the delivery of the message to the destination. This is because in the mean-field regime, i.e. for large N, and when the state is represented as the fraction of nodes of each type, the change in the energy distributions as a result of a single transmission of the message is negligible. Note that in the single-delivery scenario, once the destination receives the message, subsequent contacts between infectives and the destination will not result in any transmission of the message.

We will go further than just representing the probability of delivery in this fixed time window in that, within this time interval, we assign more reward for earlier message delivery. In an alternative scenario, the time-frame of delivery of the message is flexible, but instead, a minimum probability of delivery is mandated on the message. In this case, the goal is to meet this requirement as soon as possible. In what follows, we formally present these two cases.

Let $t = 0$ mark the moment of message generation. The network achieves $g(t)$ units of reward if a copy of the message is delivered to the destination at time t, where $0 \leq t \leq T$. There is no reward for deliveries later than T, hence $g(t)$ can be taken to be zero for $t > T$. $g(t)$ is a non-increasing function of t over $[0, T]$, since we associate more reward for earlier delivery of the message. That is, the sooner the message is delivered, the better. We also assume $g(t)$ to be differentiable; hence $g'(t) \leq 0$ for $0 < t < T$.

Mathematically, let the random variable σ represent the time at which a copy of the message is delivered to the destination. The reward associated with the delivery of the message can be represented by the random variable $g(t)\mathbf{1}_{\sigma=t}$, where $\mathbf{1}$ is the indicator function. Let $\hat{\beta}_0$ be the rate of contact of a node with the destination node, potentially different from $\hat{\beta}$, and define $\beta_0 := N\hat{\beta}_0$. Following the exponential distribution of the inter-contact times, the *expected reward*, R, to be maximized is given by:

$$R := \mathbb{E}\{g(t)\mathbf{1}_{\sigma=t}\} = \int_0^T g(t)\mathbb{P}(\sigma = t)\, dt =$$

$$\int_0^T g(t) \exp\left(-\hat{\beta}_0 \int_0^t \sum_{i=\tau}^{B} N I_i(\xi)\, d\xi\right) \cdot \hat{\beta}_0 \sum_{i=\tau}^{B} N I_i(t)\, dt.$$

Note that similar to (1), $\hat{\beta}_0$ is inversely proportional to the roaming area, which itself scales with N. Theferore, as long as we can define a meaningful density (number of nodes divided by the total roaming area of nodes), this probability is nontrivial. Another point to notice is that the summation inside the integral starts from index τ, since infective nodes with less than τ units of energy cannot forward their message to the destination upon potential contact. In order to change the form of the integration to something more conducive to analysis, we use integration by parts:

$$R = g(0) - g(T)e^{-\beta_0 \int_0^T \sum_{i=\tau}^{B} I_i(t)\, dt} + \int_0^T g'(t)e^{-\beta_0 \int_0^t \sum_{i=\tau}^{B} I_i(\xi)\, d\xi}\, dt.$$

The alternative measure of timely delivery of the message, instead of maximizing the probability of the delivery within a given time interval, involves the stricter notion of enforcing a minimum probability of delivery but in a flexible window of time. Subject to this probability, the goal is to minimize a penalty associated with the time it takes to satisfy such a requirement (along with the adverse effects on the residual energy of the nodes, which we will discuss next). This can be interpreted as an *optimal stopping time* problem. In what follows, we mathematically represent this alternative framework. The constraint $\mathbb{P}(\text{delivery}) \geq p$ in our case is: $1 - \exp\left(-\int_0^T \beta_0 \sum_{i=\tau}^{B} I_i(t)\, dt\right) \geq p$, which is equivalent to:

$$\int_0^T \sum_{i=\tau}^{B} I_i(t)\, dt \geq -\ln(1-p)/\beta_0. \tag{5}$$

Let us represent the cost associated with the time T it takes to satisfy the above *throughput constraint* in general to be $f(T)$. The only necessary property for $f(T)$ to be a meaningful penalty function for *delay* is that it should be *non-decreasing* in T. We further assume that $f(t)$ is differentiable w.r.t T (hence, $f'(T) \geq 0$).

Energy cost of the policy. In the simplest representation of the trade-off with the energy overhead, one can think of maximizing the aggregate remaining energy in the network at T, irrespective of how it is distributed. But, as we mentioned in the introduction, it is desirable for the network to avoid creating nodes with critically low energy reserves. Specifically, nodes with lower residual energy can contribute in relaying and/or generating future messages for shorter durations. In the extreme case, a sizable fraction of depleted nodes can gravely jeopardies the functionality of the network. We capture the impact of a forwarding policy on the residual energy reserves of the nodes by penalizing the nodes that have lower energy levels. Specifically, the overall penalty associated with the distribution of the residual energies of the nodes is captured by: $\sum_{i=0}^{B} a_i \left(S_i(T) + I_i(T)\right)$, in which, a_i is a *decreasing* sequence in i, i.e., a higher penalty is associated with lower residual energies at T.

The trade-offs should now be clear: by using a more aggressive forwarding policy (i.e., higher $u_i(t)$s and for longer durations), the message propagates at a faster rate and hence there is a greater chance of delivering the message to the destination earlier. However, this will lead to lesser overall remaining energy in the nodes upon delivery of the message, and it will potentially push the energy reserves of some nodes toward critically low levels, which can degrade future performance of the network.

Overall Objective and Problem Statements. The overall utility of the system is a (weighted) summation of the above two components (a measure of quality of service along with the effect of the policy on residual energies). We now concisely state the two optimization problems.

