Pricing for Past Channel State Information in Multi-Channel Cognitive Radio Networks

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Abstract—Cognitive Radio (CR) networks have received significant attention as a promising approach to improve the spectrum efficiency of current license-based regulatory system. In CR networks, a Secondary User (SU) can use a spectrum vacancy that can be detected by either sensing-before-transmission or database access. However, it is often difficult to detect a vacant spectrum opportunity because of inaccuracies due to sensing and delays to update and/or the database that holds this information. In this paper, we develop a hybrid detection framework in multi-channel CR networks, where an SU can selectively sense a channel for spectrum vacancy by accessing the spectrum history of Markovian channels. We focus on the value of the channel history information offered by the Primary Provider (PP) of each channel, and consider a market for the information exchange between multiple PPs and SUs. We investigate the interplay between of the PPs and the SUs through their pricing and buying decisions for this information, in the presence of sensing inaccuracy, i.e., false alarm and miss detection.

Index Terms—Cognitive radio, pricing, network economics, Stackelberg game, Gilbert-Elliott channel model

1 Introduction

In current spectrum management, centralized authorities, e.g., the Federal Communications Commission, regulate the use of frequency spectrum and license spectrum bandwidth to service providers. It has been observed that this command-and-control spectrum management approach has suffered from very low spectrum utilization in both time and space [2]. As an alternative, cognitive radio (CR) networks have received significant attention as a promising approach to improve spectrum efficiency of current license-based regulatory system [3]. In CR networks, license holders (*Primary Providers* (PPs)) have the right to use their assigned spectrum to serve *Primary Users* (PUs), and unlicensed users[1] (*Secondary Users* (SUs) or *Secondary Providers* (SPs)) can access unused spectrum by appropriately managing their interference on the PUs.

One of the key technologies that enable CR networks is Dynamic Spectrum Access (DSA) that allows SUs to identify space-time-frequency vacancies and to opportunistically use the holes, achieving high spectrum efficiency [4]. In general, there have been two classes of schemes to address *uncertainty* of spectrum availability: *sensing-before-transmission* [5] and *database access* [6]. In the former approach, the network performance depends on the rate of sensing errors, i.e., *false alarm* and *miss detection*. Although recent advances in radio technologies have improved the sensing accuracy from many different angles such as circuit design, radio signal detection, and collaborative sensing [7], non-negligible sensing errors will still remain for the foreseeable future [8]. In the latter approach, a spectrum availability information can be accessed directly through database. A weakness of this approach is that it takes time to access (write and read) into the database, which is often much longer than the time for scheduling transmissions. Thus, database access would be suitable for long-term channel statistics or the history of spectrum usage, and it is unlikely to provide SUs immediate availability of the spectrum within a few microseconds.

In this work, we develop hybrid DSA framework using both sensing and database access in multi-channel CR networks, where the database information assists the decision for sensing. As a first step, we consider a CR network with one SU and two PPs, where each PP uses a separate spectrum band and then extend the results to multiple SUs. The SU can sense only one of the spectrum bands before transmission due to sensing delay. When an SU has data to send, it wakes up its communication module and selectively senses one of the primary spectrum bands before transmission. If the SU senses the spectrum band as idle and successfully transmits the data, it returns to sleep the communication module until it has data again. Many applications of Internet-of-Things (IoT) such as metering [9] and event detection monitoring [10] fit this scenario. We assume that initially the SU has no history information[2]

1. An unlicensed user can be either the Secondary Provider (SP) or the Secondary User (SU), depending on the entity that decides transmission. We interchangeably use the terms of SP and SU.

2. When a transmission fails, the SU can learn the activity of the spectrum band and can utilize the information for access in the next time. In this work, we ignore the impact of learning, which is beyond the scope of this paper, and remains an interesting open problem.

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about the activity of PPs except long-term characteristics of channels (e.g., stationary distribution of the Markov channel model). Unless the activity of the PP is memoryless, history information will be helpful for the SU to decide which channel it will sense. The information can be provided by the PPs, which, however, will cause additional cost to the PPs. To incite the PPs to participate in and voluntarily provide their activity information, we introduce a pricing-based mechanism. Each PP with an assigned spectrum band can set the price of its information about its previous activities. For each channel, the SU can buy the information, and use it to figure out the most-likely vacant channel. Thus the gain from the channel information can be distributed to the PPs and the SU through the market.

To the best of our knowledge, this is the first work that considers CR networks with both the sensing and the database access, and that investigates the pricing of the information (rather than commodity) in CR networks. Our contribution can be summarized as follows.

- We develop a hybrid DSA frameworks with spectrum sensing and database access in multi-channel CR networks.
- We model the channel information market as a two-stage sequential game (i.e., Stackelberg game [11]), and we investigate the pricing behavior of the PPs for their channel activity information, and the SU decisions in the single SU case. Using the obtained results, we further extend to multiple SUs.
- We demonstrate the impact of competition and cooperation between the PPs on the performance (i.e., the revenue of the PPs, the payoff of the SU, and the social welfare).

The rest of paper is organized as follows: In Section 2, we provide a survey of related works. In Section 3, we describe the network model and two-stage game model. We investigate the behavior of the SU to maximize its reward in Section 4. Then we study the responses of the PPs under competition and cooperation, and show how the prices can be determined in Section 5. In Section 6, we extend the result to multiple SUs, and study how PPs' pricing changes and SUs' payoffs are affected. In Section 7, we verify our results through simulations. In Section 8, we discuss important issues related to our work and several interesting open problems. Finally, we conclude the paper in Section 9.

2 Related Works

A spectrum access control in cognitive radio networks has been studied from many perspectives such as multi-armed bandit problems or spectrum markets. In [12], a joint mechanism of sensing, probing, and accessing is formulated with combinatorial (non-stochastic) multi-armed bandits problems. In the perspective of a spectrum market, SUs pay PPs (or PUs) monetary cost (i.e., money-exchange) or provide communication resources (i.e., resource-exchange) for the usage of spectrum. In money-exchange spectrum trading, access control can be managed through auction-based [13] or pricing-based mechanisms [14]. In auction-based mechanisms, the SUs bid for spectrum as a function of their own desires, and each PP chooses a set of SUs in order to maximize its own revenue. In pricing-based mechanisms, PPs set a price for their spectrum and SUs buy the spectrum on the market. In resource-exchange spectrum trading [15], a PU shares its spectrum with SUs in exchanging of their own performance improvement (e.g., PU's improvement of communication reliability through ARQ transmissions by SUs [16]).

A power control game between SUs is another perspective of a spectrum access control, where a spectrum access is modeled by overlay or underlay framework. In the overlay framework [17], an SU is allowed to transmit only on currently unoccupied spectrum by PUs. In the underlay framework [18], each SU controls its transmission power such that their resultant interference level at PUs' receivers should not exceed an interference temperature threshold.

In addition to resource markets, the value of information and information markets have received much attention in various networks including sensor networks [19] and Internet of Things (IoT) applications [20]. One of information markets in IoT is the sensing information market in cognitive radio networks, where spectrum sensing devices are deployed to monitor PUs' activities on certain spectrum. The spectrum sensing information collected from the IoT devices can be valuable to SUs for dynamic spectrum access, and thus it can be sold to SUs.

In this work, our main interest is the sensing information market in cognitive radio networks. Unlike the game formulation in [20], where an SU makes a transmission decision only with spectrum information bought, we consider an SU that performs sensing before transmission, and it makes a sensing/transmission decision (i.e., which channel it chooses to sense and transmit) with spectrum information bought. As a first step to understand the impact of channel information, we consider the overlay framework with a simple spectrum access, and focus on the interaction of PPs and SUs in the information market.

3 System Model

3.1 Network Model

We first consider a cognitive radio network with two primary providers (PPs) and a secondary user (SU). There are two non-interfering frequency channels (or spectrum bands), each of which is assigned to one of the PPs, and can be used opportunistically by the SU when idle. The channels need not be adjacent and it may take some time for the SU to switch its operating channel. We assume a time-slotted system, where the SU can sense/use one of the two frequency channels in a time slot, due to the delay in switching the channel [21].

