Age-optimal Scheduling over Hybrid Channels

Jiayu Pan, Ahmed M. Bedewy, Yin Sun, *Senior Member, IEEE,* and Ness B. Shroff, *Fellow, IEEE*

Abstract—We consider the problem of minimizing the age of information when a source can transmit status updates over two heterogeneous channels. Our work is motivated by recent developments in 5G mmWave technology, where transmissions may occur over an unreliable but fast (e.g., mmWave) channel or a slow reliable (e.g., sub-6GHz) channel. The unreliable channel is modeled as a time-correlated Gilbert-Elliot channel at a high rate when the channel is in the "ON" state. The reliable channel provides a deterministic but lower data rate. The scheduling strategy determines the channel to be used for transmission in each time slot, aiming to minimize the time-average age of information (AoI). The optimal scheduling problem is formulated as a Markov Decision Process (MDP), which is challenging to solve because super-modularity does not hold in a part of the state space. We address this challenge and show that a multi-dimensional threshold-type scheduling policy is optimal for minimizing the age. By exploiting the structure of the MDP and analyzing the discrete time Markov chains (DTMCs) of the threshold-type policy, we devise a low-complexity bisection algorithm to compute the optimal thresholds. We compare different scheduling policies using numerical simulations.

Index Terms—Age of information, hybrid channels, scheduling, and mmWave communications.

1 Introduction

Timely updates of the system state are of great significance in cyber-physical systems, such as vehicular networks, sensor networks, and UAV navigations. In these systems, freshly generated data is more valuable than outdated data. *Age of information* (AoI), or simply *age*, was introduced as an end-to-end application-layer metric to measure information freshness [2]–[25]. The age at time t is defined as $\Delta(t) = t - U_t$, where U_t is the generation time of the freshest packet that has been received by time t. The difference between age and classical performance metrics of wireless networks like delay and throughput is evident even in elementary queuing systems [3]. High throughput requires frequent status updates, which would cause a long waiting time in the queue that worsens timeliness. On the other hand, delay and waiting time can be greatly reduced by decreasing the update frequency, which, however, may increase the age because the status is updated infrequently.

In future wireless networks, sub-6GHz frequency spectrum is insufficient for fulfilling the high throughput demand of emerging real-time applications such as VR/AR applications, where contents must be delivered within 5-20 ms of latency, requiring a high throughput of 400-600 Mbps [26]. To address this challenge, 5G technology utilizes high-frequency millimeter wave (mmWave) bands such as 28/38 GHz, which provide a much higher data rate than sub-6GHz [27]. Verizon and Samsung demonstrated that a throughput of nearly 4Gbps was achieved in their mmWave demo system, using a 28GHz frequency band with 800MHz bandwidth [28]. However, unlike sub-6GHz spectrum bands, mmWave channels are highly unreliable due to blocking susceptibility, strong atmospheric absorption, and low penetration. Real-world smartphone experiments have shown that even obstructions by hands could significantly degrade the mmWave throughput [29]. One solution to mitigate this effect is to let sub-6GHz coexist with mmWave to form two *heterogeneous* channels, so that the user equipment can offload data to sub-6GHz when mmWave communications are unfeasible [30]–[33]. Some work has already been done based on mmWave/sub-6GHz heterogeneous networks [34], [35]. However, how to improve information freshness in such hybrid networks has remained largely unexplored.

In this study, we consider a hybrid status updating system where a source can transmit the update packets over an unreliable but fast mmWave channel or a slow reliable sub-6GHz channel. Our objective is to find a dynamic channel scheduling policy that minimizes the long-term average expected age. The main contributions of this paper are stated as follows:

- The optimal scheduling problem for minimizing the age over heterogeneous channels is formulated as a Markov Decision Process (MDP). The state transition of this MDP is complicated for two reasons: (i) the two channels have different data rates and packet transmission times, and (ii) the state of the unreliable mmWave channel is correlated over time. We prove that there exists a multi-dimensional threshold-type scheduling policy that is optimal. This optimality result holds for all possible values of the channel parameters. One of the tools for proving this result is super-modularity [36]. Because of the complicated state transitions, super-modularity holds in a part of the state space but not in the rest of the state space. This is a key difference from the scheduling problems considered earlier in prior studies, e.g.,

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J. Pan is with the Department of ECE, The Ohio State University, Columbus, OH 43210 USA (e-mail: pan.743@osu.edu).

A. M. Bedewy is with the Department of ECE, The Ohio State University, Columbus, OH 43210 USA (e-mail: bedewy.2@osu.edu).

Y. Sun is with the Department of ECE, Auburn University, Auburn, AL 36849 USA (e-mail: yzs0078@auburn.edu).

N. B. Shroff is with the Department of ECE and the Department of CSE, The Ohio State University, Columbus, OH 43210 USA (e-mail: shroff.11@osu.edu).

[10], [22], [23], [37]–[40]. To conquer this challenge, we develop additional techniques to show that the optimal scheduling policy has a threshold-type structure over the entire state space, including the part of state space where super-modularity does not hold.

- The state transition of the discrete time Markov chain (DTMC) for the threshold-type scheduling policy is complicated. Nonetheless, we show that the thresholds of the optimal scheduling policy can be evaluated efficiently, by using closed-form expressions or a low-complexity bisection search algorithm. Compared with the algorithms for calculating the thresholds and optimal scheduling policies in, e.g., [10], [22], [23], [37]–[40], our solution algorithms have much lower computational complexities.
- In the special case that the state of the unreliable mmWave channel is independent and identically distributed (i.i.d.) over time, the optimal scheduling policy is shown to possess a simpler and interesting form. Finally, numerical results are provided to validate our results by comparing with several other policies.

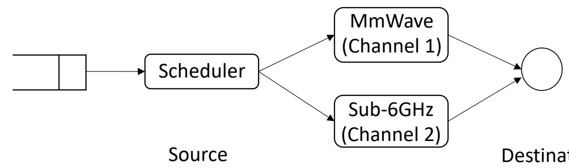

Figure 1. The system model for status updates in heterogeneous channels. The scheduler chooses mmWave (Channel 1) or sub-6GHz (Channel 2) for transmission over time.

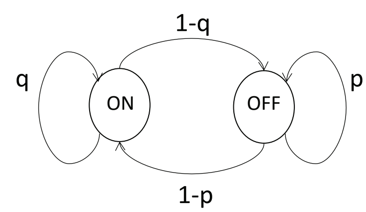

Figure 2. The Gilbert-Elliot ON-OFF Markov model for Channel 1.

2 Related Works

Age of information has become a popular research topic in recent years, e.g., [2]–[25]. A comprehensive survey of the area was recently provided in [2]. First, there has been substantial work on age performance analysis in queuing systems [3]–[8]. Average age and peak age in elementary queuing systems were analyzed in [3]–[5]. A similar setting was considered in [6] where the inter-arrival times or service times follow a Gilbert-Elliot two-state Markov chain model. A Last-Generated, First-Served (LGFS) policy was shown (near) optimal in single-source, multi-server, and multihop networks with arbitrary packet generation and arrival process [7], [8]. These results were extended to multi-source multi-server networks in [9].

Next, there has been a significant effort in age-optimal sampling [10]–[12], [21], [22]. The optimal sampling policy was provided for minimizing a monotonic age function in [10], [21], [22]. Joint Sampling and scheduling in multi-source systems were analyzed in [12] where the objective problem could be decoupled into maximum age first (MAF) scheduling [9] and an optimal sampling problem. Finally, age in wireless networks has been substantially explored in [13], [14], [16]–[20]. Scheduling in a broadcast network with random arrivals was provided where Whittle index policy can achieve (near) age optimality [13]. Some other age-optimal scheduling for cellular networks were considered in [14], [16]–[18], [25]. A class of age-optimal scheduling policies was analyzed in the asymptotic regime when the number of sources and channels both grow to infinity [19]. An age-optimal multi-path routing strategy was introduced in [20].

Recently, there have been studies on integrating two heterogeneous channels: mmWave and sub-6GHz [32]–[35], [41]–[44]. In [33], a comprehensive tutorial is proposed, where the user equipment should be equipped with the low-band sub-6GHz and multiple mmWave links to combat the high interruption rates of mmWave. In [32], the paper designs integrated mmWave and sub-6GHz architectures to achieve enhanced mobile (eMBB) services and ultra-reliable low-latency communication (URLLC). The experimental results demonstrate that this new scheme can achieve higher reliability compared to only using a single channel and less handover failure rate compared to the conventional handover scheme. In [42], the authors design the small-sized antenna with a radio frequency PIN diode so that the antenna can switch between 3.5 GHz and 28 GHz frequency band. Multiple-input multiple-output (MIMO) is formed to enhance the functionality.