Problem 1: Fixed Terminal Time The system seeks to maximize the following overall utility function:

$$R = g(0) - g(T)e^{-\beta_0 n_D \int_0^T \sum_{i=\tau}^{B} I_i(t)\, dt} \tag{6}$$

$$+ \int_0^T g'(t)e^{-\beta_0 n_D \int_0^t \sum_{i=\tau}^{B} I_i(\xi)\, d\xi}\, dt - \sum_{i=0}^{B} a_i \left(S_i(T) + I_i(T)\right)$$

by dynamically selecting the vector $(u_\tau(t), \ldots, u_B(t))$ subject to the state dynamics of (2), and the control constraints $0 \leq u_i \leq 1$ for all $\tau \leq i \leq B$ and all $0 \leq t \leq T$, the initial state conditions (3), and the state constraints in (4).

Problem 2: Optimal Stopping Time Similarly, the system's objective is to maximize the overall utility function:

$$R = \max_{0 \leq u_i \leq 1} -f(T) - \sum_{i=0}^{B} a_i(S_i(T) + I_i(T)) \tag{7}$$

by dynamically regulating $(u_\tau(t), \ldots, u_B(t))$ subject to satisfying the QoS requirement of (5), state equations (2), control constraints $0 \leq u_i \leq 1$ for all $\tau \leq i \leq B$ and all $0 \leq t \leq T$, and the state constraints in (4), given the initial state conditions in (3).

3. OPTIMAL FORWARDING POLICIES

In what follows for both of the problems developed in the previous section, we establish that the optimal dynamic forwarding decisions follow a simple structure. Specifically, we show that the nodes should opportunistically forward the message to any node that they encounter until a threshold time that depends on the current remaining energy of the node. Once the threshold is passed, they should completely cease forwarding until the time-to-live of the message is reached.[3] In the language of control theory, we show that optimal controls for each energy level are bang-bang with at most one jump from maximum to zero.[4]

THEOREM 1. *For all i, an optimal control u_i is in the class of $\mathcal{U}(t-t_i)$ where $\mathcal{U}(t)$ is the reverse step function*[5] *and $0 \leq t_i < T$.*[6, 7]

In what follows we provide the proof of the theorem using tools from classical optimal control theory, specifically Pontryagin's Maximum Principle. We provide the full proof for the fixed terminal time scenario (6) in §3.1, and specify the modifications in the proof for the optimal stopping time problem in §3.2.

3.1 Fixed Terminal Time Scenario

This theorem is proved in the following two steps:
1- Using optimal control theory we are able to show that each optimal control assumes the maximum value (1) when a *switching function* is positive, and the minimum value (0) when the switching function is negative. Standard optimal control results, however, do not specify the nature of the optimal control when the corresponding switching function is at 0. It is also not a priori clear whether these switching functions even have a finite number of zero-crossing points.
2- The main contribution in this part is to establish, using the specifics of the problem, that each switching function is 0 only at, at most, one point and is positive before its (potential) zero-crossing epoch and negative subsequently. This is achieved by showing that the time derivative of the switching function will be strictly negative at all (potential) zero-crossing points. Hence, each optimal control will have a *bang-bang* structure with one drop from 1 to 0 in $[0,T]$.

PROOF. Consider the system in (2) and the objective function in (6). To make the formulation better suited to Pontryagin's Maximum Principle, we introduce the following new state variable:

$$\dot{E} = \sum_{i=\tau}^{B} I_i, \qquad E(0) = 0.$$

Step 1: Using the above defined state, the Hamiltonian is

$$\begin{aligned}\mathcal{H} := g'(t)e^{-\beta_0 n_D E} - \sum_{i=r}^{B}[\beta\lambda_i S_i \sum_{j=\tau}^{B} u_j I_j] \\ + \sum_{i=r}^{B}[\beta\rho_{i-r} S_i \sum_{j=\tau}^{B} u_j I_j] + \sum_{i=\tau}^{B}[\beta u_i \rho_{i-\tau} I_i \sum_{j=r}^{B} S_j] \\ - \sum_{i=\tau}^{B}[\beta u_i \rho_i I_i \sum_{j=r}^{B} S_j] + \lambda_E \sum_{i=\tau}^{B} I_i\end{aligned} \tag{8}$$

where the co-state functions λ_i, ρ_i and λ_E satisfy

$$\begin{aligned}
\dot{\lambda}_i &= -\frac{\partial \mathcal{H}}{\partial S_i} = \beta\lambda_i \sum_{j=\tau}^{B} u_j I_j - \beta\rho_{i-r} \sum_{j=\tau}^{B} u_j I_j \\
&\quad - \beta \sum_{j=\tau}^{B} u_j \rho_{j-\tau} I_j + \beta \sum_{j=\tau}^{B} u_j \rho_j I_j \qquad r \leq i \leq B \\
\dot{\lambda}_i &= -\frac{\partial \mathcal{H}}{\partial S_i} = 0 \qquad i < r \\
\dot{\rho}_i &= -\frac{\partial \mathcal{H}}{\partial I_i} = -\lambda_E + \beta u_i \sum_{j=r}^{B} \lambda_j S_j - \beta u_i \sum_{j=r}^{B} \rho_{j-r} S_j \\
&\quad - \beta u_i \rho_{i-\tau} \sum_{j=r}^{B} S_j + \beta u_i \rho_i \sum_{j=r}^{B} S_j \qquad \tau \leq i \leq B \\
\dot{\rho}_i &= -\frac{\partial \mathcal{H}}{\partial I_i} = 0 \qquad i < \tau \\
\dot{\lambda}_E &= -\frac{\partial \mathcal{H}}{\partial E} = g'(t)\beta_0 n_D e^{-\beta_0 n_D E}
\end{aligned} \tag{9}$$

with the final constraints:

$$\begin{aligned}&\lambda_i(T) = -a_i, \quad \rho_i(T) = -a_i, \quad \forall i = 0, \dots, B \\ &\lambda_E(T) = \beta_0 n_D g(T) e^{-\beta_0 n_D E(T)}.\end{aligned} \tag{10}$$