We denote the two channels by channel A and channel B. For each channel $i \in \{A, B\}$, let PP i denote the primary provider that the channel is assigned to, and let $d^i(t)$ denote the state of channel i (or the activity of PP i) at time slot t. Channel i is either *busy* ($d^i(t) = 1$) if the channel is in use at time slot t by PP i, or *idle* ($d^i(t) = 0$), otherwise. We interchangeably use the state of channel i and the activity of PP i throughout the paper. We assume that the state of channel i is independent from the other, and changes across time following an independent Markov process with transition probability matrix $P^i = \{p^i_{jk}\}$, where $p^i_{jk} := \mathrm{Prob}\{d^i(t) = k | d^i(t-1) = j\}$. Clearly, we have $p^i_{00} + p^i_{01} = 1$ and $p^i_{10} + p^i_{11} = 1$. We assume that the transition matrices (i.e., P^A and

P^B) are known to all the PPs and the SU. Let π_j^i denote the steady-state probability distribution of channel i in state j. From the balance equations, it can be easily obtained as $\vec{\pi}^i = (\pi_0^i, \pi_1^i)$ where $\pi_0^i = \frac{p_{10}^i}{p_{10}^i + p_{01}^i}$, and $\pi_1^i = \frac{p_{01}^i}{p_{10}^i + p_{01}^i}$.

When the SU has data to send, the SU chooses one of the channels, senses the channel, and communicates over the channel if it senses it as being idle. Let $x^i(t) \in \{0, 1\}$ be the sensing decision of the SU: $x^i(t) = 1$ if the SU decides to sense channel i, and $x^i(t) = 0$, otherwise. We assume that the SU can sense only one channel at a time slot, i.e., $x^A(t) + x^B(t) \leq 1$, due to channel switching delay. Suppose that the SU senses channel i at time slot t. The sensing result is denoted by $\tilde{d}^i(t) \in \{0, 1\}$: $\tilde{d}^i(t) = 0$ if the SU senses channel i as idle, in which case the SU transmits data on channel i during time slot t, $\tilde{d}^i(t) = 1$ if the SU senses channel i as busy, in which case the SU remains silent on channel i. There is a possibility that the SU senses busy but the channel is indeed idle, in which case the time slot is wasted. This is called as *false alarm*. We denote the false alarm probability by $f := \mathrm{Prob}\{\tilde{d}^i = 1 \mid d^i = 0\}$. Similarly, it is possible that the SU senses idle but the channel is indeed busy. In this case, a collision occurs and both transmissions from the PP and the SU fail. This is called a *miss detection*. We denote the miss detection probability by $m := \mathrm{Prob}\{\tilde{d}^i = 0 \mid d^i = 1\}$. These probabilities can be different according to the type of device and the location of the SU. The SU who transmits on channel i receives a reward $r^i > 0$ for its successful transmission, and pays a penalty $a^i > 0$ (to the PP) for a collision caused by miss detection. The reward and penalty are inclusive of the payment from the users, the fee of using the channels, the cost of collision, etc. Further, we assume no channel error,[3] i.e., a packet transmission is successful if there is no collision, and PPs have prior knowledge about the location and the device type of the SUs for their service subscription, which can be used for estimating false alarm and miss detection probabilities, as well as the rewards that can be interpreted as achievable bandwidth. We also assume that long-term statistics of each channel and collision penalty are common knowledge.[4] Algorithm 1 (line 4-8) shows the basic procedure of the SU for sensing and data transmission.

3.2 Game Model

We now consider a market, where the PPs try to sell their channel information to the SU. We assume that an SU does not neither keep tracking the channel states, nor resell the information to other SU. Monitoring and sensing scenarios of IoT devices, where few devices wake up at the same time, are a good example. At the beginning of given time slot t, the SU has the statistical information of both the channels but has no knowledge about the actual channel states $d^i(k)$ for all $k \leq t-1$. In contrast, PU i has the history of its activity, i.e., channel states $d^i(k)$ for $k \leq t-1$.

3. This error-free assumption can be relaxed by multiplying channel-error probability to the channel reward, i.e., $r^i(1-p_e^i)$, where p_e^i is the channel-error probability of channel i. This relaxation does not impact the main results of this work.

4. We make an additional remark on common knowledge with multiple SUs for future extension. Generally, unlike SUs, PPs have a lot of resources and participate the market for a long time. Thus, PPs have an ability to estimate SUs' private information through learning from SUs' response iteratively, or doing user survey or market research.

Algorithm 1. Procedure of SU

1: Collect the price of information α^A and α^B
2: Calculate payoffs U_{ij} for each $(i,j) \in \{0,1\}^2$
3: Determine $(w^A, w^B) = \arg\max_{(i,j)\in\{0,1\}^2} U_{ij}$
4: Determine (x^A, x^B) as in (6)
5: Sense channel i if $x^i = 1$, store the sensing result in $\tilde{d}^i$
6: **if** senses idle channel, i.e., $\tilde{d}^i = 0$ **then**
7:     Make a transmission on channel i
8: **end if**
9: Earn reward r^i or pay penalty a^i depending on the result of the transmission

At the beginning of time slot t, PPs send their current activity information to a common storage (or database) via a control channel which, we assume, takes less than a half time slot. The SU makes the buying decision for the information. In other words, in the middle of the time slot t, the SU decides whether it buys the information or not. If it decides to buy, it should be able to retrieve the information within the second half of time slot t, such that the SU can make use of the information to determine which channel to sense at time slot $t+1$. Note that considering the Markovian property of the channel, the channel state information at t is sufficient for the SU to make the best prediction on the channel state at $t+1$. Thus, we assume that for each of storing and retrieving the information, a half time slot, which is supposed to be tens of milliseconds, is sufficiently large for accessing through the control channel: for example, a careful time coordination can let the PP store its information through a short signal tone, and radio-tuning time for channel switching can be also minimized by exploring new radio techniques, e.g., OFDM subcarrier nulling [22].

Since the channel information is beneficial to the SU, the PPs may sell the information to the SU at a reasonable price. We model the behaviors of the PPs and the SU as a two-stage sequential game with known strategy, i.e., a Stackelberg game [11]. At the first stage of the game, PP i decides on price α^i for its state information as a *leader*, and at the second stage, the SU as a *follower* determines (i) whether it buys the information, and (ii) which channel to sense. Fig. 1 illustrates the interactions. PP i determines price α^i (independently or in a cooperative manner) in stage 1. Then, in stage 2(i) the SU decides whether to buy the information of each channel i, and in stage 2(ii), decides which channel it senses. In Fig. 1, the SU bought the information $d^A(t-1)$ of channel A at price α^A, and sensed channel B. Let $w^i(t)$ denote the buying decision of the SU: $w^i(t) = 1$ if the SU decides to buy the information of channel i, and $w^i(t) = 0$ otherwise.

Under our model, we assume that the strategy of the SU is known a priori, and the PPs will choose the prices that maximize their revenue. To elaborate, given the buying decision $(w^A, w^B) \in \{0,1\}^2$, the expected payoff of the SU is known as

$$U_{w^A w^B} = V_{w^A w^B} - w^A \alpha^A - w^B \alpha^B, \tag{1}$$

where $V_{w^A w^B}$ denotes the expected reward of the SU provided the buying decision (w^A, w^B).

In this work, we investigate the interplay between the optimal decisions of the SU and the pricing of the PPs. We first study the decisions of the SU and then focus on the

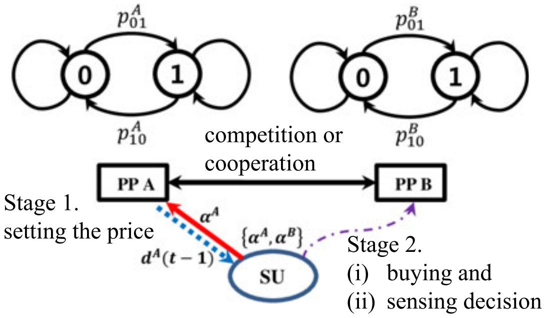


Fig. 1. Channel and game models. PP i sets price α^i for the channel information $d^i(t-1)$. The SU decides to buy the information, and then, selects the channel to sense. Depending on the sensing result, the SU transmits on the channel that it sensed.

pricing of the PPs under two different scenarios of competition and cooperation.

4 Optimal Decisions of Secondary User

At the beginning of time slot t, when the SU has data to send, it would sense the channel with the highest expected reward because it can sense only one channel. Since the state of channel i follows the on-off Markov model (i.e., Gilbert-Elliott channel), the SU can make more precise estimation on the expected reward for channel i with the channel state information at $t-1$ than when it predicts the expected reward with the stationary probabilities. In this section, we formulate the reward estimation of the SU, and investigate the value of the channel information by comparing the performance of the SU with and without the information. We omit subscript if there is no confusion.