In this paper, we assume that the source is equipped with an antenna that can switch between the two frequency bands. Therefore, a packet can be transmitted using only one channel at a time, i.e., the two channels cannot be used simultaneously. In the literature, both cases are considered: (i) only using one channel at at time, e.g., in [42], [44], (ii) using both channels simultaneously, e.g., in [32]. Typically, if the source uses a single antenna (possibly with MIMO to enhance the functionality) that can switch between the two frequency bands, the first case holds. If the second case holds, the transceiver needs multiple single-band antennas. An example of designing such a band-switching antenna is described in [42]. The antenna is composed of a microstrip patch and a meandered structure that are connected with a PIN diode with ON/OFF states. When the PIN diode is in the ON state, it allows the current to flow from the microstrip patch to the meandered structure to operate at 3.5GHz (sub-6GHz); when the PIN diode is in the OFF state, it limits the radiating structure to only the microstrip patch, resulting in resonance at 28GHz (mmWave). At any time, only one frequency band can resonate on the antenna. Compared to multiple antennas, one of the advantages of such a single antenna is to reduce the transceiver size and avoid electromagnetic interference. Similar to [42], another example is provided in [44], where a low-pass filter replaces the PIN diode to achieve frequency reconfigurations. Different from the PIN diode, the low-pass filter is a microstrip, so the antenna can operate on up to three frequency bands (2.8GHz, 28GHz and 38GHz). In this paper, our focus is on the first case that only allows one frequency band at a time.

However, the age-optimal scheduling problem via heterogeneous channels has been largely unexplored yet. Technical results for similar models were reported in [23], [24]. In these studies, it is assumed that the first channel is unreliable but consumes a lower cost, and the second channel has the same delay as the first channel, but depletes a higher cost. Optimal scheduling policies were derived to achieve the optimal trade-off between age performance and cost. Our study is different from [23], [24] in two aspects: (i) The study in [23], [24] show the optimality of a threshold-type policy and efficiently computes the optimal threshold when the first channel is i.i.d. [23], but our work allows a Markovian channel which generalizes the i.i.d. channel model in [23]. Since mmWave is highly unreliable, in a discrete time system, the mmWave is modeled as an ON-OFF unreliable channel. A typical way of modeling the ON-OFF unreliable channel is to assume that it is i.i.d. However, the channel state may not be i.i.d. In some situations, such as vehicular network, a vehicle can transmit the sample with good connectivity at a wide area, but with bad connectivity when there is building blockage. In this case, the channel state is positive correlated in general. Therefore, we use an extended model: Markovian channel model, to represent the mmWave channel. (ii) In addition, our study assumes that the second sub-6GHz channel has a larger delay than the first mmWave channel, which complies with the property of dual mmWave/sub-6GHz channels in real applications. These two differences between mmWave and sub-6GHz make the MDP formulation more complex than those of [23], [24]. Thus, the techniques in e.g., [10], [22], [23], [37]–[40] that can show a nice structure of the optimal policy or solve the optimal policy with low complexity do not apply to our model.

3 System Model and Problem Formulation

3.1 System Models

Consider a single-hop network as illustrated in Fig. 1, where a source sends status update packets to the destination. We assume that time is slotted with slot index $t \in \{0, 1, 2...\}$. The source can generate a fresh status update packet at the beginning of each time slot. The packets can be transmitted either over the mmWave channel or over the sub-6GHz channel. The packet transmission time of the mmWave channel is 1 time slot, whereas the packet transmission time of the sub-6GHz channel is d time slots ($d \geq 2$)[1] because of its lower data rate. The two channels have different advantages, which is the key feature of our study.

The mmWave channel, called *Channel 1*, follows a two-state Gilbert-Elliot model that is shown in Fig. 2. We say that Channel 1 is ON in time slot t, denoted by $l_1(t) = 1$, if the source is connected to Channel 1 with rate 1 in time slot t; otherwise Channel 1 is said to be OFF, denoted by $l_1(t) = 0$. If a packet is not successfully transmitted, then it is dropped, and a new status update packet is generated at the beginning of the next time slot. The self transition probability of the ON state is q, and the self transition probability of the OFF state is p, where $0 < q < 1$ and $0 < p < 1$. We assume that at the beginning of time slot t, the source knows $l_1(t-1)$ perfectly.

The sub-6GHz channel, called *Channel 2*, has a steady connection. As mentioned above, the packet transmission time of Channel 2 is d time slots. Define $l_2(t) \in \{0, 1, ..., d-1\}$ as the state of Channel 2 in time slot t, where $l_2(t)$ is the remaining transmission time of the packet being sent over Channel 2 at the beginning of time slot t, and $l_2(t) = 0$ means that Channel 2 is currently idle and ready for sending the next packet. In time slot t, the source has immediate knowledge about the state $l_2(t)$ of Channel 2. On the other hand, because the packet transmission time of Channel 1 is 1 time slot, Channel 1 is always ready for transmission at the beginning of each time slot.

As is illustrated in Section 2, a packet can be transmitted using only one channel at a time, i.e., the two channels cannot be used simultaneously. The scheduler decides which channel to use for transmitting a packet at each time slot. We also assume that the scheduler can choose idle (neither channel) since it has been shown that channel idling could reduce the average age in some systems [10], [12], [22]. Hence, the scheduling decision at the beginning of time slot t can be denoted by $u(t) \in \{1, 2, none\}$. The action $u(t) = 1$ or 2 means that the source generates a packet and assigns it to Channel 1 or Channel 2, respectively. The action $u(t) = none$ means that no new packet is assigned to any channel at time slot t. Hence, u(t) = $none$ can occur if (i) a packet was assigned to Channel 2 earlier and has not completed its transmission, i.e., $l_2(t) \in \{1, 2, \ldots, d-1\}$ such that no packet can be assigned for transmission, or (ii) $l_2(t) = 0$, but both channels are kept idle on purpose.

The age of information (AoI) $\Delta(t)$ is the time difference between the current time slot t and the generation time of the freshest delivered packet [3]. By this definition, when a packet is delivered, the age drops to the transmission time duration of the delivered packet. Specifically, if Channel 1 is selected in time slot t and Channel 1 is ON, then the age drops to 1 at time slot $t+1$. If the remaining service time of Channel 2 at time slot t is 1, then age drops to d at time slot $t+1$. When there is no packet delivery at time slot t, the age increases by one in each time slot. Hence, the time-evolution of the age is given by

$$\Delta(t+1) = \begin{cases} 1 & \text{if } u(t) = 1 \text{ and } l_1(t) = 1, \\ d & \text{if } l_2(t) = 1, \\ \Delta(t) + 1 & \text{Otherwise.} \end{cases} \tag{1}$$

3.2 Problem Formulations

We use $\pi = \{u(0), u(1)...\}$ to denote a scheduling policy. A scheduling policy is said to be *admissible* if (i) $u(t) = none$ whenever $l_2(t) \geq 1$ and (ii) $u(t)$ is determined by the current and history information that is available at the scheduler. Let $\Delta_\pi(t)$ denote the AoI induced by policy π. The expected time-average age of policy π is

$$\limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[\Delta_\pi(t)].$$

1. If $d = 1$, one can readily see that it is better to choose sub-6GHz than mmWave. Thus, in this paper we study the nontrivial case of $d \geq 2$.