Maximization of the Hamiltonian yields:

$$u_i = 1 \text{ for } \varphi_i > 0 \quad \& \quad u_i = 0 \text{ for } \varphi_i < 0 \tag{11}$$

where the φ_i's, called *switching functions*, are defined as:

$$\varphi_i := \frac{\partial \mathcal{H}}{\partial u_i} = \beta I_i \Big[-\sum_{j=r}^{B} \lambda_j S_j + \sum_{j=r}^{B} \rho_{j-r} S_j + \rho_{i-\tau} \sum_{j=r}^{B} S_j - \rho_i \sum_{j=r}^{B} S_j\Big]$$

for $\tau \leq i \leq B$, or more simply:

$$\varphi_i = \beta I_i \left(\sum_{j=r}^{B} (-\lambda_j + \rho_{j-r} + \rho_{i-\tau} - \rho_i) S_j \right) \qquad \tau \leq i \leq B. \tag{12}$$

This reveals an accessible intuition about the logic behind the decision process: at any given time, by activating u_i, through contacts between infectives with energy level i and susceptible of any energy level greater than r, infectives with energy i turn into infectives with energy $i-\tau$, and susceptibles of energy level j turn into infectives of energy level $j-r$. The optimal control provides the answer to whether

[3] Implicit is the assumption that any node with sufficient energy that contacts the destination node at any time delivers the message if the destination has not yet received it.

[4] Note that as infective nodes transmit, their energy level sinks; the threshold of each infective should therefore be measured with regards to the *current* level of energy (and not, for example, the starting level). The optimal control is indeed expected to be non-increasing in time: if the control is increasing over a segment, just flipping that part of the control in time would result in earlier propagation of the message and a higher throughput with the same final state energies.The following result, however, goes beyond that intuition in that it establishes that the optimal controls are at their maximum value for a period and then drop abruptly to zero, none of which is a priori clear.

[5] A function that is 1 from $[0,t)$ and 0 from $[t,T]$

[6] Note that the theorem does not exclude the possibility of $t_i = 0$ but excludes $t_i = T$. That is, nodes of some energy levels might never forward the message, but nodes of all energy levels cease forwarding it before the time-to-live.

[7] At times when $I_i(t) = 0$, the optimal control $u_i(t)$ can take any arbitrary value, so our $u_i(t)$ can still have the above structure. As $I_i(t) = 0$ makes $u_i(t)$ trivial, henceforth we concern ourselves with $u_i(t)$ at times when $I_i(t) \neq 0$.

such an action is *beneficial*, taking into account the advantages (positive terms) and disadvantages (negative terms).

At $t = T$, for $\tau \le i \le B$, we have:

$$\varphi_i(T) = \beta I_i(T) \sum_{j=r}^{B} (a_j - a_{j-r} - a_{i-\tau} + a_i)\, S_j(T). \quad (13)$$

Recall that a_i is a decreasing sequence in i. Hence, for all i, $\varphi_i(T) < 0$.[8] This shows that $u_i(t) = 0$ in a subinterval that extends till $t = T$.

Step 2: This step is accomplished in 2 parts: First, the derivative of the switching function at a (potential) zero crossing point is computed, and after simplification, it is upper-bounded using the definition of the switching function. Then, the upper-bound is shown to be negative, thus forcing the time derivative of the switching function (at a potential zero crossing point) to be negative. The last part follows from the key insight that it is not possible to convert all of the susceptibles to infectives in a finite interval of time and hence, the total fraction of infectives (with energy reserves at terminal time) is strictly less than the sum of the susceptibles and infectives with energy reserves greater than r, τ at any time before T.

The theorem is deduced from the following lemma and (11).

LEMMA 1. *For all i, φ_i is never zero on an interval of non-zero length, it crosses zero, at most, at one point, and ends at a strictly negative value.*

PROOF. All φ_i are continuous and piecewise differentiable functions of time, with potential points of non-differentiability at their potential zero-crossing points.

In what follows, we show that the time derivative of φ_i at a potential zero point is strictly negative. Note that this, together with $\varphi_i(T) < 0$, is sufficient to yield the statement of the lemma. We have:

$$\dot{\varphi}_i = \dot{I}_i \frac{\varphi_i}{I_i} - \varphi_i \beta \sum_{j=\tau}^{B} u_j I_j + \beta I_i \sum_{j=r}^{B} (-\dot{\lambda}_j + \dot{\rho}_{j-r} + \dot{\rho}_{i-\tau} - \dot{\rho}_i) S_j.$$

Therefore, at a time at which $\varphi_i = 0$, we have:

$$\dot{\varphi}_i|_{\varphi_i=0} = \beta I_i \sum_{j=r}^{B} (-\dot{\lambda}_j + \dot{\rho}_{j-r} + \dot{\rho}_{i-\tau} - \dot{\rho}_i) S_j. \quad (14)$$

From the expressions for the time derivative of the co-states in (9) combined with the expression for the switching functions in (12), we can write:

$$\begin{aligned}
\dot{\varphi}_i|_{\varphi_i=0} =& \beta I_i \sum_{j=r}^{B} \Bigg(-\dot{\lambda}_j - \lambda_E - \frac{\varphi_{j-r} u_{j-r}}{I_{j-r}} - \lambda_E \\
& - \frac{\varphi_{i-\tau} u_{i-\tau}}{I_{i-\tau}} + \lambda_E + \frac{\varphi_i u_i}{I_i} \Bigg) S_j \\
=& \beta I_i \sum_{j=r}^{B} \Bigg(-\dot{\lambda}_j - \lambda_E - \frac{\varphi_{j-r} u_{j-r}}{I_{j-r}} - \frac{\varphi_{i-\tau} u_{i-\tau}}{I_{i-\tau}} \Bigg) S_j \\
\le& \beta I_i \sum_{j=r}^{B} (-\dot{\lambda}_j - \lambda_E) S_j \\
=& \beta I_i(t) \Bigg(\mathcal{H}(t) - g'(t) e^{-\beta_0 n_D E} - \lambda_E \sum_{j=\tau}^{B} I_i - \lambda_E \sum_{j=r}^{B} S_j \Bigg).
\end{aligned}$$