(1) Expected reward of the SU without previous channel state information: We first consider when the previous channel state information ($\{d^i(t-1)\}$) is not available, i.e., when $w^A = w^B = 0$. Since no channel state information is available, the SU will make its sensing decision based on long-term characteristics of the channels. Given the steady-state probability π_j^i that channel i is in state $j \in \{0(idle), 1(busy)\}$, we define the gain G^i of sensing channel i as

$$G^i = \pi_0^i \cdot (1-f) \cdot r^i - \pi_1^i \cdot m \cdot a^i,$$

where f and m denote the false alarm probability and miss detection probability of channel i, respectively, and r^i and a^i denote the reward upon a successful transmission and the penalty upon a collision for transmitting on channel i, respectively. The first term is the expected reward when the channel is idle and the SU's sensing is correct (i.e., no false alarm), and the second term is the expected penalty when the channel is busy and the sensing result is wrong (i.e., miss detection).

Since the SU makes a sensing decision by comparing G^A and G^B, the expected reward of the SU can be written as

$$V_{00} = \max\{G^A, G^B, 0\}. \tag{2}$$

From (1), we can obtain the payoff of the SU as $U_{00} = V_{00}$.

(2) Expected reward of the SU with previous channel state information: We next consider the case that the SU makes the buying decision $(w^A, w^B) = (1,1)$ and obtains the channel state information of both the channels, i.e., $d^A(t-1)$ and $d^B(t-1)$. Letting $\gamma_i = d^i(t-1)$ and from the channel state transition matrix P^i, the SU can estimate the expected channel gain $G_{\gamma_i}^i$ at time slot t given the previous channel state γ_i as

$$G_{\gamma_i}^i = p_{\gamma_i 0}^i \cdot (1-f) \cdot r^i - p_{\gamma_i 1}^i \cdot m \cdot a^i,$$

where $p_{\gamma_i 0}^i$ and $p_{\gamma_i 1}^i$ denote the transition probability from γ_i to 0 (idle) and to 1 (busy), respectively. The SU will make the sensing decision by comparing $G_{\gamma_A}^A$ and $G_{\gamma_B}^B$.

At a given time slot t, the channel state at time $t-1$ is one of the four cases, i.e.,$\left(d^A(t-1), d^B(t-1)\right) \in \{(0,0), (0,1), (1,0), (1,1)\}$. Since the channels are independent, we can obtain the expected reward as

$$V_{11} = \textstyle\sum_{\gamma_A=0}^{1} \sum_{\gamma_B=0}^{1} \pi_{\gamma_A}^A \pi_{\gamma_B}^B \max\{G_{\gamma_A}^A, G_{\gamma_B}^B, 0\}, \tag{3}$$

where $\pi_{\gamma_A}^A \pi_{\gamma_B}^B$ denotes the probability of state (γ_A, γ_B) at time slot $t-1$. From (1), the payoff of the SU will be $U_{11} = V_{11} - \alpha^A - \alpha^B$.

Similarly, when the SU has the previous state information of only one channel, the expected reward and the payoff of the SU can be calculated as follows:

- When $\gamma_A = d^A(t-1)$ is known with $(w^A, w^B) = (1,0)$,

$$V_{10} = \textstyle\sum_{\gamma_A=0}^{1} \pi_{\gamma_A}^A \max\{G_{\gamma_A}^A, G^B, 0\},\ U_{10} = V_{10} - \alpha^A.$$

- When $\gamma_B = d^B(t-1)$ is known with $(w^A, w^B) = (0,1)$,

$$V_{01} = \textstyle\sum_{\gamma_B=0}^{1} \pi_{\gamma_B}^B \max\{G^A, G_{\gamma_B}^B, 0\},\ U_{01} = V_{01} - \alpha^B. \tag{4}$$

We expect that the reward will be higher with the channel information. The following proposition shows that with additional information, the reward increases.

Proposition 4.1 (Non-negative gain of information). *The expected gains satisfy (i) $V_{00} \leq V_{10} \leq V_{11}$, and (ii) $V_{00} \leq V_{01} \leq V_{11}$.*

The results come from the definitions of the expected gains and the Jensen's inequality. We refer Appendix A. for the proof. We now can specify the buying decision of the SU that maximizes its payoffs as

$$\left(w^A(t), w^B(t)\right) = \arg\max_{(j,k)\in\{0,1\}^2} U_{jk}, \tag{5}$$

where $U_{ij} = V_{ij} - w^i \cdot \alpha^A - w^j \cdot \alpha^B$. With the buying decision w^i of channel i, the expected gain $(1-w^i) \cdot G^i + w^i \cdot G_{\gamma_i}^i$ will change according to the previous channel state γ_i. The SU will sense the channel with the highest expected gain, i.e.,

$$\begin{aligned}(x^A(t), x^B(t)) &\\ = \arg\max_{\{(0,1),(1,0),(0,0)\}} &\{x^A \cdot [(1-w^A) \cdot G^A + w^A \cdot G_{\gamma_A}^A] \\ &+ x^B \cdot [(1-w^B) \cdot G^B + w^B \cdot G_{\gamma_B}^B]\}.\end{aligned} \tag{6}$$

5 Pricing Game of Primary Providers

Based on the optimal decisions of the SU in Section 4, we consider the behaviors of the PPs. We assume that the gain of the PPs from leasing the channel is constant (e.g., a flat fee) and the loss from collision is perfectly

(i) If the single-info gain of the SU is greater than the marginal-info gain, i.e., if $V_{10}-V_{00}\geq V_{11}-V_{01}$, then

$$\alpha^A_{\text{comp}}\left(\alpha^B_{\text{comp}}\right)=\begin{cases}V_{11}-V_{01} & \text{if } \alpha^B_{\text{comp}}\in[0,V_{11}-V_{10})\\ \alpha^B_{\text{comp}}+V_{10}-V_{01} & \text{if } \alpha^B_{\text{comp}}\in[V_{11}-V_{10},V_{01}-V_{00}]\\ V_{10}-V_{00} & \text{if } \alpha^B_{\text{comp}}\in(V_{01}-V_{00},\infty).\end{cases}\tag{12}$$

(ii) If the marginal-info gain of the SU is greater than the single-info gain, i.e., if $V_{10}-V_{00}\leq V_{11}-V_{01}$, then

$$\alpha^A_{\text{comp}}\left(\alpha^B_{\text{comp}}\right)=\begin{cases}V_{11}-V_{10} & \text{if } \alpha^B_{\text{comp}}\in[0,V_{10}-V_{00})\\ -\alpha^B_{\text{comp}}+V_{11}-V_{00} & \text{if } \alpha^B_{\text{comp}}\in[V_{10}-V_{00},V_{11}-V_{01}]\\ V_{01}-V_{00} & \text{if } (V_{11}-V_{01},\infty).\end{cases}\tag{13}$$

(a) The price α^A_{comp} of PP A as a function of α^B_{comp}.

(i) If the single-info gain of the SU is greater than the marginal-info gain, i.e., if $V_{01}-V_{00}\geq V_{11}-V_{10}$, then

$$\alpha^B_{\text{comp}}\left(\alpha^A_{\text{comp}}\right)=\begin{cases}V_{11}-V_{10} & \text{if } \alpha^A_{\text{comp}}\in[0,V_{11}-V_{01})\\ \alpha^A_{\text{comp}}+V_{01}-V_{10} & \text{if } \alpha^A_{\text{comp}}\in[V_{11}-V_{01},V_{10}-V_{00}]\\ V_{01}-V_{00} & \text{if } \alpha^A_{\text{comp}}\in(V_{10}-V_{00},\infty).\end{cases}\tag{14}$$

(ii) If the marginal-info gain of the SU is greater than the single-info gain, i.e., if $V_{01}-V_{00}\leq V_{11}-V_{10}$, then

$$\alpha^B_{\text{comp}}\left(\alpha^A_{\text{comp}}\right)=\begin{cases}V_{11}-V_{01} & \text{if } \alpha^A_{\text{comp}}\in[0,V_{01}-V_{00})\\ -\alpha^A_{\text{comp}}+V_{11}-V_{00} & \text{if } \alpha^A_{\text{comp}}\in[V_{01}-V_{00},V_{11}-V_{10}]\\ V_{10}-V_{00} & \text{if } (V_{11}-V_{10},\infty).\end{cases}\tag{15}$$

(b) The price α^B_{comp} of PP B as a function of α^A_{comp}.

Fig. 2. Price of PP i when the fixed price of the other PP is known.

compensated by the penalty of the SU (e.g., see [23]). The PPs can earn additional revenue by selling their channel state information, and thus they will try to maximize their revenue by setting the price to the highest value as long as the SU buys the information. We assume that the price is non-negative, and investigate the pricing of the PPs in two different scenarios of competition and cooperation between the PPs.