Table I
Value of State Transition Probability

$P_{\mathbf{ss}'}(u)$	Action and State Transition
p	$u=1, \mathbf{s}=(\delta,0,0), \mathbf{s}'=(\delta+1,0,0)$ $u=2, \mathbf{s}=(\delta,0,0), \mathbf{s}'=(\delta+1,0,d-1)$ $u=none, \mathbf{s}=(\delta,0,0), \mathbf{s}'=(\delta+1,0,0)$ $u=none, \mathbf{s}=(\delta,0,l_2), \mathbf{s}'=(\delta+1,0,l_2-1), l_2\geq 2$ $u=none, \mathbf{s}=(\delta,0,1), \mathbf{s}'=(d,0,0)$
$1-p$	$u=1, \mathbf{s}=(\delta,0,0), \mathbf{s}'=(1,1,0)$ $u=2, \mathbf{s}=(\delta,0,0), \mathbf{s}'=(\delta+1,1,d-1)$ $u=none, \mathbf{s}=(\delta,0,0), \mathbf{s}'=(\delta+1,1,0)$ $u=none, \mathbf{s}=(\delta,0,l_2), \mathbf{s}'=(\delta+1,1,l_2-1), l_2\geq 2$ $u=none, \mathbf{s}=(\delta,0,1), \mathbf{s}'=(d,1,0)$
q	$u=1, \mathbf{s}=(\delta,1,0), \mathbf{s}'=(1,1,0)$ $u=2, \mathbf{s}=(\delta,1,0), \mathbf{s}'=(\delta+1,1,d-1)$ $u=none, \mathbf{s}=(\delta,1,0), \mathbf{s}'=(\delta+1,1,0)$ $u=none, \mathbf{s}=(\delta,1,l_2), \mathbf{s}'=(\delta+1,1,l_2-1), l_2\geq 2$ $u=none, \mathbf{s}=(\delta,1,1), \mathbf{s}'=(d,1,0)$
$1-q$	$u=1, \mathbf{s}=(\delta,1,0), \mathbf{s}'=(\delta+1,0,0)$ $u=2, \mathbf{s}=(\delta,1,0), \mathbf{s}'=(\delta+1,0,d-1)$ $u=none, \mathbf{s}=(\delta,1,0), \mathbf{s}'=(\delta+1,0,0)$ $u=none, \mathbf{s}=(\delta,1,l_2), \mathbf{s}'=(\delta+1,0,l_2-1), l_2\geq 2$ $u=none, \mathbf{s}=(\delta,1,1), \mathbf{s}'=(d,0,0)$
0	Otherwise

Our objective in this paper is to solve the following optimal scheduling problem for minimizing the expected time-average age:

$$\bar{\Delta}_{\text{opt}} = \inf_{\pi\in\Pi} \limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}[\Delta_\pi(t)], \quad (2)$$

where Π is the set of all admissible policies. Problem (2) can be equivalently expressed as an infinite time-horizon average-cost MDP problem [38], [45], which is illustrated below.

- **Markov State:** The system state in time slot t is defined as

$$\mathbf{s}(t) = (\Delta(t), l_1(t-1), l_2(t)), \quad (3)$$

  where $\Delta(t) \in \{1,2,3,...\}$ is the AoI in time slot t, $l_1(t-1)\in\{0,1\}$ is the $ON-OFF$ state of Channel 1 in time slot $t-1$, and $l_2(t) \in \{0,1,...,d-1\}$ is the remaining transmission time of Channel 2 at the beginning of time slot t. Let **S** denote the state space which is countably infinite. The time-evolution of $\Delta(t)$ is determined by the state and action in time slot $t-1$.
- **Action:** As mentioned before, if Channel 2 is busy (i.e., $l_2(t) > 0$), the scheduler always chooses an idle action, i.e., $u(t) = none$. Otherwise, the action $u(t) \in \{1, 2, none\}$.
- **Cost function:** Suppose that a decision $u(t)$ is applied at a time slot t, we encounter a cost $C(\mathbf{s}(t), u(t)) = \Delta(t)$.
- **Transition probability:** We use $P_{\mathbf{ss}'}(u)$ to denote the transition probability from state **s** to **s**$'$ for action u. The value of $P_{\mathbf{ss}'}(u)$ is summarized in Table I.
  We provide an explanation of the transition probabilities $P_{\mathbf{ss}'}(u)$ in Table I. Due to the Markovian state transition properties of Channel 1, there are four possible values of state transition probabilities: $p, 1-p, q$ and $1-q$. For example, $P_{\mathbf{ss}'}(u) = p$ if both the current and previous states of Channel 1 is OFF. Thus, there are two possible age state evolutions: if the remaining time slot of Channel 2 is 1, the age δ decreases to d; otherwise, the age δ increases by one time slot. The transition probabilities of other cases, i.e., $P_{\mathbf{ss}'}(u) = 1-p, q$ and $1-q$ in Table I can be explained in the similar way.

4 Main Results

In this section, we show that there exists a threshold-type policy that solves Problem (2). We then provide a low-complexity algorithm to obtain the optimal policy and optimal average age.

4.1 Optimality of Threshold-type Policies

As mentioned in Section 3.2, the action space of the MDP allows $u(t) = none$ even if Channel 2 is idle, i.e., $l_2(t) = 0$. In the following lemma, we show that the action $u(t) = none$ can be abandoned when $l_2(t) = 0$. Define

$$\Pi' = \{\pi \in \Pi : u(t) \neq none, \text{ if } l_2(t) = 0\}. \quad (4)$$

Lemma 1. *For any $\pi \in \Pi$, there exists a policy $\pi' \in \Pi'$ that is no worse than π.*

Remark 1. *In [10], [12], [22], it was shown that in certain systems, the zero wait policy (transmitting immediately after the previous update has been received) might not be optimal. However, in our model, the zero wait policy is indeed optimal. The reason is that in our model, the minimum non-zero waiting time is one time slot which is the same as the delay of Channel 1. If $l_2(t) = 0$, it is better to choose Channel 1 than keeping both channels idle, because, by choosing Channel 1, fresh packets could be delivered over Channel 1.*

The proof of Lemma 1 is provided in Appendix A of the supplementary material. By Lemma 1, the scheduler only needs to choose from the actions $u(t) \in \{1, 2\}$ when $l_2(t) = 0$. This lemma simplifies the MDP problem.

The parameters of the hybrid channels are (p, q, d), where p, q are the self transition probabilities of Channel 1 and d is the transmission delay of Channel 2. For the ease of presenting our main results, we divide the possible values of channel parameters (p, q, d) into four complementary regions $\mathbf{B}_1, \ldots, \mathbf{B}_4$.

Definition 1. *The regions $\boldsymbol{B}_1, \ldots, \boldsymbol{B}_4$ are defined as*

$$\begin{aligned} \boldsymbol{B}_1 &= \{(p,q,d) : F(p,q,d) \leq 0, H(p,q,d) \leq 0\},\\ \boldsymbol{B}_2 &= \{(p,q,d) : F(p,q,d) > 0, G(p,q,d) \leq 0\},\\ \boldsymbol{B}_3 &= \{(p,q,d) : F(p,q,d) > 0, G(p,q,d) > 0\},\\ \boldsymbol{B}_4 &= \{(p,q,d) : F(p,q,d) \leq 0, H(p,q,d) > 0\}, \end{aligned} \quad (5)$$

where

$$\begin{aligned} F(p,q,d) &= \frac{1}{1-p} - d,\\ G(p,q,d) &= 1 - dq,\\ H(p,q,d) &= \frac{1-q}{1-p} + 1 - d. \end{aligned} \quad (6)$$

Note that the inequality $1/(1-p) > d$ also represents a comparison between the channel delay d and the average

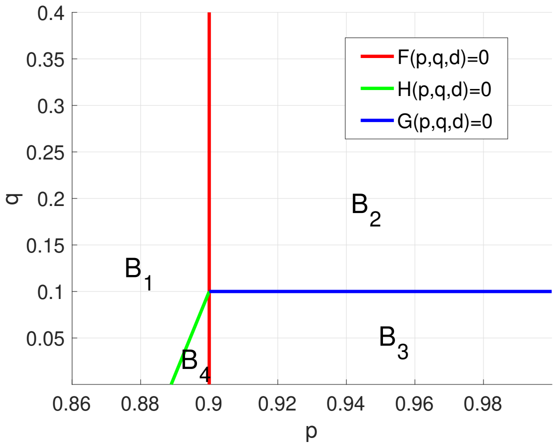

Figure 3. The Diagram of the regions $\mathbf{B}_1, \ldots, \mathbf{B}_4$ with an example of $d = 10$. In the diagram, each function F, G, H divides the whole plane $((p, q) \in (0, 1) \times (0, 1))$ into two half-planes respectively. Each region $\mathbf{B}_1, \ldots, \mathbf{B}_4$ is the intersection of some two half-plane areas. Since we emphasize the differences of the four regions, we provide the partial but enlarged diagram.

waiting time for an ON channel state given that the last channel state is OFF. Similarly, $1 - dq > 0$ represents a comparison between d and the average waiting time for an ON channel state given that the last channel state is ON. Finally, $(1 - q)/(1 - p) + 1 > d$ represents a comparison between d and the average waiting time of Channel 1 under steady-state distribution of the Gilbert-Elliot model. These comparisons interpret all the boundary functions F, G, H of the regions $\mathbf{B}_1 - \mathbf{B}_4$. The four regions $\mathbf{B}_1, \ldots, \mathbf{B}_4$ are depicted in Fig. 3, for the case that $d = 10$.