The inequality follows because terms $\varphi_{j-r} u_{j-r}/I_{j-r}$ and $\varphi_{i-\tau} u_{i-\tau}/I_{i-\tau}$ are non-negative, as imposed by the optimizations in (11)– to see this, note that $\underline{u}_i = 0$ is a feasible solution of the optimization. The equality follows from expanding the summation and regrouping, noting that (from (8)),

$$\sum_{j=r}^{B} -\dot{\lambda}_j(t) S_j(t) = \mathcal{H}(t) - g'(t) e^{-\beta_0 n_D E(t)} - \lambda_E(t) \sum_{j=\tau}^{B} I_j(t).$$

Theferore, all we need to show in order to finish the proof of the lemma, is the following lemma, which we will prove next.[9]

LEMMA 2. *For all $t \in [0, T)$, we have:*

$$\mathcal{H}(t) - g'(t) e^{-\beta_0 n_D E} - \lambda_E \sum_{j=\tau}^{B} I_i - \lambda_E \sum_{j=r}^{B} S_j < 0. \quad (15)$$

PROOF. Note that for a general $g(t)$, the system is **not** *autonomous*[10] as the Hamiltonian has explicit dependence on t. Nevertheless, in general we have [15, p.86] that the Hamiltonian is continuous in time and $\frac{d\mathcal{H}}{dt} = \frac{\partial \mathcal{H}}{\partial t}$, i.e., the time derivative of $\mathcal{H}$ is equal to the partial derivative of $\mathcal{H}$ with respect to t (when only explicit dependence on t is considered). Therefore, in our case: $\frac{d\mathcal{H}}{dt}(t) = g''(t) e^{-\beta_0 n_D E(t)}$. This yields:

$$\begin{aligned}
\mathcal{H}(t) =& \mathcal{H}(T) - \int_t^T g''(\nu) e^{-\beta_0 n_D E(\nu)}\, d\nu \\
=& \mathcal{H}(T) + g'(t) e^{-\beta_0 n_D E(t)} - g'(T) e^{-\beta_0 n_D E(T)} \\
& + \int_t^T g'(\nu) \beta_0 n_D \dot{E}(\nu) e^{-\beta_0 n_D E(\nu)}\, d\nu \quad (16)
\end{aligned}$$

where the second equality follows from integration by parts. Following the discussion after (13), $u_i(T) = 0$ for all i, and therefore from (8), $H(T)$ simplifies to

$$\mathcal{H}(T) = g'(T) e^{-\beta_0 n_D E(T)} + \lambda_E(T) \sum_{i=\tau}^{B} I_i(T). \quad (17)$$

Also, for $\lambda_E(t)$ we have:

$$\lambda_E(t) = \lambda_E(T) - \int_t^T g'(\nu) \beta_0 n_D e^{-\beta_0 n_D E(\nu)}\, d\nu. \quad (18)$$

From (16), (17) and (18), the expression in (15) becomes

$$\lambda_E(T) \left(\sum_{i=\tau}^{B} I_i(T) - \sum_{i=r}^{B} S_i(t) - \sum_{i=\tau}^{B} I_i(t) \right) \quad (19a)$$

$$+ \int_t^T g'(\nu) \beta_0 n_D \sum_{i=\tau}^{B} I_i(\nu) e^{-\beta_0 n_D E(\nu)}\, d\nu \quad (19b)$$

$$+ (\sum_{j=r}^{B} S_j + \sum_{i=\tau}^{B} I_i) \int_t^T g'(\nu) \beta_0 n_D e^{-\beta_0 n_D E(\nu)}\, d\nu. \quad (19c)$$

The lemma now follows from the observations below:
(A) $\sum_{i=\tau}^{B} I_i(T) - \sum_{i=r}^{B} S_i(t) - \sum_{i=\tau}^{B} I_i(t) < 0$. This the result whose intuition was given in the preliminary outline

[8]To see this, note that each term is negative as $a_{j-r} \ge a_j$ and $a_{i-\tau} \ge a_i$.

[9]Note that for $I_i(t) = 0$, the value of $u_i(t)$ is irrelevant.

[10]An autonomous optimal control is one whose dynamic differential equations and its objective function do not have parameters which explicitly vary with time t.

of the proof. To see why this holds mathematically, observe that we have

$$\sum_{i=\tau}^{B} I_i(T) \leq \sum_{i=\tau}^{B} I_i(t) + \sum_{i=r}^{B} S_i(t) - \sum_{i=r}^{B} S_i(T), \quad \text{and}$$

$$\frac{d}{dt}\sum_{i=r}^{B} S_i(t) = -\beta \sum_{i=r}^{B} S_i \sum_{j=\tau}^{B} u_j I_j \leq -\beta \sum_{i=r}^{B} S_i$$

$$\Rightarrow \sum_{i=r}^{B} S_i(T) \geq \left(\sum_{i=r}^{B} S_i(t)\right) e^{-\beta(T-t)} > 0.$$

This demonstrates the negativity of (19a).
(B) $g'(t) \leq 0$, which shows that both (19b) and (19c) are non-positive. □

This concludes the proof of the theorem.