5.1 Competitive Scenario

In a competitive scenario, the two PPs do not share their price information with each other, while the SU knows both the prices. As a result, each PP will lower its price to attract the SU to buy. We show that there are Nash equilibria where both the PPs do not change their prices.

Let α^i_{comp} be the price of previous channel state information of PP i in the competitive scenario. Once α^A_{comp} and α^B_{comp} are decided, the SU will make the buying decision as specified in (5). For example, if $\arg\max U_{jk}=(1,0)$, the SU will buy only the previous state information of channel A. We consider the price setting of PP A assuming that the price of PP B is known to PP A, and vice versa. This will be used later to estimate the pricing behavior of the other PP. Recall that V_{jk} is the expected reward of the SU with and without the previous state information of the two channels. For channel A, we denote $V_{10}-V_{00}$ by *single-info* gain, and $V_{11}-V_{01}$ by *marginal-info* gain. Similarly, for channel B, we denote $V_{01}-V_{00}$ and $V_{11}-V_{10}$ by single-info gain and by marginal-info gain, respectively.

Fig. 2 shows the price of previous channel state information of PPs. We first derive the price of previous channel state information of PPs when the single-info gain is greater than the marginal-info gain, i.e.,

$$V_{01}-V_{00}\geq V_{11}-V_{10}.\tag{7}$$

We further divide it into two cases:

$$\text{case 1)}\ \ \alpha^B_{\text{comp}}\leq V_{01}-V_{00},\tag{8}$$

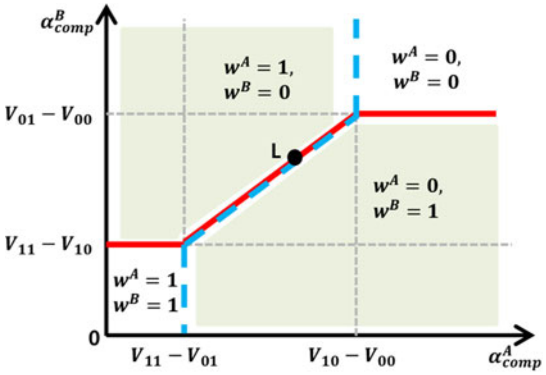
$$\text{case 2)}\ \ \alpha^B_{\text{comp}}\geq V_{01}-V_{00}.\tag{9}$$

In case 1), given α^B_{comp}, we have $V_{01}-\alpha^B_{\text{comp}}\geq V_{00}$, which implies $U_{00}\leq U_{01}$ from (2) and (4), and thus $(w^A,w^B)=(0,0)$ cannot be a buying decision of the SU. Hence, to sell its information, PP A will set its price such that the payoff when the SU buys the information of PP A (i.e., either U_{11} or U_{10}) is no smaller than U_{01}. This strategy results in $\max\{U_{11},U_{10}\}\geq U_{01}$, or equivalently, $\max\{V_{11}-\alpha^A_{\text{comp}}-V_{01}, V_{10}-\alpha^A_{\text{comp}}-V_{01}+\alpha^B_{\text{comp}}\}\geq 0$. Note that $V_{11}-V_{10}\geq 0$ from Proposition 4.1. From (7) and (8), and since PP A will maximize its revenue, we have

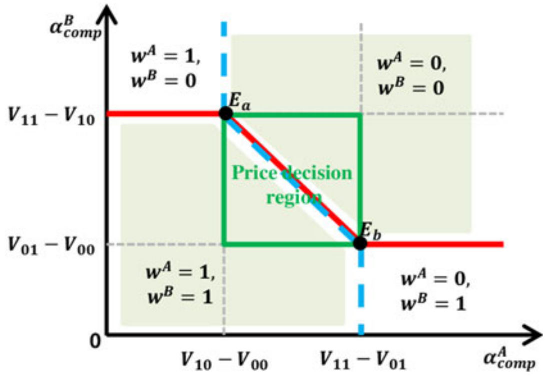
$$\begin{aligned}\alpha^A_{\text{comp}}&=\max\{V_{11}-V_{01},V_{10}-V_{01}+\alpha^B_{\text{comp}}\}\\&=\begin{cases}V_{11}-V_{01}, & \text{if } \alpha^B_{\text{comp}}\leq V_{11}-V_{10},\\ V_{10}-V_{01}+\alpha^B_{\text{comp}}, & \text{if } V_{11}-V_{10}\leq\alpha^B_{\text{comp}}\leq V_{01}-V_{00}.\end{cases}\end{aligned}\tag{10}$$

Case 2) is the case that $(w^A,w^B)=(0,1)$ cannot be a buying decision of the SU, since $U_{00}>U_{01}$ from (11), (2), and (4). Therefore, PP A will set its price such that either U_{11} or U_{10} is no smaller than U_{00}, and this strategy results in $\max\{V_{11}-\alpha^A_{\text{comp}}-\alpha^B_{\text{comp}}-V_{00},V_{10}-\alpha^A_{\text{comp}}-V_{00}\}\geq 0$. For maximizing its revenue, PP A will have

$$\alpha^A_{\text{comp}}=\begin{cases}V_{11}-V_{00}-\alpha^B_{\text{comp}}, & \text{if } \alpha^B_{\text{comp}}\leq V_{11}-V_{10},\\ V_{10}-V_{00}, & \text{if } \alpha^B_{\text{comp}}\geq V_{11}-V_{10}.\end{cases}\tag{11}$$



(a) When the single-info gain of the SU is greater than the marginal-info, i.e., when $V_{10} - V_{00} \geq V_{11} - V_{01}$.



(b) When the marginal-info gain of the SU is greater than the single-info gain, i.e., when $V_{11} - V_{01} > V_{10} - V_{00}$.

Fig. 3. Pricing of the PPs and the corresponding buying decisions of the SU.

However, the first condition of (11) conflicts with (7) and (11), and thus does not hold. Combining (10) and the second condition of (11), we have (12).

Similarly, we can show (13) for the case when the marginal-info gain is greater than the single-info gain, i.e., $V_{11} - V_{01} > V_{10} - V_{00}$. Also, (14) and (15) can be shown by the symmetry between PP A and PP B.

We note that in (12), (13), (14), and (15), PPs need to know $\{V_{00}, V_{01}, V_{10}, V_{11}\}$, which can be obtained from (2), (3), and (4) with stationary transition probability matrix P^i, false alarm and miss detection probabilities, reward r^i, and penalty a^i. The information is common knowledge or can be calculated at PP as explained in Section 3.1.

Fig. 3 shows the pricing of PP i given the other PP's price and the corresponding buying decision of the SU. The buying decision of the SU is classified into four different regions according to PPs' prices. In Fig. 3a, the dashed line represents PP A's price $\alpha^A_{\text{comp}}(\alpha^B_{\text{comp}})$ when PP B's price α^B_{comp} is fixed and known, and the solid line represents PP B's price $\alpha^B_{\text{comp}}(\alpha^A_{\text{comp}})$ when PP A's price α^A_{comp} is fixed and known. At the point $(\alpha^A_{\text{comp}}, \alpha^B_{\text{comp}}) = (V_{11} - V_{01}, V_{11} - V_{10})$, buying decisions $(w^A, w^B) = (1, 1), (1, 0)$, and $(0, 1)$ give the same payoff to the SU. We assume that if the SU has multiple buying decisions, it makes a choice that maximizes the social welfare. Therefore, at the point $(V_{11} - V_{01}, V_{11} - V_{10})$, the SU will information of both PPs. Let us consider the other price points $(\alpha^A_{\text{comp}}, \alpha^B_{\text{comp}})$ where the two lines overlap (e.g., Point L in Fig. 3a). At these points, buying decisions $(w^A, w^B) = (1, 0)$ and $(0, 1)$ give the same payoff to the SU. We assume that the SU has no preference to any PPs so that each PP will be chosen with the same probability $(0.5, 0.5)$. Therefore, if PP A slightly decreases its price and PP B keeps α^B_{comp} (i.e., if the point moves to the left), then the SU will buy the information of PP A only. Similarly, if PP B slightly decreases its price and PP A keeps α^A_{comp} (i.e., if the point moves to the bottom), then the SU will buy the information of PP B only. Since each PP does not know the price of the other, they will lower the price to sell their information to the SU until they arrive at point $(V_{11} - V_{01}, V_{11} - V_{10})$.

In Fig. 3b, at all the price points where the two lines overlap (i.e., between E_a and E_b), buying decisions $(w^A, w^B) = (1, 1)$ and $(0, 0)$ give the same payoff to the SU. The SU will buy both of information since it maximizes the social welfare. If either PP A or B increases its price, the SU will not buy any information, and thus both of them will not increase their price. On the other hand, if either PP A or B decreases its price, its revenue will decrease, and thus it will not change its price.