Consider a stationary policy $\mu(\delta, l_1, l_2)$. As mentioned in Lemma 1, when $l_2 = 0$, the decision $\mu(\delta, l_1, 0)$ can be 1 (Channel 1) or 2 (Channel 2). Given the value of l_1, $\mu(\delta, l_1, 0)$ is said to be *non-decreasing in the age* δ, if

$$\mu(\delta, l_1, 0) = \begin{cases} 1 & \text{if } \delta < \lambda; \\ 2 & \text{if } \delta \geq \lambda. \end{cases} \tag{7}$$

Conversely, $\mu(\delta, l_1, 0)$ is said to be *non-increasing in the age* δ, if

$$\mu(\delta, l_1, 0) = \begin{cases} 2 & \text{if } \delta < \lambda; \\ 1 & \text{if } \delta \geq \lambda. \end{cases} \tag{8}$$

One can observe that scheduling policies in the form of (7) and (8) are both with a *threshold-type*, where λ is the threshold on the age δ at which the value of $\mu(\delta, l_1, 0)$ changes.

One optimal solution to Problem (2) is of a special threshold-type structure, as stated in the following theorem:

Theorem 1. *There exists an optimal solution $\mu^*(\delta, l_1, 0)$ to Problem* (2)*, which satisfies the following properties:*

(a) *if $(p, q, d) \in \mathbf{B}_1$, then $\mu^*(\delta, 0, 0)$ is non-increasing in the age δ and $\mu^*(\delta, 1, 0)$ is non-increasing in the age δ;*
(b) *if $(p, q, d) \in \mathbf{B}_2$, then $\mu^*(\delta, 0, 0)$ is non-decreasing in the age δ and $\mu^*(\delta, 1, 0)$ is non-increasing in the age δ;*
(c) *if $(p, q, d) \in \mathbf{B}_3$, then $\mu^*(\delta, 0, 0)$ is non-decreasing in the age δ and $\mu^*(\delta, 1, 0)$ is non-decreasing in the age δ;*
(d) *if $(p, q, d) \in \mathbf{B}_4$, then $\mu^*(\delta, 0, 0)$ is non-increasing in the age δ and $\mu^*(\delta, 1, 0)$ is non-decreasing in the age δ.*

Proof. See Section 6.2 for the proof. □

As is shown in Theorem 1, for all possible parameters (p, q, d) of the two channels, the optimal action $\mu^*(\delta, l_1, 0)$ of channel selection is a monotonic function of the age δ. Whether $\mu^*(\delta, l_1, 0)$ is non-decreasing or non-increasing in δ depends on the region of the channel parameters (p, q, d) and the previous state l_1 of Channel 1.

The study in [23] assumed that the first channel is unreliable and consumes a lower cost, and the second channel the same delay as the first channel but a higher cost. They studied the scheduling policy for optimizing the trade-off between age and cost. The optimal scheduling policy in Theorem 1 is quite different from that in [23]: The study in [23] assumes the first channel to be i.i.d., but our result allows a Markovian Channel 1, which is a generalization of the i.i.d. case. Observe that in [23], the first channel is no better than the second channel with regard to delay and reliability. However, in our study, the two channels (i.e., Channel 1 and 2) have their own advantages in delay and reliability. Therefore, the optimal solution in our study is non-decreasing in age for some values of (p, q, d) and non-increasing in age for the remaining values of (p, q, d). In conclusion, our study allows for general channel parameters (p, q, d) and our optimal decision $\mu^*(\delta, l_1, 0)$ is non-increasing in age or non-decreasing in age depending on the choices of channel parameters.

4.1.1 *Insights Behind the Regions $\mathbf{B}_1 - \mathbf{B}_4$*

The regions $\mathbf{B}_1 - \mathbf{B}_4$ were introduced in Theorem 1 for proving that the action value function $Q(\mathbf{s}, u)$ is *super-modular* or *sub-modular*, where $\mathbf{s} = (\delta, l_1, 0)$ denotes the state of the MDP and u is the action. For example, in the case of $l_1 = 0$, if $1/(1 - p) > d$ and $1/q \leq d$ (i.e., $(p, q, d) \in \mathbf{B}_2$), Lemma 9 in Section 6.2 showed that $Q(\delta, 0, 0, u)$ is sub-modular in (δ, u) (in the discounted case). As a result, the optimal action $\mu^*(\delta, 0, 0)$ is increasing in δ.

However, in the case $l_1 = 1$ of Theorem 1, there are additional technical challenges: For example, if $(p, q, d) \in \mathbf{B}_2$, $Q(\delta, 1, 0, u)$ is neither super-modular nor sub-modular. A new method was developed in Lemma 10 in Section 6.2 to conquer this challenge. Technically, super-/sub-modularity is a sufficient but not necessary condition for the monotonicity of $\mu^*(\delta, l_1, 0)$. When neither super-modularity nor sub-modularity holds, we are able to show that the optimal decision $\mu^*(\delta, l_1, 0)$ does not change with δ. By this, we proved the monotonicity of $\mu^*(\delta, l_1, 0)$ for all values of δ and l_1, without requiring $Q(\mathbf{s}, u)$ to be super-modular or sub-modular over the entire state space $\mathbf{s} \in \mathbf{S}$.

The following is one of the key technical contributions of the paper: we proved that the optimal action $\mu^*(\delta, l_1, 0)$ is monotonic in δ even if super-/sub-modularity does not hold. This is a key difference from prior studies, e.g., [10], [22], [23], [37]–[39], where super-modularity (or sub-modularity) holds for the entire state space.

4.2 Optimal Scheduling Policy

In Theorem 1, we have characterized the threshold structure for an optimal policy in region $\mathbf{B}_1, \ldots, \mathbf{B}_4$. A threshold-type policy is fully identified by its thresholds λ_0, λ_1, where λ_0 is the threshold given that the previous state of Channel 1

Algorithm 1: Bisection method for solving (21)

Given function h_i. $l = 0$, l' sufficiently large, tolerance ϵ small. The value $i \in \{1, 2, 3, 4\}$.
repeat
  $\beta = \frac{1}{2}(l + l')$
  if $h_i(\beta) < 0$: $l' = \beta$. **else** $l = \beta$
until $l' - l < \epsilon$
return $\beta_i = \beta$

is OFF (i.e., $l_1 = 0$) and λ_1 is the threshold given that the previous state of Channel 1 is ON (i.e., $l_1 = 1$). Thus, for a given region $\mathbf{B}_i$ ($i = 1, \ldots, 4$), the MDP problem (2) reduces to

$$\bar{\Delta}_{\text{opt}} = \min_{\lambda_0 \in \mathbb{N}^+, \lambda_1 \in \mathbb{N}^+} \bar{\Delta}_i(\lambda_0, \lambda_1), \tag{9}$$

where $\bar{\Delta}_i(\lambda_0, \lambda_1)$ is the long term average cost of the threshold-type policy such that: (1) the threshold (monotone) structure is determined by Theorem 1 and $\mathbf{B}_i$; (2) the thresholds are λ_0, λ_1. Note that a threshold type policy is stationary and thus can be modeled as a discrete-time Markov chain (DTMC). Then, (9) can be solved by deriving the steady-state distribution of the DTMC.

We use λ_0^* and λ_1^* to denote the thresholds of $\mu^*(\delta, 0, 0)$ and $\mu^*(\delta, 1, 0)$, respectively. In this section, we provide the optimal scheduling policy and the thresholds.

4.2.1 Optimal Scheduling Policy for $(p, q, d) \in \boldsymbol{B}_1$

Theorem 2. *If $(p, q, d) \in \boldsymbol{B}_1$, then an optimal scheduling policy is*

$$\mu^*(\delta, 0, 0) = 1, \delta \geq 1; \tag{10}$$
$$\mu^*(\delta, 1, 0) = 1, \delta \geq 1. \tag{11}$$

In this case, the optimal objective value of (2) *is*

$$\bar{\Delta}_{opt} = \frac{(1-q)(2-p) + (1-p)^2}{(2-q-p)(1-p)}. \tag{12}$$

We provide an insight to Theorem 2: As will be shown by Lemma 11 and Lemma 12 in Section 6.3, if $(p, q, d) \in \mathbf{B}_1$, then $\mu^*(1, 0, 0) = 1$ and $\mu^*(1, 1, 0) = 1$. According to Theorem 1 (a), if $(p, q, d) \in \mathbf{B}_1$, $\mu^*(\delta, 0, 0)$ and $\mu^*(\delta, 1, 0)$ are both non-increasing in δ. Thus, $\mu^*(\delta, 0, 0) = 1$ and $\mu^*(\delta, 1, 0) = 1$ for all $\delta \geq 1$. That is, the optimal scheduler always chooses Channel 1. The DTMC for a policy always choosing Channel 1 is easy to analyze, so we omit the derivation steps and provide

$$\bar{\Delta}_{\text{opt}} = \bar{\Delta}_1(1, 1) = \frac{(1-q)(2-p) + (1-p)^2}{(2-q-p)(1-p)}. \tag{13}$$

This resucolorblackctly implies Theorem 2. Therefore, Theorem 2 provides a much stronger result than Theorem 1(a).