We now investigate the relationship between the threshold-times for optimal controls corresponding to different energy-levels. Since lower levels of residual energies are penalized more and the energy consumed in each transmission and reception is the same irrespective of the energy levels of the nodes, it may appear that the threshold-times will be monotonically increasing functions of the energy levels. We now prove that this is indeed the case if the terminal-time penalty sequence is strictly convex (i.e., the difference between the penalties associated with consecutive energy levels increases with a decrease in energy levels). Interestingly enough, in §4 we construct counter-examples, using partly convex and partly concave and also fully concave sequences, such that this intuition is negated and the monotonically increasing order of the threshold times is violated. This demonstrates that indeed naive intuition is misleading and the order predicted by the theorem does not extend in general when the strict convexity condition is absent.

THEOREM 2. *Assuming that sequence a_i in (6) is non-negative, decreasing and strictly convex, then the sequence of t_i in Theorem 1 is increasing in i.*

PROOF. It suffices to show that if $\varphi_i(t) = 0$, we have $\varphi_k(t) \leq 0$ for any $k \leq i$. It then follows from the proof of the previous theorem that the threshold time for optimal control $u_k(\cdot)$, if any, precedes that of $u_i(\cdot)$.

We show that if $\varphi_i(t) = 0$ for $t = \sigma_i$, then φ_k equals:

$$\beta I_k \left(\sum_{j=r}^{B} \left(-\rho_{i-\tau} + \rho_i + \rho_{k-\tau} - \rho_k\right) S_j\right)\Bigg|_{t=\sigma_i}.$$

We subsequently show that for each j, the term that multiplies S_j is non-positive at each time later than or equal to the zero-crossing point referred to above. We show that the above is non-positive at $t = T$ (utilizing the convexity of the penalty sequence $\{a_i\}$), and subsequently prove that this holds for all times after, and including, $t = \sigma_i$. This argument does not follow from standard optimal control theory, and it is therefore one of the theoretical contributions of this paper.

We now proceed with the proof: From (12) we have:

$$\varphi_i(\sigma_i) = \beta I_i \left(\sum_{j=r}^{B} \left(-\lambda_j + \rho_{j-r} + \rho_{i-\tau} - \rho_i\right) S_j\right)\Bigg|_{t=\sigma_i} = 0.$$

Therefore, at $t = \sigma_i$ we can write[11]

$$\sum_{j=r}^{B} \left(-\lambda_j + \rho_{j-r}\right) S_j = -\sum_{j=r}^{B} \left(\rho_{i-\tau} - \rho_i\right) S_j$$

Evaluating φ_k at σ_i, using the above replacement yields:

$$\begin{aligned}\varphi_k(\sigma_i) =& \beta I_k \left(\sum_{j=r}^{B} \left(-\lambda_j + \rho_{j-r} + \rho_{k-\tau} - \rho_k\right) S_j\right)\Bigg|_{t=\sigma_i} \\ =& \beta I_k \left(\sum_{j=r}^{B} \left(-\rho_{i-\tau} + \rho_i + \rho_{k-\tau} - \rho_k\right) S_j\right)\Bigg|_{t=\sigma_i}.\end{aligned}$$

For $0 \leq k \leq i$, define:

$$\psi_{i,k}(\sigma_i) := \begin{cases} -\rho_{i-\tau} + \rho_i + \rho_{k-\tau} - \rho_k & \tau \leq k \\ -\rho_{i-\tau} + \rho_i - \rho_k & 0 \leq k \leq \tau \end{cases}$$

The theorem now follows from the following lemma.

LEMMA 3. *For any $k \leq i$, we have $\psi_{i,k}(\sigma_i) \leq 0$.*

PROOF. We present the proof for $k \geq 2\tau$. The case of $0 \leq k \leq 2\tau$ follows similarly. At $t = T$ following (10), we have:

$$\begin{aligned}\psi_{i,k}(T) =& -\rho_{i-\tau}(T) + \rho_i(T) + \rho_{k-\tau}(T) - \rho_k(T) \\ =& a_{i-\tau} - a_i - (a_{k-\tau} - a_k)\end{aligned}$$

which following the properties assumed for a_i (a_i being decreasing and strictly convex in i), yields $\psi_{i,k}(T) < 0$. This also holds on a sub-interval of nonzero length that extends to $t = T$, owing to the time-continuity of $\psi_{i,k}$. We prove the lemma by contradiction:Going backward in time from $t = T$ towards $t = \sigma_i$, suppose the lemma is violated first at time $\bar{\sigma}$, that is, for at least one $k < i$ we have:

$$\begin{aligned}&(-\rho_{i-\tau} + \rho_i + \rho_{k-\tau} - \rho_k) < 0 \text{ for } \sigma_i < \bar{\sigma} < t \leq T; \text{ and} \\ &(-\rho_{i-\tau} + \rho_i + \rho_{k-\tau} - \rho_k) = 0 \text{ at } \bar{\sigma}\end{aligned}$$

and the inequality holds for the rest of the levels. In what follows we show that the time derivative of $\psi_{i,k}$ is non-negative over the interval of $[\bar{\sigma}, T]$. Note that this leads to a contradiction with the existence of $\bar{\sigma}$ and hence proves the lemma, since:

$$\psi_{i,k}(\bar{\sigma}) = \psi_{i,k}(T) - \int_{t=\bar{\sigma}}^{T} \dot{\psi}_{i,k}(\nu)\, d\nu \Rightarrow \psi_{i,k}(\bar{\sigma}) \leq \psi_{i,k}(T) < 0.$$