The price points at which the PPs no longer change their prices become a Nash equilibrium point as shown in the following proposition.

Proposition 5.1 (Nash equilibrium). *Under competition, the Nash equilibria of channel state information prices are as follows:*

(i) *If* $V_{10} - V_{00} \geq V_{11} - V_{01}$, *then* $(\alpha^A_{\text{comp}}, \alpha^B_{\text{comp}}) = (V_{11} - V_{01}, V_{11} - V_{10})$.

(ii) *If* $V_{10} - V_{00} \leq V_{11} - V_{01}$, *then* $(\alpha^A_{\text{comp}}, \alpha^B_{\text{comp}}) = \{(\alpha^A, \alpha^B) \mid \alpha^A + \alpha^B = V_{11} - V_{00}\}$.

For an arbitrary point of the above proposition, no PPs has a motivation to deviate from the point called Nash equilibrium. More specifically, a PP will not decrease its price since it gets lower payoff by decreasing its price. If a PP increase its price, the SU will buy only opponent's information, and thus it gets zero payoff. We refer to Appendix B. for the proof. As shown in Proposition 5.1, if $V_{11} - V_{01} \geq V_{10} - V_{00}$, multiple Nash equilibria exist. Since our problem is a one-shot game, it is difficult for each PP to decide its price without knowledge about the price of the other PP. Since PP A sets its price in $[V_{10} - V_{00}, V_{11} - V_{01}]$ and PP B sets its price in $[V_{01} - V_{00}, V_{11} - V_{10}]$, the prices will be some point within the green square box shown in Fig. 3b. It is not very clear exactly what point the prices will be. To avoid such uncertainty, we assume that the PPs behave conservatively, i.e., they prefer a smaller guaranteed profit than a large uncertain profit. Under this assumption, the PPs want to secure its revenue regardless of the other's price, and then their prices will be settled down at $(V_{10} - V_{00}, V_{01} - V_{00})$, since below which each PP can sell its information. The price point is not a Nash equilibrium and the players will change their behavior if they know the other's a priori price. Also, at the price, the PPs will have no greater revenue than at any of the Nash equilibrium points.

5.2 Cooperative Scenario

In a cooperative scenario, the two PPs share their information and cooperate with each other as long as they achieve a higher individual revenue than their revenues under competition. In this case, it is not difficult to maximize the revenues of the PPs. Instead, it is necessary to predetermine how to share the total revenue between the PPs. In this work, we use the Shapley value [11] due to its desirable properties of efficiency, symmetry, linearity, etc. The Shapley value solves the distribution problem of the total surplus revenue produced under cooperation. Specifically, the total revenue is distributed among the users participating in the cooperation depending on the level of contribution to the cooperation. Let ϕ^i denote the revenue of PP i when they cooperate. Let $N = \{A, B\}$, and let $\Pi^A = \{\{A, B\}, \{A\}\}$ and $\Pi^B = \{\{A, B\}, \{B\}\}$ be the set of all cooperation combinations containing, A and B, respectively. Further, let $v(S)$ be the revenue that can be earned by cooperation combination $S \in \{\{A, B\}, \{A\}, \{B\}, \emptyset\}$. In the following paragraphs, we calculate the revenues of the PPs by the Shapley value in two distinct cases.

(1) When the single-info gain is greater than the marginal-info gain (when $V_{10} - V_{00} \geq V_{11} - V_{01}$): We can obtain $v(\{A\}) = V_{11} - V_{01}$, $v(\{B\}) = V_{11} - V_{10}$, and $v(\{A, B\}) = \max\{2V_{11} - V_{10} - V_{01}, V_{10} - V_{00}, V_{01} - V_{00}\}$. It can be shown that $v(\{A, B\})$ is the maximum sum of revenues that the PPs can earn under cooperation. Note that under competition, the total revenue of the PPs at Nash equilibrium points is $2V_{11} - V_{10} - V_{01}$ with the buying decision of $(w^A, w^B) = (1, 1)$, $V_{10} - V_{00}$ with the buying decision of $(w^A, w^B) = (1, 0)$, or $V_{01} - V_{00}$ with $(w^A, w^B) = (0, 1)$. Under cooperation, the PPs will maximize their total revenue by selecting the prices such that $\max\{2V_{11} - V_{10} - V_{01}, V_{10} - V_{00}, V_{01} - V_{00}\}$. Hence, letting $\theta = \max\{2V_{11} - V_{10} - V_{01}, V_{10} - V_{00}, V_{01} - V_{00}\}$, we can obtain the Shapley value of PP A as $\phi^A = \sum_{S \in \Pi^A} [v(S) - v(S - A)] \frac{(|S|-1)!(|N|-|S|)!}{|N|!} = \frac{\theta + V_{10} - V_{01}}{2}$. Similarly, we can calculate the Shapley value ϕ^B of PP B as $\phi^B = \frac{\theta + V_{01} - V_{10}}{2}$. Then, the price of anarchy (i.e., the ratio of their total profits between cooperation and competition) is $\frac{\theta}{2V_{11} - V_{10} - V_{01}}$ which is larger than or equal to 1, and increases when the information from one PP is much more valuable than the other. The equality holds when $V_{10} - V_{00} = V_{11} - V_{01}$ or $\theta = 2V_{11} - V_{10} - V_{01}$.

(2) When the marginal-info gain is greater than the single-info gain (when $V_{11} - V_{01} \geq V_{10} - V_{00}$): Now, under the conservative-behavior assumption of the PPs, we have $v(\{A\}) = V_{10} - V_{00}$, $v(\{B\}) = V_{01} - V_{00}$, and $v(\{A, B\}) = V_{11} - V_{00}$, where $v(\{A, B\})$ is a point on the tilted line (E_a, E_b) in Fig. 3b. Clearly, $v(\{A, B\})$ is the largest revenue that the PPs can achieve. Then the Shapley values can be calculated as

$$\phi^A = \frac{V_{11} + V_{10} - V_{01} - V_{00}}{2}, \; \phi^B = \frac{V_{11} + V_{01} - V_{10} - V_{00}}{2},$$

and the price of anarchy is $\frac{V_{11} - V_{00}}{V_{10} + V_{01} - 2V_{00}}$ which is larger than or equal to 1, and increases the values of information from PPs are close to each other. The equality holds when $V_{11} - V_{01} = V_{10} - V_{00}$.

Interestingly, although the PPs achieve a larger revenue by cooperating with each other, the social welfare of the system, defined as the sum of the revenue of all the PPs and the payoff of the SU, may decrease as described in Proposition 5.2.

Proposition 5.2 (Non-increasing social welfare). *The social welfare under cooperation is no greater than the social welfare under (conservative) competition.*

When the SU makes a buying decision (w^A, w^B), the value of the expected reward of the SU, V_{w^A, w^B}, is distributed to the SU and PPs depending on the buying decision. In other words, the PPs get $w^i \cdot \alpha^i_{comp}(\text{or}\phi^i)$, the SU gets U_{w^A, w^B}, and the social welfare is V_{w^A, w^B} from (1). Therefore, from Proposition 4.1, the social welfare is maximized when $(w^A, w^B) = (1, 1)$, and this buying decision is made under competition. However, under cooperation, the PPs' pricing sometimes leads the SU to buy only one of information (i.e., $(w^A, w^B) = (1, 0)$ or $(0, 1)$), and this results in the decrease of the social welfare even though it increases their own profit. We refer to Appendix C. for the detailed proof. Proposition 5.2 implies that cooperation between the PPs may decrease the social welfare by excessively reducing the payoff of the SU. In order to prevent substantial loss of the social welfare, it may need an authority (e.g., government) to regulate cooperation between the PPs such that the prices are determined at Nash equilibria, and the social welfare is maximized with buying decision $(w^A, w^B) = (1, 1)$.

6 Extension to Multiple Secondary Users with Single Price

We now extend our results to multiple SUs with two competitive PPs. We assume that to ensure no bias fairness each PP adopts a single price α^i policy for all the SUs despite different information values (i.e., willingness to pay) of each SU.

Let r^i_j be the reward of SU j from channel i, and let a^i denote the collision penalty for channel i. Let f_j and m_j denote the false alarm probability and the miss detection probability of SU j, respectively, that depends on the sensitivity of the SU device. We let α^i_j denote the price obtained in the previous section, i.e., the price that SU j will pay for the channel information of PP i when SU j is the only user in the system.