4.2.2 Optimal Scheduling Policy for $(p, q, d) \in \boldsymbol{B}_2$

While the result of case $(p, q, d) \in \mathbf{B}_1$ is easy to describe, the result of case $(p, q, d) \in \mathbf{B}_2$ is not. As shown by Theorem 3, the optimal decision $\mu^*(\delta, l_1, 0)$ is not constant in age δ.

Theorem 3. *If $(p, q, d) \in \boldsymbol{B}_2$, then an optimal scheduling policy is given by:*

$$\mu^*(\delta, 0, 0) = \begin{cases} 1 & \text{if } \delta < \lambda_0^*; \\ 2 & \text{if } \delta \geq \lambda_0^*, \end{cases} \tag{14}$$

$$\mu^*(\delta, 1, 0) = \begin{cases} 2 & \text{if } \delta < \lambda_1^*; \\ 1 & \text{if } \delta \geq \lambda_1^*, \end{cases} \tag{15}$$

where λ_0^* is unique, but λ_1^* may take multiple values, given by

$$\begin{cases} \lambda_0^* = s_1(\beta_1), & \lambda_1^* = 1 & \text{if } \bar{\Delta}_{opt} = \beta_1, \\ \lambda_0^* = s_2(\beta_2), & \lambda_1^* = 1 & \text{if } \bar{\Delta}_{opt} = \beta_2, \\ \lambda_0^* = 1, & \lambda_1^* \in \{2, 3, \ldots, d\} & \text{if } \bar{\Delta}_{opt} = f_0/g_0, \\ \lambda_0^* = 1, & \lambda_1^* \in \{d+1, \ldots\} & \text{if } \bar{\Delta}_{opt} = (3/2)d - 1/2, \end{cases} \tag{16}$$

$\bar{\Delta}_{opt}$ is the optimal objective value of (2)*, determined by*

$$\bar{\Delta}_{opt} = \min\Big\{\beta_1, \beta_2, \frac{f_0}{g_0}, \frac{3}{2}d - \frac{1}{2}\Big\}, \tag{17}$$

$s_1(\cdot)$, $s_2(\cdot)$, β_1, and β_2 are given in Definition 2 below,

$$f_0 = q\sum_{i=1}^{d} i + (1-q)\sum_{i=d+1}^{2d} i + \frac{b'_d q + 1}{1 - b_d}\sum_{i=d}^{2d-1} i + d, \tag{18}$$

$$g_0 = \frac{b'_d q + 1}{1 - b_d} d + d + 1, \tag{19}$$

$$\begin{bmatrix} b'_d \\ b_d \end{bmatrix} = \begin{bmatrix} q & 1-q \\ 1-p & p \end{bmatrix}^d \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{20}$$

Proof. See Section 6.3. □

In order to prove Theorem 3, we have conducted steady-state analysis of four DTMCs, each of which corresponds to one case in (16). These four DTMCs have diverse state transmission matrices and have to be analyzed separately. For each case, the optimal thresholds λ_0^*, λ_1^* are either constants or can be computed by using a low-complexity bisection search method to compute the root of (21) given in below.

Definition 2. *The value of β_i is the root of*

$$f_i\big(s_i(\beta_i)\big) - \beta_i g_i\big(s_i(\beta_i)\big) = 0, \ i \in \{1, 2, 3, 4\}, \tag{21}$$

where

$$s_i(\beta_i) = \max\left\{\left\lceil \frac{-k_i(\beta_i)}{1 - d(1-p)} \right\rceil, d\right\}, \ i \in \{1, 3, 4\},$$

$$s_2(\beta_2) = \max\left\{\min\left\{\left\lceil \frac{-k_2(\beta_2)}{1 - d(1-p)} \right\rceil, d\right\}, 2\right\}, \tag{22}$$

$$k_i(\beta_i) = l'_i - \beta_i o_i, \tag{23}$$

and $\lceil x \rceil$ is the smallest integer that is greater or equal to x. For the ease of presentation, 16 closed-form expressions of $f_i(\cdot)$, $g_i(\cdot)$, l'_i, and o_i for $i = 1, \ldots, 4$ are provided in Table III of our supplementary material.

Note that β_3 and β_4 in Definition 2 will be used in later subsection where $(p, q, d) \in \mathbf{B}_3$. For notational simplicity, we define

$$h_i(\beta) = f_i(s_i(\beta)) - \beta g_i(s_i(\beta)), \ i \in \{1, 2, 3, 4\}. \tag{24}$$

The function $h_i(\beta)$ has the following nice property:

Lemma 2. *For all $i \in \{1, 2, 3, 4\}$, the function $h_i(\beta)$ satisfies the following properties:*

(1) $h_i(\beta)$ is continuous, concave, and strictly decreasing on β;

(2) $h_i(0) > 0$ and $\lim_{\beta\to\infty} h_i(\beta) = -\infty$.

Proof. See Appendix B of our supplementary material. □

Lemma 2 implies that (21) has a unique root on $[0, \infty)$. Therefore, we can use a low-complexity bisection method to compute $\beta_1, \ldots, \beta_4$ as illustrated in Algorithm 1.

Lemma 2 is motivated by Lemma 2 in [11] and Lemma 2 in [21]. In [11] and [21], since the channel is error free, the age state at the end of each transmission is independent from history information. Thus, Lemma 2 in [11] and Lemma 2 in [21] are related to a per-sample (single transmission) control. However, our study does not have such a property and Lemma 2 arises from solving (9) for optimizing the thresholds λ_0^* and λ_1^*.

The advantage of Theorem 2, Theorem 3 is that the solution is easy to implement. In Theorem 2, we showed that the optimal policy is a constant policy that always chooses Channel 1. In Theorem 3, $\bar{\Delta}_{\text{opt}}$ is expressed as the minimization of only a few precomputed values, and the optimal threshold-type policy are then obtained based on the value of $\bar{\Delta}_{\text{opt}}$.

Since we can use a low complexity algorithm such as bisection method to obtain β_1, β_2 in Theorem 3, Theorem 3 provides a solution that has much lower complexity than other solutions for MDPs such as relative value iteration and policy iteration.

We now provide the sketch of the proof when $(p, q, d) \in \mathbf{B}_2$:

First, by computing the steady-state distributions of some DTMCs with different thresholds, we have obtained the average age performance for four cases, given by

$$\bar{\Delta}_2(\lambda_0, 1) = \begin{cases} f_1(\lambda_0)/g_1(\lambda_0) & \lambda_0 \in \{d+1, \ldots\}, \\ f_2(\lambda_0)/g_2(\lambda_0) & \lambda_0 \in \{2, \ldots d\}, \end{cases} \quad (25)$$

$$\bar{\Delta}_2(1, \lambda_1) = \begin{cases} (3/2)d - 1/2 & \lambda_1 \in \{d+1, \ldots\}, \\ f_0/g_0 & \lambda_1 \in \{1, \ldots d\}. \end{cases} \quad (26)$$

Note that each one of the four expressions in (25) and (26) corresponds to each one of the four cases in (16), respectively. One of our technical contributions is that only studying the steady-state analysis of the four types of DTMCs in (25) is sufficient to solve (9). The proof of this statement and the detailed expressions of the DTMC structure of the four cases in (25) and (26) are relegated to Section 6.3[2]. Therefore, the optimal average age $\bar{\Delta}_{\text{opt}}$ chooses the smallest value of the four cases from (25) and (26),

$$\bar{\Delta}_{\text{opt}} = \min\Big\{\beta_1', \beta_2', \frac{f_0}{g_0}, \frac{3}{2} - \frac{1}{2}\Big\}, \quad (27)$$

where β_1', β_2' are defined as follows:

$$\beta_i' = \min_{\lambda_0 \in \{d+1,\ldots\}} \frac{f_i(\lambda_0)}{g_i(\lambda_0)}, i \in \{1, 3, 4\}, \quad (28)$$

$$\beta_2' = \min_{\lambda_0 \in \{2,\ldots d\}} \frac{f_2(\lambda_0)}{g_2(\lambda_0)}. \quad (29)$$

2. Although (9) is a two-dimensional optimization problem in (λ_0, λ_1), (9) has been simplified as (25), which is a one-dimensional optimization problem.