We now investigate $\dot{\psi}_{i,k}$ over $[\bar{\sigma}, T]$:

$$\begin{aligned}\dot{\psi}_{i,k} =& -\dot{\rho}_{i-\tau} + \dot{\rho}_i + \dot{\rho}_{k-\tau} - \dot{\rho}_k \\ =& (\lambda_E + \frac{\varphi_{i-\tau} u_{i-\tau}}{I_{i-\tau}}) + (-\lambda_E - \frac{\varphi_i u_i}{I_i}) \\ &+ (-\lambda_E - \frac{\varphi_{k-\tau} u_{k-\tau}}{I_{k-\tau}}) + (\lambda_E + \frac{\varphi_k u_k}{I_k}) \\ =& \frac{\varphi_{i-\tau} u_{i-\tau}}{I_{i-\tau}} - \frac{\varphi_i u_i}{I_i} - \frac{\varphi_{k-\tau} u_{k-\tau}}{I_{k-\tau}} + \frac{\varphi_k u_k}{I_k} \geq -\frac{\varphi_i u_i}{I_i} - \frac{\varphi_{k-\tau} u_{k-\tau}}{I_{k-\tau}}.\end{aligned}$$

The last inequality follows from (11). For the remaining terms, note that following from the definition of σ_i and as we showed in the proof of Theorem 1, we have $\varphi_i(t) \leq 0$ over the interval of $[\sigma_i, T]$. Now we show that $\varphi_{k-\tau}(t) \leq 0$

[11] unless $I_i = 0$, for which the control, u_i, is irrelevant.

over the interval of $[\bar{\sigma}, T]$. From (12), we have:

$$\begin{cases} \varphi_i & = \beta I_i \left(\sum_{j=r}^{B} \left(-\lambda_j + \rho_{j-r} + \rho_{i-\tau} - \rho_i \right) S_j \right) \\ \varphi_{k-\tau} & = \beta I_{k-\tau} \left(\sum_{j=r}^{B} \left(-\lambda_j + \rho_{j-r} + \rho_{k-2\tau} - \rho_{k-\tau} \right) S_j \right) \end{cases}$$

Following the definition of $\bar{\sigma}$, we have $\rho_{k-2\tau} - \rho_{k-\tau} \le \rho_{i-\tau} - \rho_i$ over the interval of $[\bar{\sigma}, T]$. Hence:

$$\sum_{j=r}^{B} \left(-\lambda_j + \rho_{j-r} + \rho_{i-\tau} - \rho_i \right) S_j \le 0$$

$$\Rightarrow \sum_{j=r}^{B} \left(-\lambda_j + \rho_{j-r} + \rho_{k-2\tau} - \rho_{k-\tau} \right) S_j \le 0,$$

and therefore, $\varphi_i \le 0 \Rightarrow \varphi_{k-\tau} \le 0$. This concludes the lemma, and hence the theorem. □

3.2 Optimal Stopping Time Scenario

We will use the variable final time version of Pontryagin's Maximum Principle for the optimal stopping time version of the problem. First, we transform the *path* constraint in (5) to a final state constraint, which is better suited to the PMP formulation. This can be done simply by introducing a new variable, E, such that:

$$\dot{E} = \sum_{i=\tau}^{B} I_i(t), \quad E(0) = 0, \quad E(T) \ge -\ln(1-p)/\beta_0.$$

Now consider a new Hamiltonian identical to (8) with the exception that the first term is removed. According to PMP, there exist absolutely continuous co-state functions $\vec{\lambda}$, $\vec{\rho}$, λ_E and a constant $\lambda_0 \ge 0$ such that at any given time, an optimal u_i is a maximizer of the new Hamiltonian. The co-state functions satisfy the same differential equations as in (9), with the exception that here we have $\dot{\lambda}_E = 0$.

The final conditions for the co-states also change to the following:

$$\lambda_i(T) = -\lambda_0 a_i, \quad \rho_i(T) = -\lambda_0 a_i, \quad \forall i = 0, \ldots, B \quad (20a)$$

$$\lambda_E(T) \ge 0, \quad \lambda_E(T)(E(T) + \ln(1-p)/\beta_0) = 0 \quad (20b)$$

together with $H(T) = \lambda_0 f'(T)$ and for every $t \in [0, T]$:

$$(\lambda_0, \vec{\lambda}(t), \vec{\rho}(t), \gamma(t)) \ne \vec{0}. \quad (21)$$

The last condition along with $\lambda_0 \ge 0$ leads to $\lambda_0 > 0$. Maximization of the Hamiltonian yields:

$$u_i = \begin{cases} 1 & \varphi_i > 0 \\ 0 & \varphi_i < 0 \end{cases} \quad (22)$$

where φ_is, the switching functions, are defined as before, and with a similar argument it can be shown that $\varphi_i(T) < 0$. Therefore, we have $H(T) = \lambda_E(T) \sum_{i=\tau}^{B} I_i(T)$. Hence,

$$\lambda_0 f'(T) = \lambda_E(T) \sum_{i=\tau}^{B} I_i(T) \quad (23)$$

Now, we claim that both Theorems 1 and 2 apply in this case as well. The proofs are almost identical. Here we only list the alterations.

▷ **Changes in the proof of Theorem 1**:
The proof is identical up to the following point, which we develop onward:

$$\begin{aligned} \dot{\varphi}_i|_{\varphi_i=0} \le & \beta I_i \sum_{j=r}^{B} (-\dot{\lambda}_j - \lambda_E) S_j = \beta I_i \sum_{j=r}^{B} (-\dot{\lambda}_j S_j - \lambda_E S_j) \\ = & \beta I_i (\mathcal{H} - \lambda_E \sum_{i=\tau}^{B} I_i - \lambda_E \sum_{j=r}^{B} S_j) \\ = & \beta I_i (\mathcal{H}(T) - \lambda_E \sum_{i=\tau}^{B} I_i - \lambda_E \sum_{j=r}^{B} S_j) \\ = & \beta I_i (\gamma(T) \sum_{i=\tau}^{B} I_i(T) - \lambda_E \sum_{i=\tau}^{B} I_i - \lambda_E \sum_{j=r}^{B} S_j). \end{aligned}$$

Note that $\dot{\lambda}_E = 0$ and hence λ_E is a constant, i.e., $\lambda_E = \lambda_E(T)$. Furthermore, it is a strictly positive constant because first $\lambda_E(T) \ge 0$ from (20b); and second, λ_E cannot be zero, because if it is, following (23), it implies $\lambda_E = \lambda_0 \equiv 0$, which together with the ODE of the co-state functions and (20a), leads to $(\lambda_0, \vec{\lambda}(T), \vec{\rho}(T), \lambda_E) = \vec{0}$. This would contradict (21). Hence $\dot{\varphi}_i|_{\varphi_i=0} < 0$, and the rest of the proof is similar to the one presented in the previous section.