PP i can expect the behavior of each SU given the prices (α^A, α^B): for SU j, if the single-info gain $V^j_{10} - V^j_{00}$ is greater then the marginal-info gain $V^j_{11} - V^j_{01}$, then it will behave as shown in Fig. 3a, and otherwise, behave as shown in Fig. 3b. In our competitive scenario, the PPs do not know each other's price and even pricing policy. Given this situation, each PP will decide its single price to maximize its revenue as follow.

Proposition 6.1. *Under competition, PP i can maximize its revenue with the channel state information price*

$$\alpha^i = \max{}^{(k)}_j \alpha^i_j,$$
$$\text{for } k = \text{argmax}_{n=1,2,\ldots} \{n \cdot \max{}^{(n)}_j \alpha^i_j\},$$

where $\max^{(n)}\{\cdot\}$ is the nth highest value.

Proof. We note that if price α^i is higher than price α^i_j, then SU j will not buy information of channel i. Hence, if price α^i is higher than $\max^{(1)}_j \alpha^i_j$, no SU will buy the information of channel i. Suppose that we set price α^i such that $\max^{(2)}_j \alpha^i_j < \alpha^i \leq \max^{(1)}_j \alpha^i_j$, then there is one SU $j (= \text{argmax}^{(1)}_j \alpha^i_j)$ that will buy the information of channel i,

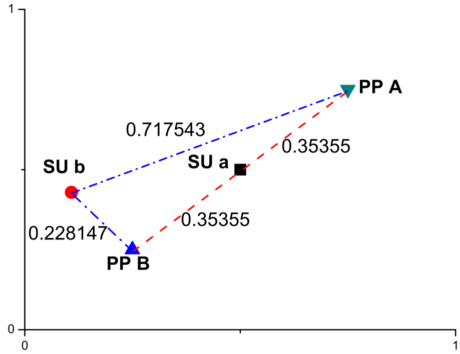


Fig. 4. Location of the PPs and the SUs.

where $\mathrm{argmax}^{(n)}\{\cdot\}$ is the nth highest valued argument. Similarly, if $\alpha^i \in (\max_j^{(k+1)}\alpha_j^i, \max_j^{(k)}\alpha_j^i]$, then there are at least k SUs that will buy the information of channel i. PP i can achieve more than $k \cdot \max_j^{(k)}\alpha_j^i$ revenue. In our model, we assume that PP i does not know the price and the strategy of the other PPs. So it would sets its price in a conservative manner as in Proposition 5.1. To this end, PP i will set the price of the previous channel state information as $\alpha^i = \max_j^{(k)}\alpha_j^i$, where $k = \mathrm{argmax}_{n=1,2,\ldots}\{n\cdot \max_j^{(n)}\alpha_j^i\}$ and receives at least k as its reward. □

Once PP i determines its price as α^i, it gets at least $\max_{n=1,2,\ldots}\{n \cdot \max_j^{(n)}\alpha_j^i\}$ regardless of the other PP's price.

If SU j is the only SU in the system, it will buy the previous channel information of PP i as α_j^i. However, when multiple SUs exist and the information price of PP i is set as $\alpha^i < \alpha_j^i$, the SU j can buy the same information with lower price and get higher payoff from PP i. On the other hand, if the information price of PP i is set as $\alpha^i > \alpha_j^i$, the SU j will not buy the information and get lower payoff from PP i. We will demonstrate this effect in our simulation results.

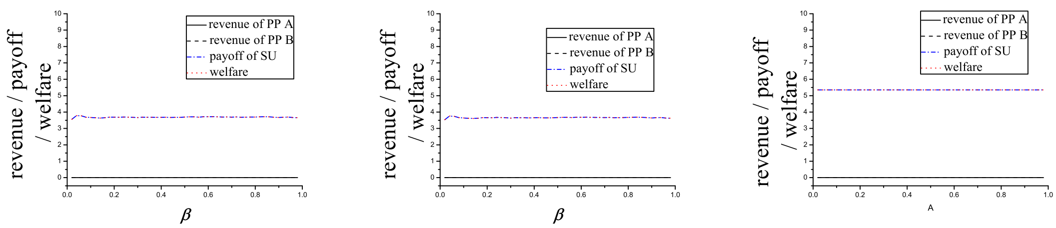
7 SIMULATION RESULTS

We have shown that the previous state information of the channels is valuable to the SUs, and studied the buying and the sensing decision of the SUs and the pricing decision of the PPs under competition and under cooperation. In this section, we simulate the behaviors of the PPs and the SUs, and observe their performance: the revenues of the PPs, the payoff of the SUs, and the social welfare. We assume that the PPs behave conservatively, i.e., when the marginal-info gain is greater than the single-info gain as in Fig. 3b, the PPs set their price at the single-info gain. We also assume that under an equal payoff, the SUs make a choice that maximizes the social welfare, e.g., if $U_{11} = U_{10}$, for an SU, it decides $(w^A, w^B) = (1,1)$ since $V_{11} \geq V_{10}$.

We consider a Markov chain channel model as shown in Fig. 1. We consider a special case that the channel behavior of PP A is symmetric, i.e., $p_{01}^B = p_{10}^B = \beta$, for ease of exposition. Changing β, we measure the revenues of the PPs, the payoff of the SUs, and the social welfare in different scenarios: i) when the channel information is not provided, ii) when the PPs compete with each other, iii) when the PPs cooperate with each other, and iv) when there are multiple SUs under competition of the PPs. There are three types of cognitive radio devices, and false alarm and miss detection probabilities of device 1, 2, and 3 are $(0.1, 0.1)$, $(0.15, 0.1)$, and $(0.1, 0.15)$, respectively. Each SU has one of three devices at random, and located within 1×1 square uniformly at random. PPs A and B are located at $(0.75, 0.75)$ and $(0.25, 0.25)$, respectively. We denote d_{ij} by the distance between PP i and SU j. The reward of SU j from PP i depends on the distance as $r_j^i = w \log\frac{P\cdot d_{ij}^{-1}}{N}$, where w, P, and N are bandwidth, transmission power, and noise of channel i, respectively. In this simulation, we set $w = 2.0$, $P = 1.0$, and $N = 0.01$. The penalties of channel A and B are both 50. For simulations of i)-iii), we consider a single SU, i.e., SU a in Fig. 4, whose error probabilities and location are $f = 0.1$, $m = 0.15$, and $(x, y) = (0.5, 0.5)$, respectively. We first verify our analysis through simulations, by measuring the revenues of the PPs, the payoff of the SU, and the social welfare. The results of analysis and simulations agree with each other and we omit them due to the limited space. We now provide more detailed simulation results.

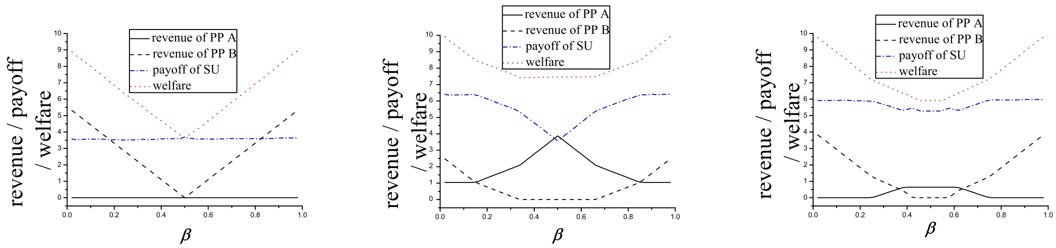
(1) Revenue, payoff, welfare when the channel information is not provided: We observe the performance of the PPs and SU a when the channel information is not provided. We consider three different types of channel B: Case 1) $(p_{01}^B, p_{10}^B) = (0.5, 0.5)$ for the case when the previous state information (of channel B) is not helpful to predict the current state (of channel B), Case 2) $(p_{01}^B, p_{10}^B) = (0.85, 0.85)$, for the case when the previous state information (of channel B) is useful to predict the current channel state (of channel B), and Case 3) $(p_{01}^B, p_{10}^B) = (0.55, 0.75)$, for a more general case with asymmetric transition probability. Fig. 5 shows the revenues of the PPs, the payoff of the SU, and the social welfare (the sum of the revenues of the PPs and the payoff of the SU). Since the PPs do not provide the channel information, they achieve zero revenue and the social welfare is the same as the payoff of the SU.