Note that β_3' and β_4' in (28) and (29) will be mentioned in the next subsection where $(p, q, d) \in \mathbf{B}_3$. Finally, in Section 6.3, we show that

$$\beta_i' = \beta_i, \ i \in \{1, 2, 3, 4\}. \quad (30)$$

Thus, Theorem 3 is solved by (25) − (30).

Finally, we provide a conjecture. Recall that by Theorem 3, the optimal decision $\mu^*(\delta, 1, 0)$ in region $\mathbf{B}_2$ chooses Channel 2 if the age δ is lower than a finite threshold and chooses Channel 1 if δ is larger than the threshold. Here, we conjecture that $\mu^*(\delta, 1, 0) = 1$ for all $\delta \geq 1$ in parameter region $\mathbf{B}_2$, which is a special case of Theorem 3 (by letting the threshold be 0). By enumerating over the possible values of the parameters p, q, d, we have the following empirical observation:

Conjecture 1. *If $(p, q, d) \in \mathbf{B}_2$, then we have the following condition:*

$$\min\{\frac{f_0}{g_0}, \frac{3}{2}d - \frac{1}{2}\} > \bar{\Delta}_{opt}. \quad (31)$$

This conjecture comes from a large number of numerical experiments that we have run. Proving this conjecture is an open problem. We leave this proof to our future work. By Theorem 3, f_0/g_0 and $3/2d - 1/2$ are the expected average age value of the only two cases in which $\mu^*(1, 1, 0) = 2$. Therefore, Conjecture 1 implies that $\mu(1, 1, 0) = 2$ is suboptimal and the optimal policy is $\mu^*(1, 1, 0) = 1$. When $(p, q, d) \in \mathbf{B}_2$, $\mu^*(\delta, 1, 0)$ is non-increasing in $\delta \geq 1$. This and Conjecture 1 suggest that $\mu^*(\delta, 1, 0) = 1$ for all $\delta \geq 1$. In conclusion, Conjecture 1 and Theorem 3 together suggests the following result:

Conjecture 2. *If $(p, q, d) \in \mathbf{B}_2$, then the optimal scheduling policy is given by*

$$\mu^*(\delta, 0, 0) = \begin{cases} 1 & \text{if } \delta < \lambda_0^*; \\ 2 & \text{if } \delta \geq \lambda_0^*, \end{cases} \quad (32)$$

$$\mu^*(\delta, 1, 0) = 1, \ \delta \geq 1, \quad (33)$$

where λ_0^* is given by

$$\begin{cases} \lambda_0^* = s_1(\beta_1), & \text{if } \bar{\Delta}_{opt} = \beta_1, \\ \lambda_0^* = s_2(\beta_2), & \text{if } \bar{\Delta}_{opt} = \beta_2, \end{cases} \quad (34)$$

$\bar{\Delta}_{opt}$ is the optimal objective value of (2), determined by

$$\bar{\Delta}_{opt} = \min\Big\{\beta_1, \beta_2\Big\}. \quad (35)$$

Conjecture 2 provides an important message: the optimal policy in region $\mathbf{B}_2$ is overall non-decreasing in age δ. This message will also be validated by the simulation in Section 5.

4.2.3 Optimal Scheduling Policy for $(p, q, d) \in \mathbf{B}_3$

According to Theorem 1, the optimal decision $\mu^*(\delta, l_1, 0)$ is non-decreasing in age δ. Similar to the case $(p, q, d) \in \mathbf{B}_2$ in Theorem 3, the optimal solution $\mu^*(\delta, l_1, 0)$ is not constant. Therefore, we need to solve the optimal thresholds λ_0^* and λ_1^* by deriving the steady-state distribution of the DTMC. The final result is presented as follows:

Theorem 4. *If $(p,q,d)\in \boldsymbol{B}_3$, then an optimal scheduling policy is given by*

$$\mu^*(\delta,0,0)=\begin{cases}1 & \text{if } \delta<\lambda_0^*;\\ 2 & \text{if } \delta\geq\lambda_0^*,\end{cases} \tag{36}$$

$$\mu^*(\delta,1,0)=\begin{cases}1 & \text{if } \delta<\lambda_1^*;\\ 2 & \text{if } \delta\geq\lambda_1^*,\end{cases} \tag{37}$$

where λ_0^* is unique, but λ_1^* may take multiple values, given by

$$\begin{cases}\lambda_0^*=s_1(\beta_1), & \lambda_1^*\in\{d+1,\dots\} \quad \text{if } \bar{\Delta}_{opt}=\beta_1,\\ \lambda_0^*=s_2(\beta_2), & \lambda_1^*\in\{d+1,\dots\} \quad \text{if } \bar{\Delta}_{opt}=\beta_2,\\ \lambda_0^*=s_3(\beta_3), & \lambda_1^*\in\{2,\dots,d\} \quad \text{if } \bar{\Delta}_{opt}=\beta_3,\\ \lambda_0^*=s_4(\beta_4), & \lambda_1^*\in\{2,\dots,d\} \quad \text{if } \bar{\Delta}_{opt}=\beta_4,\\ \lambda_0^*=1, & \lambda_1^*\in\{1,2,\dots,d\}, \quad \text{if } \bar{\Delta}_{opt}=(3/2)d-1/2,\end{cases} \tag{38}$$

$\bar{\Delta}_{opt}$ is the optimal objective value of (2)*, determined by*

$$\bar{\Delta}_{opt}=\min\Big\{\beta_1,\beta_2,\beta_3,\beta_4,\frac{3}{2}d-\frac{1}{2}\Big\}, \tag{39}$$

$s_1(\cdot),\dots,s_4(\cdot)$ and $\beta_1,\dots,\beta_4$ are given in Definition 2.

Proof. See Section 6.3. □

From Theorem 4 and Lemma 2, the optimal thresholds λ_0^*,λ_1^* can be either expressed by constants or computed with low complexity. To show Theorem 4, we have analyzed five different DTMCs. Each of the DTMC corresponds to one case in (38). As is explained in Section 6.3, the solution to each case in (38) is closed-form or related with a one-dimensional optimization problem. Different from Theorem 3 which needs to compute β_1 and β_2 in (16), Theorem 4 needs to compute $\beta_1,\dots,\beta_4$ in (38). By Definition 2 and Lemma 2, $\beta_1,\dots,\beta_4$ can be solved by using low complexity bisection search algorithm (Algorithm 1). Therefore, despite Theorem 4 containing a number of cases, the optimal thresholds described in (38) can be efficiently solved.

4.2.4 Optimal Scheduling Policy for $(p,q,d)\in \boldsymbol{B}_4$

From Theorem 1, $\mu^*(\delta,0,0)$ is non-increasing in age δ and $\mu^*(\delta,1,0)$ is non-decreasing in δ. The result of $(p,q,d)\in \mathbf{B}_4$ is similar to that of Theorem 2.

Theorem 5. *If $(p,q,d)\in \boldsymbol{B}_4$, then an optimal scheduling policy is*

$$\mu^*(\delta,0,0)=1,\delta\geq 1, \tag{40}$$

$$\mu^*(\delta,1,0)=\begin{cases}1,\delta\geq 1 & \text{if } \bar{\Delta}_{opt}=\bar{\Delta};\\ 2,\delta\geq 1 & \text{if } \bar{\Delta}_{opt}=f_0'/g_0',\end{cases} \tag{41}$$

where $\bar{\Delta}_{opt}$ is the optimal objective value of (2)*, determined by*

$$\bar{\Delta}_{opt}=\min\Big\{\bar{\Delta},\frac{f_0'}{g_0'}\Big\}, \tag{42}$$

the constants $\bar{\Delta}, f_0', g_0'$ are given by

$$\bar{\Delta}=\frac{(1-q)(2-p)+(1-p)^2}{(2-q-p)(1-p)}, \tag{43}$$

$$f_0'=\sum_{i=1}^{d} i+\frac{1-b_d'}{b_d'}\times\sum_{i=d}^{2d-1} i+\sum_{i=d}^{\infty} ip^{i-d}, \tag{44}$$

$$g_0'=\frac{d}{b_d'}+1/(1-p). \tag{45}$$

Proof. See Section 6.3. □

Table II
Channel selection by the optimal scheduling policy $\mu^*(\delta,l_1,0)$