▷ **Changes in the proof of Theorem 2**:
Everything remains the same except that the final value of $\psi_{i,k}(T)$ is multiplied by λ_0, that is

$$\psi_{i,k}(T) = \lambda_0 \left(a_{i-\tau} - a_i - (a_{k-\tau} - a_k) \right).$$

As we argued in the previous item, we have $\lambda_0 > 0$ and henceforth, the rest of the arguments follow identically.

Practical Issues and Implementation

A corollary of Theorem 1 is that the forwarding policy can be completely represented by a vector of threshold times corresponding to different energy levels. This vector is of size $B - \tau$ and can be calculated once at the source node of the message and added to it as a small overhead. Each node that receives the message simply retrieves the threshold levels and forwards the message if its age is less than the threshold entry corresponding to its current energy level. The one-time calculation of the threshold levels for each message at the origin can be done by estimating the current distribution of the energy levels in the network. Note that the required information is the fractions of nodes with each level of energy, and not the identity of the nodes. This estimation can be done if the distribution of the energies at the time at which the network starts operation is known (e.g. all nodes start with full batteries) and origin nodes keep a history of the past messages. The robustness of our policy with respect to inaccuracy in the estimation of the initial energy profile of the network is an interesting direction for future research. The search for optimum thresholds is now an optimization with only $B - \tau$ variables. Moreover, following Theorem 2, the search can be limited to a small subset of the space of $[0, T]^{B-\tau}$. Finally, as we show in our numerical section, heuristically, a common threshold for all energy levels can be first optimized and its solution used as a good initial solution for the multi-variable optimization. Qualitatively, our results show that each node should have different *modes of action* depending on its residual battery, and that these modes of actions themselves vary for each newly generated message. These observations should give clues to improving state-of-the-art DTN routing policies.

In developing our theoretical model, as for any analytical model, we made a series of technical assumptions for tractability and/or avoidance of unnecessary clutter. (a) We ignored the energy dissipated in scanning the media in search of new nodes. Incorporating the media scanning energy dissipation is straightforward, however it would unduly complicate the model and is left to our future research. (b) *homogeneous mixing*, i.e., the assumption that the inter-contact times are similarly distributed for all pairs, which may not hold in practice. This assumption can partly be addressed technically by using the ideas in [9], which suggest that our results likely generalize to spatially inhomogeneous cases. (c) We assumed that during the interval $[0, T]$, only one message is routed in the network. This assumption is valid if the load in the network is low and the routing time intervals of different messages do not overlap (i.e., the interval between the generation of new messages is longer than T). Generalization to the routing of multiple messages with overlapping routing intervals can be a future direction of research.

4. NUMERICAL INVESTIGATIONS

In this section, we investigate the structure of the optimal control in systems with the above dynamics. We investigated the single-delivery fixed terminal time problem with $g(t) = 1$, i.e., no time discrimination in $[0, T]$, in the simulations, and unless otherwise stated, our test system has parameters: $B = 5$ (i.e., five energy levels), $\tau = 2$, $r = 1$, and $T = 10$. Note that $\tau > r$, as demanded by our system model. For β (and β_0), our benchmark is 0.223 (motivated by [8]). It is instructive to note that βT denotes the average number of contacts of each node in the system in the time interval $[0, T]$, and therefore our choice of β is limited by the time-to-live (T) of the message. In our case, each node contacts more than two other nodes on average within the TTL of the message, a not-unreasonable assumption.

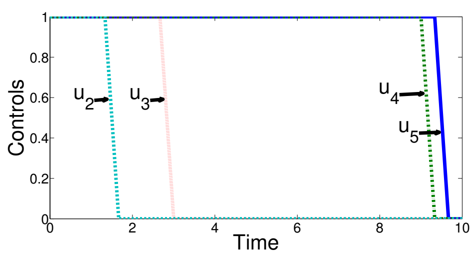

Figure 1: **An illustrative example for Theorems 1 and 2. The controls are plotted for a system with parameters: $B = 5$, $r = 1$, $t = 2$. The initial distribution is $\mathbf{I_0} = (0, 0, 0.25, 0.02, 0.02, 0.01)$ and $\mathbf{S_0} = (0, 0, 0, 0.2, 0.3, 0.2)$, and the battery penalties are $a_i = \kappa(B - i)^2$, with $\kappa = 0.005$.**

First, we illustrate Theorems 1 and 2 by examining a system with 5 energy levels and convex final-state cost coefficients (fig. 1). As can be seen, the optimal control for each energy level demonstrates bang-bang behavior with one drop-off point, and furthermore, because of the convexity of the final state costs and in accordance with Theorem 2, the drop-off times of the different energy level controls follow the ordering of the energy levels.