(2) Revenue, payoff, welfare under competition: With the channel information, the SU a can make a better prediction on the channel state, which in turn increases the social



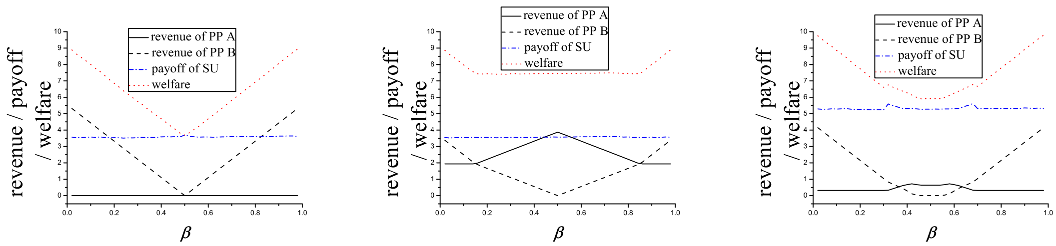
(a) $(p_{01}^B, p_{10}^B) = (0.5, 0.5)$ (b) $(p_{01}^B, p_{10}^B) = (0.85, 0.85)$ (c) $(p_{01}^B, p_{10}^B) = (0.55, 0.75)$

Fig. 5. Revenues of the PPs, payoff of the SU, and the social welfare when β varies with no information.



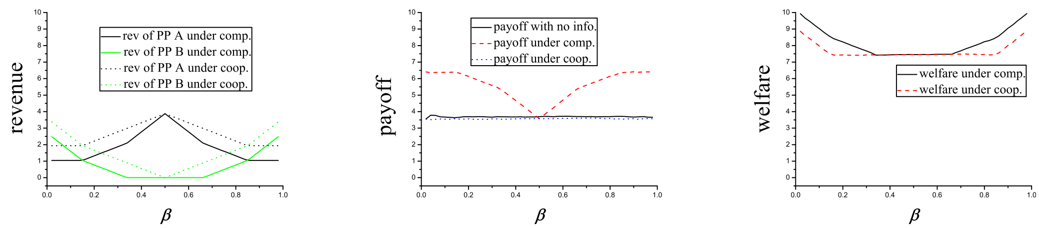
(a) $(p^B_{01}, p^B_{10}) = (0.5, 0.5)$ (b) $(p^B_{01}, p^B_{10}) = (0.85, 0.85)$ (c) $(p^B_{01}, p^B_{10}) = (0.55, 0.75)$

Fig. 6. Revenues of the PPs, payoff of the SU, and the social welfare when β varies under competition.



(a) $(p^B_{01}, p^B_{10}) = (0.5, 0.5)$ (b) $(p^B_{01}, p^B_{10}) = (0.85, 0.85)$ (c) $(p^B_{01}, p^B_{10}) = (0.55, 0.75)$

Fig. 7. Revenues of the PPs, payoff of the SU, and the social welfare when β varies under cooperation.



(a) Revenues of the PPs (b) Payoff of the SU (c) Social welfare

Fig. 8. Revenues of the PPs, payoff of the SU, and the social welfare under differenet pricing scenarios.

welfare as well as the PP's revenue. We run simulations to see the behavior of the PPs and the SU in two scenarios of competition and cooperation between the PPs. Fig. 6 demonstrates the results under the competition. Comparing Fig. 5 with Fig. 6, the payoffs of the SU improve, and the PPs earn additional revenues by selling their information, and the social welfare also increases. Note that when $(p^B_{01}, p^B_{10}) = (0.5, 0.5)$, the channel information of PP B is not helpful because the current channel state is independent from the previous channel state, and thus the revenue of PP B remains zero (because the SU does not buy the information). Further, when $\beta = 0.5$, the channel information of PP A is not beneficial either and the social welfare is equal to the value achieved without channel information.

(3) Revenue, payoff, welfare under cooperation: When the PPs cooperate with each other, it is expected that the revenues of the PPs will increase. Fig. 7 shows such results. In all the cases, the revenues of each PP are higher than those under competition, which also confirms that the Shapley value provides a fair distribution of the obtained gain to the PPs. However, cooperation does not always lead to an increase of the social welfare. To elaborate, the social welfares are smaller than the values under competition for Case 2 when $\beta \in [0, 0.34]$ in Fig. 7b, and for Case 3 when $\beta \in [0.26, 0.30]$ in Fig. 7c. This is because, when $V_{10} > V_{11} - \alpha^B$ (or when $V_{01} > V_{11} - \alpha^A$), the pricing of the PPs under competition leads to social welfare V_{11}, and the pricing under cooperation leads to social welfare V_{10}.

We now provide direct comparison of different pricing: no pricing (no channel information), pricing of channel information under competition, and pricing of channel information under cooperation. Fig. 8 demonstrates the performance when $(p^B_{01}, p^B_{10}) = (0.85, 0.85)$. It shows that *the channel information improves the payoff of the SU.* Further, it shows that the cooperation increases the revenues of the PPs, but decreases the payoff of the SU and the social welfare as we showed in Proposition 5.2.

(4) Revenue, payoff with multiple SUs under competition: We observe the effect of multiple SUs under competition of PPs. We consider two SUs denoted by SU a and b, which are located at $(0.5, 0.5)$ and $(0.10834, 0.42884)$, respectively as seen in Fig. 4. The two SUs have same devices of which error probabilities are 0.1 for the false alarm and 0.15 for the miss detection. We denote scenario 1 as the scenario that for each channel i, the information price is determined for each SU j independently (i.e., the SU j buys the information of channel i with the price α^i_j) as in Proposition 5.1. For scenario 2, we have that the SUs buy the information of channel i with same price α^i as in Proposition 6.1.

Fig. 9 and 10 show the revenues of PPs and the payoffs of SUs under scenario 1 and 2, respectively. For PP i, without loss of generality, we assume that $\alpha^i_a \geq \alpha^i_b$. While PP i earns

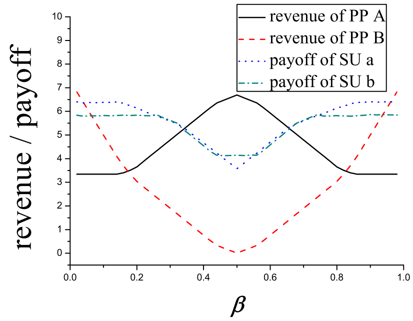


Fig. 9. Revenues of PPs and payoffs of SUs when β varies under scenario 1.

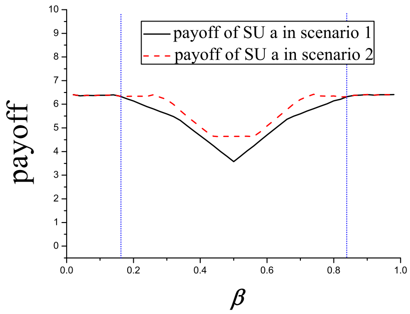


Fig. 11. Payoff of SU a when β varies under scenarios 1 and 2.

the revenue $\alpha_a^i + \alpha_b^i$ in scenario 1, in scenario 2 its revenue is α_a^i or $2\alpha^b$. Therefore, the both of PPs' revenues in scenario 2 are less than those in scenario 1. Fig. 11 compares the payoff of SU a under scenarios 1 and 2. For $\beta \in [0.16, 0.84]$, we have $\alpha_a^A \geq 2\alpha_b^A$ since SU a is closer to PP A than SU b. Since, $\alpha^A = \alpha_a^A$ if $\alpha_a^A \geq 2\alpha_b^A$, the price will be set as $\alpha^A = \alpha_b^A < \alpha_a^A$. Hence, the SU a can buy the same information with a lower price, so its payoffs are higher in the interval. The payoffs of SU b are shown in Fig. 12. The results are the opposite of SU a and can be explained similarly.

8 Discussion

In the section, we discuss some important issues and interesting open problems.

The results in this paper can be extended to more general scenarios with multiple players. For example, we can consider K PPs, in which case, each PP should calculate $V_{i_1 i_2 \ldots i_K}^j$ of SU j for all her previous channel information $i_k \in \{0, 1\}$ of PP k. Although they can compute the variables following the same line of our analysis with two-provider model, its computational complexity will be much higher as the number of PPs increases. For the scenario of K PPs and N SUs, the computational complexity is $O(N2^K)$. For multiple SUs scenarios, non-zero probability of collision between SUs makes the problem more challenging. In this work, we have derived our results on the premise of perfect collision resolution or the existence of a centralized arbitrator. However, even in the presence of a centralized arbitrator, it is an interesting question how each SU accesses to the sensed spectrum hole, in particular, when the access is associated with the information pricing. To this end, solving the joint optimization problem of channel accessing and channel information pricing, in particular in a distributed manner, is crucial and a very interesting open problem. We believe that our results provide an initial basis to that direction.