$\mathbf{B}_1$	$l_1=0$	Choose mmWave Channel
	$l_1=1$	Choose mmWave Channel
$\mathbf{B}_2$	$l_1=0$	Choose mmWave for low AoI δ, choose sub-6GHz for high AoI δ
	$l_1=1$	Choose sub-6GHz for low AoI δ, choose mmWave for high AoI δ. Conjecture 2: Always choose mmWave
$\mathbf{B}_3$	$l_1=0$	Choose mmWave for low AoI δ, choose sub-6GHz for high AoI δ
	$l_1=1$	Choose mmWave for low AoI δ, choose sub-6GHz for high AoI δ
$\mathbf{B}_4$	$l_1=0$	Choose mmWave channel
	$l_1=1$	Choose mmWave channel if $\bar{\Delta}\leq f_0'/g_0'$, choose sub-6GHz channel if $\bar{\Delta}>f_0'/g_0'$

As is illustrated in Theorem 5, the proposed optimal decision $\mu^*(\delta,0,0)$ for $(p,q,d)\in\mathbf{B}_4$ is constant in age δ, depending on whether $\bar{\Delta}_{\text{opt}}=\bar{\Delta}$ or $\bar{\Delta}_{\text{opt}}=f_0'/g_0'$ from (42). The value $\bar{\Delta}$ is the expected age of the steady-state DTMC that always chooses Channel 1. The value f_0'/g_0' is the expected age of the steady-state DTMC that chooses Channel 1 if $l_1=0$ and chooses Channel 2 if $l_1=1$. If $\bar{\Delta}_{\text{opt}}=\bar{\Delta}$, then it is optimal to always choose Channel 1; if $\bar{\Delta}_{\text{opt}}=f_0'/g_0'$, then we will select Channel 1 when $l_1=0$ and Channel 2 when $l_1=1$. Recall that in Theorem 1, $\mu^*(\delta,0,0)$ is non-increasing in δ. Further, in Theorem 5, we have shown that $\mu^*(\delta,0,0)$ is a constant, thus implying that $\mu^*(\delta,l_1,0)$ is overall non-decreasing in δ.

We briefly summarize the results for Theorems 2—5: An optimal solution to (2) is presented for the 4 complementary regions $\mathbf{B}_1,\dots,\mathbf{B}_4$ of the channel parameters (p,q,d). If $(p,q,d)\in\mathbf{B}_1\cup\mathbf{B}_4$, the solution is constant in age (Theorem 2 and Theorem 5). Otherwise, for $(p,q,d)\in\mathbf{B}_2\cup\mathbf{B}_3$, there exists an optimal scheduling policy that has a threshold structure depending on the current age value $\Delta(t)$ and the previous state of Channel 1 (Theorem 3 and Theorem 4). The optimal thresholds can be computed efficiently. In region $\mathbf{B}_2$, by Theorem 3, the optimal scheduling policy may be non-increasing in age, i.e., when the state of Channel 1 at previous time slot is OFF, it chooses channel 2 when the age of current time $\Delta(t)$ is smaller than a finite threshold and chooses Channel 1 when $\Delta(t)$ is larger than the threshold. However, Conjecture 2 claims that, in this case, the optimal policy should always chooses Channel 1 (by letting the threshold be 0). Therefore, combining Theorems 2—5 and Conjecture 2, the optimal scheduling policy $\mu^*(\delta,l_1,0)$ should be *non-decreasing* in the age δ for any values of l_1,p,q,d. To be specific, it either always chooses Channel 1, always chooses Channel 2, or chooses Channel 1 when $\Delta(t)$ is small and chooses Channel 2 when $\Delta(t)$ is large, which will also be validated in simulation (Section 5). To conclude, we provide Table II for illustrating the channel selection by the optimal scheduling policy in each region $\mathbf{B}_1-\mathbf{B}_4$.

4.3 Optimal Scheduling policy for i.i.d. Channel

We finally consider a special case in which Channel 1 is i.i.d., i.e., $p+q=1$. First, according to the following lemma, if $p+q=1$, the 4 regions $\mathbf{B}_1,\ldots,\mathbf{B}_4$ will reduce to 2 regions $\mathbf{B}_1,\mathbf{B}_3$.

Lemma 3. *If $p+q=1$, then $(p,q,d)\in\mathbf{B}_1$ or $(p,q,d)\in\mathbf{B}_3$. Moreover,*

$$\mathbf{B}_1=\left\{(p,q,d):\frac{1}{1-p}\le d\right\}, \tag{46}$$

$$\mathbf{B}_3=\left\{(p,q,d):\frac{1}{1-p}> d\right\}. \tag{47}$$

Proof. By (6) and $1-q=p$, we have: (i) $F(p,q,d)=H(p,q,d)$, and (ii) $F(p,q,d)>0$ is equivalent to $G(p,q,d)>0$. From the two above results and the definition of $\mathbf{B}_1,\ldots,\mathbf{B}_4$ in (5), we directly get (46) and (47). Moreover, the definitions of (46) and (47) imply that $(p,q,d)\in\mathbf{B}_1$ or $(p,q,d)\in\mathbf{B}_3$. □

From Theorem 2, if $(p,q,d)\in\mathbf{B}_1$, then the optimal policy is always choosing Channel 1. From Theorem 4, if $(p,q,d)\in\mathbf{B}_3$, then the optimal policy chooses one of the five cases that are depicted in (38). However, we can reduce the five cases to two cases: If Channel 1 is i.i.d., then the state information of Channel 1 is not useful. Thus, $\lambda_0^*=\lambda_1^*$. Note that from Definition 2, we have $s_2(\beta)\le d$ and $s_i(\beta)\ge d+1$ for $i\in\{1,3,4\}$. Thus, only the first case and the last case in (38) can possibly appear for i.i.d. channel.

So in i.i.d. case, Theorem 2 and Theorem 4 reduce to the following:

Corollary 1. *Suppose that $p+q=1$, i.e., Channel 1 is i.i.d., then*

(a) If $1-p\ge 1/d$, then the optimal policy is always choosing Channel 1. In this case, the optimal objective value of (2) is $\bar{\Delta}_{opt}=1/(1-p)$.

(b) If $1-p<1/d$, then the optimal policy is non-decreasing in age and the optimal thresholds $\lambda_0^*=\lambda_1^*$. The threshold λ_0^* may take multiple values, given by

$$\begin{cases}\lambda_0^*=s_1(\beta_1) & \text{if } \bar{\Delta}_{opt}=\beta_1,\\ \lambda_0^*\in\{1,2,\ldots,d\} & \text{if } \bar{\Delta}_{opt}=(3/2)d-1/2,\end{cases} \tag{48}$$

$\bar{\Delta}_{opt}$ is the optimal objective value of (2), determined by

$$\bar{\Delta}_{opt}=\min\left\{\beta_1,\frac{3}{2}d-\frac{1}{2}\right\}. \tag{49}$$

Corollary 1(a) suggests that if the transmission rate of Channel 1 is larger than the rate of Channel 2 (which is $1/d$), then the age-optimal policy always chooses Channel 1. Corollary 1(b) implies that if the transmission rate of Channel 1 is smaller than the rate of Channel 2, then the age-optimal policy is non-decreasing threshold-type on age.

5 Numerical Results

In this section, we first illustrate the channel selected by the optimal scheduling policy in Fig. 4 and Fig. 5. Recall that by Theorem 2, the optimal decision is to choose mmWave in region $\mathbf{B}_1$. In the simulation, we further observe that choosing mmWave is still optimal in region $\mathbf{B}_4$. Therefore, we only provide the results for $\mathbf{B}_2$ (in Fig. 4) and $\mathbf{B}_3$ (in Fig. 5), respectively. In these figures, it is easy to observe that: (i) in $\mathbf{B}_2$, $\mu^*(\delta,1,0)=1$, and $\mu^*(\delta,0,0)$ is non-decreasing in age, (ii) in $\mathbf{B}_3$, both $\mu^*(\delta,1,0)$ and $\mu^*(\delta,0,0)$ are non-decreasing in age. These comply with our main results Theorems 2—5. Note that in Fig. 4, $\mu^*(\delta,1,0)=1$, which validates Conjecture 2. In addition, we explore a more general system model where the packet arrival follows Bernoulli process with probability 0.5. We use value iteration to simulate the new optimal policy. Fig 6 describes the optimal scheduling policy of such a system model. The optimal policy is still threshold based. Note that when $p<0.8$, the considered region is $\mathbf{B}_1$. For the case of periodic arrival process, in $\mathbf{B}_1$, the optimal policy is choosing mmWave. However, for the case of Bernoulli arrival process, Fig 6 implies that the optimal policy is choosing mmWave when the age is small, and choosing sub-6GHz when the age is large.