Then, we investigate the case where the final state costs are not convex. One sample configuration is when we have a sharp drop-off between two terminal-time coefficients, with coefficients on either side being close to each other. The motivation for such a setting could be the case where we

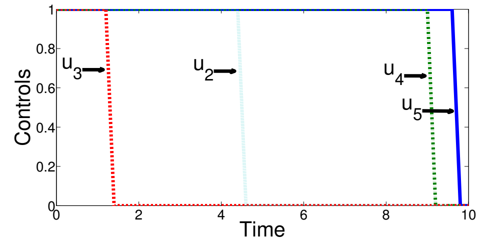

Figure 2: **In this example, the parameters were exactly the same as those used in fig. 1, with the difference that the final state coefficients were $a_0 = 4.4$, $a_1 = 4.2$, $a_2 = 4$, $a_3 = 1.2$, $a_4 = 1.1$, $a_5 = 1$. As can be seen, the drop-off in final state costs between energy levels 3 and 2 motivates nodes in level 3 to be more conservative in propagating the message.**

	Drop-off Times of Controls	
	Energy Level 3	Energy Level 2
$\kappa = 0.5$	9.6	9.8
$\kappa = 1.5$	9.6	9
$\kappa = 2$	9.6	8.4

Table 1: **An example for non-ordered drop-off points of the optimal controls for concave final state coefficients in the settings of Theorem 2. Here we construct the final-state cost coefficients in the form $a_i = \kappa(B - i)^\alpha$ with $T = 10$, $B = 3$, $r = 0$, $t = 1$ and $\kappa = 0.01$, and vary α over the values $\{0.5, 1.5, 2\}$. The initial distribution is $\mathbf{I_0} = (0.1, 0.1, 0.1, 0.2)$ and $\mathbf{S_0} = (0, 0, 0.2, 0.3)$. As can be seen, for $\alpha = 0.5$, where the final cost coefficients become concave, the ordering of the drop-off in energy levels 2 and 3 is reversed. For $\alpha = \{1.5, 2\}$, where the final costs are strictly convex, the ordering is preserved, as predicted by Theorem 2.**

only care about having a certain fixed amount of energy at terminal time, and any variation above or below that value is of very little importance to us. It can be seen in fig. 2 that Theorem 2 does not necessarily hold for such a setting, as nodes on either side of the drop-off would be incentivised to propagate the message (because of the low loss incurred for propagation in terms of final states), but those nodes in states on the cusp of the drop-off in final state coefficients would be extremely conservative, as there is a large penalty associated with any further propagation of the message.

Subsequently, it is shown that Theorem 2 does not hold for even wholly concave terminal-time state coefficients. To this end, a concave cost function is constructed that has an optimal control whose drop-off times are not ordered (Table 1). Therefore, the convexity of the final cost coefficients is integral to the result of Theorem 2.

To better illustrate the efficacy of the bang-bang controls, the performance of the system is compared with that of 3 heuristic algorithms:

1. The control is constant throughout $[0, T]$ uniformly for all energy levels (Static in Time and Across Energy Levels).
2. The control can vary within the field of uniform one-jump bang-bang controls for all uniform energy levels (Static Across Energy Levels).
3. The control can vary between energy levels, but the controls of each are constant in time (Static in Time)

In fig. 3, the system utility of the optimal control and the

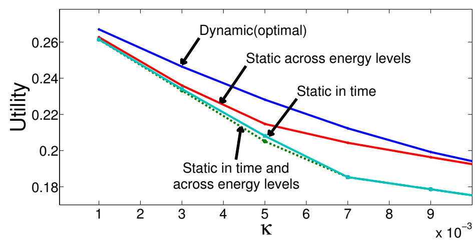

Figure 3: The performance of the heuristics is compared with the dynamic (bang-bang) optimum for different relative weights κ of the utility function for a system with parameters $B = 5$, $r = 1$, $t = 2$ and with the same initial distribution and utility function as the system in fig. 1. Here, we took β to be 0.78, so that each user has, on average, around 8 contacts in the time period. As κ goes to both extremes (0 and infinity), the utility function becomes trivial and the performance of the heuristics matches the optimal.

heuristics are plotted as a function of the relative weighting of the two parts of the utility function for a sample system. It turns out that the difference in utility becomes large whenever the drop-off times of the optimal controls are spread out across T, forcing the heuristics into choosing overly conservative policies. It is instructive to note that the static-in-time heuristic, which can take different actions for different energy levels, is outperforming the other heuristics throughout, which further emphasizing the importance of having separate controls for different energy levels, a key feature of our approach. It can be seen that this policy can be chosen as a *simple approximation* to the optimal without a significant loss in performance.

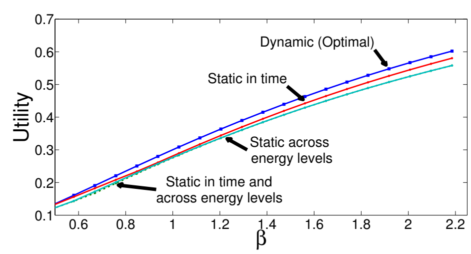

Figure 4: In this figure, β (the mean rate of contact) is varied and the performance of the optimal control and the heuristics are studied. The same parameters as those outlined in fig. 1 are used for the utility function and the initial states. For low rates of contact, the optimality of the control is less important as contacts are too infrequent to affect the energy distribution significantly. As β increases, any constraint on the policy translates to sub-optimality in the controls. As can be seen, the static-in-time heuristic again performs much better than the other two heuristics, but the sub-optimality of even this heuristic increases with an increase in β.

Finally, to better understand the structure of the optimal control, the relative performance of the optimal policy and the heuristics are illustrated in fig. 4 for a case where β is varied. It can be seen that the sub-optimality of all the heuristics increases with β, which illustrates the added importance of optimal decision-making at each instance in situations where there are more contacts.

5. CONCLUSION

We formulated the problem of optimal energy-dependent message forwarding in energy-constrained DTNs as a multi-variable optimal control problem using a deterministic stratified epidemic model. We analytically established that optimal forwarding decisions for two generalized notions of QoS are composed of simple threshold-based policies, where the thresholds depend on the current value of the remaining energies in each node. We then analytically characterized the dependence of these thresholds on the remaining energy reserves of the nodes.

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