There are also important issues related with unique features of information pricing. For example, unlike commodity, an SU may *learn* channel states and rewards that it has experienced, and can make more accurate estimation on the coming channel state. Some SU may try to *resell* past channel state information that it has bought from PPs or experienced in the past. Unlike commodity, the seller still holds the information after selling. Also, there could be non-negligible communication *overhead* to transfer information between the players. In this work, we have not considered such information features, since, in our monitoring-and-sensing IoT device scenarios, an SU sporadically wakes up for a transmission, and thus it is unlikely to have multiple active SUs at the same time. Also, communication overhead can be minimized by exploiting the presence of the database: the overhead for writing can be minimized using a short signal-tone transmission, or by the database's capability to overhear PPs' channels with high accuracy, and the overhead for reading can be reduced by broadcasting the information in an encoded form. Nonetheless, developing novel techniques to compare the performance of information buying and learning, to allowing SUs to resell, and to take into account the overhead of information transfer is promising and an interesting open problem.

In this work, we have assumed stationary Gilbert-Elliott channel model, but our framework can be applied to more general channel models with memory, e.g., channel models with multiple states or rewards. When the channel states are non-stationary, however, the gain of information pricing will be not clear without proper schemes for detecting the change and learning new distribution parameters, e.g., [24]. The frequency of changes and the accuracy of detecting and learning will impact of the performance gain.

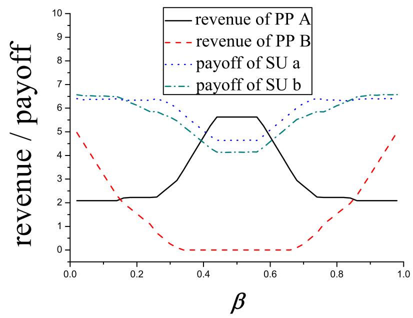


Fig. 10. Revenues of PPs and payoffs of SUs when β varies under scenario 2.

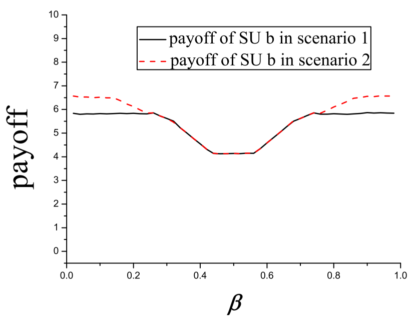


Fig. 12. Payoff of SU b when β varies under scenarios 1 and 2.

9 Conclusion

In this paper, we develop a hybrid DSA framework with spectrum sensing and database access, and show that the previous channel state information can benefit both the PPs and the SU under a game-theoretic market model. We investigate how the PPs make the pricing decision of the channel information in order to maximize their revenues under competition and under cooperation, and how the SU responds through the buying decision of the channel information and the sensing decision. We characterize Nash equilibria in both competitive and cooperative scenarios, and show that the cooperation between the PPs may decrease the social welfare (though it improves the revenues of the PPs). We extend the above results to multiple SUs, and investigate the impact of multiple SUs. We verify our results through numerical simulation with various parameters.

Appendix

A. Proof of Proposition 4.1

We show condition (i) in the proof. Condition (ii) can be shown following the same line of analysis. Note that

$$\begin{aligned} V_{11} &= \sum_{\gamma_A=0}^{1}\sum_{\gamma_B=0}^{1}\pi^A_{\gamma_A}\pi^B_{\gamma_B}\max\Big\{G^A_{\gamma_A}, G^B_{\gamma_B}, 0\Big\} \\ &\geq \sum_{\gamma_A=0}^{1}\pi^A_{\gamma_A}\max\Big\{\sum_{\gamma_B=0}^{1}\pi^B_{\gamma_B}G^A_{\gamma_A}, \sum_{\gamma_B=0}^{1}\pi^B_{\gamma_B}G^B_{\gamma_B}, 0\Big\} \\ &= \sum_{\gamma_A=0}^{1}\pi^A_{\gamma_A}\max\Big\{G^A_{\gamma_A}, G^B, 0\Big\} = V_{10}, \end{aligned}$$

where the inequality comes from the Jensen's inequality. The equality holds either when $\max\{G^B_0, G^B_1\} \leq G^A_{\gamma_A}$ or when $G^A_{\gamma_A} \leq \min\{G^B_0, G^B_1\}$ for all $\gamma_A \in \{0,1\}$. Similarly, the inequality $V_{10} \geq V_{00}$ can be obtained from

$$V_{10} = \sum_{\gamma_A=0}^{1}\pi^A_{\gamma_A}\max\Big\{G^A_{\gamma_A}, G^B, 0\Big\} \geq V_{00}.$$

The equality holds either when $G^B \leq G^A_{\gamma_A}$ or when $G^A_{\gamma_A} \leq G^B$ for all $\gamma_A \in \{0,1\}$.

B. Proof of Proposition 5.1

We first prove (i) when the single-info gain is greater than the marginal-info gain. If PP A sets its price larger than $V_{10} - V_{00}$, the SU will not buy the information of PP A since the buying decisions of not buying PP A, i.e., $(w^A, w^B) = (0,0)$ or $(0,1)$, give more payoff to the SU. Suppose that PP A sets the price α^A_{comp} between α^A_m and α^A_s. If PP B sets its price slightly smaller than the value $\alpha^B_{\text{comp}}(\alpha^A_{\text{comp}})$ of (14), then the SU will buy PP B's information only since the action $(w^A, w^B) = (0,1)$ gives the maximum revenue to the SU. Then the PP A lowers its price until the SU make a decision $(w^A, w^B) = (1,0)$. This movement of PPs (i.e., decreasing their prices) will be repeated until the prices equal to (α^A_m, α^B_m). At (α^A_m, α^B_m), the price decisions of the PPs become irrelevant; suppose that $\alpha^A_{\text{comp}} < \alpha^A_m$. Then for any $\alpha^B_{\text{comp}} < \alpha^B_m$, the SU will have a positive gain for buying the channel information of PP A, since $V_{11} - V_{01} - \alpha^A_{\text{comp}} > 0$, for buying the channel information of PP A. Hence, the SU will always buy the PP A's information regardless of PP B's price. Similarly, the price of PP B can be determined to $V_{11} - V_{10}$. Since the prices will not change from (α^A_m, α^B_m), it is a Nash equilibrium.

Next we prove (ii) when the marginal-info gain is greater than the single-info gain. Suppose that PP A sets its price α^A_{comp} between α^A_s and α^A_m. Then if PP B sets its price slightly smaller than the value $\alpha^B_{\text{comp}}(\alpha^A_{\text{comp}})$ of (15), the SU will buy both channels' information since the action $(w^A, w^B) = (1,1)$ gives the maximum revenue to the SU. If PP B sets its price higher than the $\alpha^B_{\text{comp}}(\alpha^A_{\text{comp}})$, then the SU will act in the way $(w^A, w^B) = (0,0)$. Hence, once the PPs set their price between (α^A_s, α^B_s) to (α^A_m, α^B_m), they will neither increase nor decrease their price. This result means that all the points over the line (E_a, E_b) are Nash Equilibria.

C. Proof of Proposition 5.2

First we consider the case that the single-info gain is greater than or equal to the marginal-info gain. Under competition, the social welfare is $(V_{11} - V_{01}) + (V_{11} - V_{10}) + (V_{11} - (V_{11} - V_{01}) - (V_{11} - V_{10})) = V_{11}$. Under cooperation, we can think of two case: 1) $\theta = \max\{2V_{11} - V_{10} - V_{01}, V_{10} - V_{00}, V_{01} - V_{00}\} = 2V_{11} - V_{10} - V_{01}$, and 2) $\theta = V_{10} - V_{00}$ or $V_{01} - V_{00}$. In the former, the social welfare under cooperation is equal to that under competition. However, in the latter, the social welfare is $(V_{10} + V_{00}) + (V_{10} - (V_{10} + V_{00})) = V_{10}$ or $(V_{01} + V_{00}) + (V_{01} - (V_{01} + V_{00}))) = V_{01}$, which are smaller than V_{11}.

Next we consider when the marginal-info gain is greater than the single-info gain. When the PPs are conservative, the social welfare under competition is $(V_{10} - V_{00}) + (V_{01} - V_{00}) + (V_{11} - (V_{10} - V_{00}) - (V_{01} - V_{00})) = V_{11}$. The social welfare under cooperation is $(V_{11} - V_{00}) + (V_{11} - (V_{11} - V_{00})) = V_{11}$. Considering both the cases, the social welfare under cooperation is smaller or equal to that under competition.

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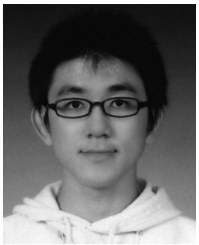

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