We then simulate the optimal thresholds under the two

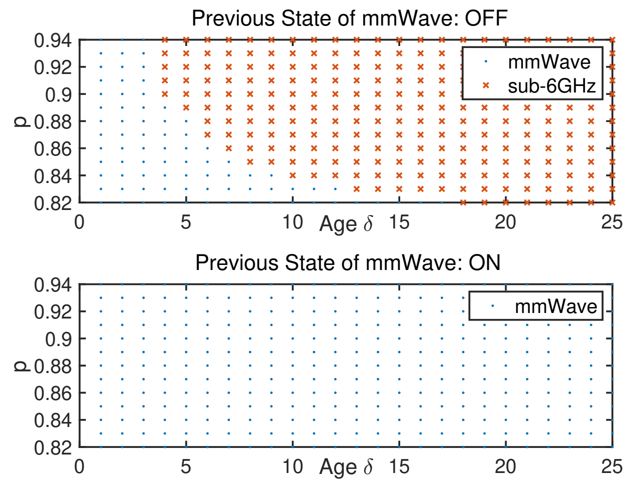

Figure 4. Optimal channel selection given the channel state of mmWave at the previous time slot, where $d=5, q=0.9$

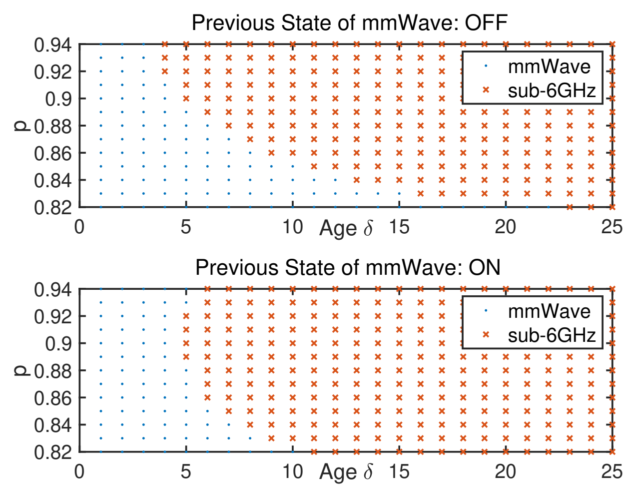

Figure 5. Optimal channel selection given the channel state of mmWave at the previous time slot, where $d=5, q=0.1$.

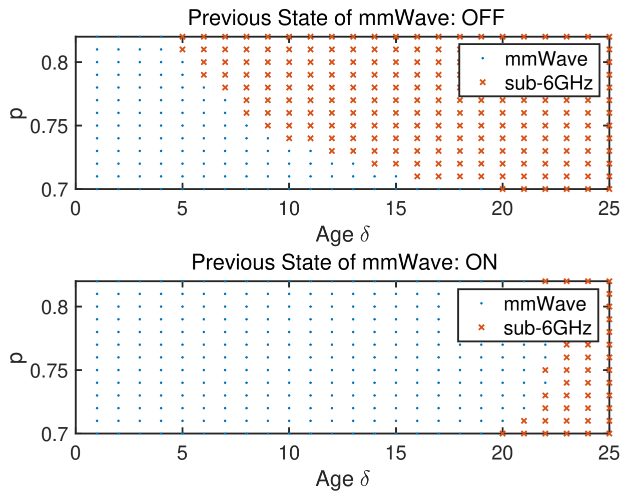

Figure 6. Optimal channel selection given the channel state of mmWave at the previous time slot with $d = 5, q = 0.3$, where the packet arrival follows Bernoulli process with probability 0.5.

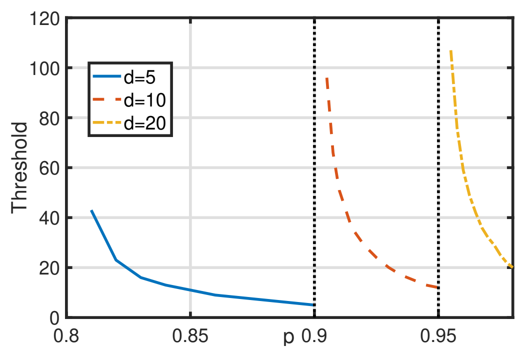

Figure 7. Threshold λ_0^* of the optimal scheduling policy for i.i.d. mmWave channel state, where the packet transmission time of the sub-6GHz channel is $d = 5, 10, 20$. The considered regions are $\mathbf{B}_1$ and $\mathbf{B}_3$ with boundary $p = 1 - 1/d$. In $\mathbf{B}_1$, the optimal decision is mmWave. In $\mathbf{B}_3$, the optimal decision is non-decreasing in age.

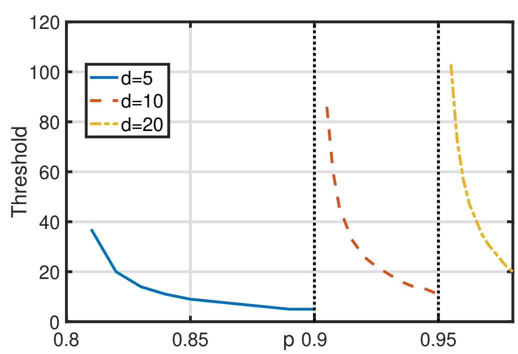

Figure 8. Threshold λ_0^* of the optimal scheduling policy, where the parameter $q = 0.8$ and the packet transmission time of the sub-6GHz channel is $d = 5, 10, 20$. The considered regions are $\mathbf{B}_1$ and $\mathbf{B}_2$ with boundary $q = 1/d$. In $\mathbf{B}_1$, the optimal decision $\mu^*(\delta, l_1, 0) = 1$. In $\mathbf{B}_2$, $\mu^*(\delta, 1, 0) = 1$, and $\mu^*(\delta, 0, 0)$ is non-decreasing in age.

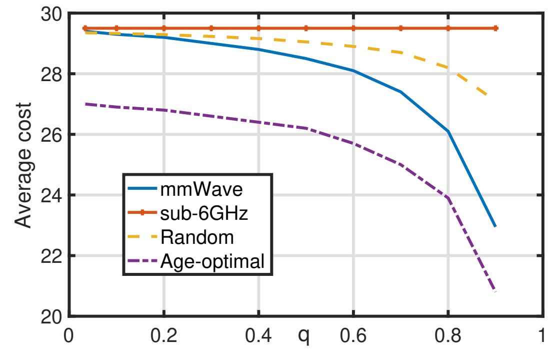

Figure 9. Time-average expected age vs. the parameter q of the mmWave channel, where $d = 20$ and $p = 0.966$. The considered regions are $\mathbf{B}_2$ and $\mathbf{B}_3$ with boundary $q = 1/d$. In $\mathbf{B}_2$, $\mu^*(\delta, 1, 0) = 1$, and $\mu^*(\delta, 0, 0)$ is non-decreasing in age. In $\mathbf{B}_3$, both $\mu^*(\delta, 1, 0)$ and $\mu^*(\delta, 0, 0)$ are non-decreasing in age.

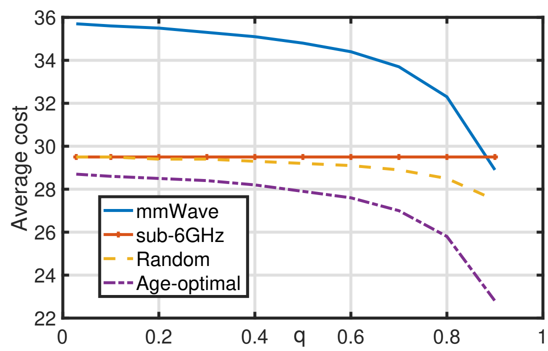

Figure 10. Time-average expected age vs. the parameter q of the mmWave channel, where $d = 20$ and $p = 0.972$. The considered regions are $\mathbf{B}_2$ and $\mathbf{B}_3$ with boundary $q = 1/d$. In $\mathbf{B}_2$, $\mu^*(\delta, 1, 0) = 1$, and $\mu^*(\delta, 0, 0)$ is non-decreasing in age. In $\mathbf{B}_3$, both $\mu^*(\delta, 1, 0)$ and $\mu^*(\delta, 0, 0)$ are non-decreasing in age.

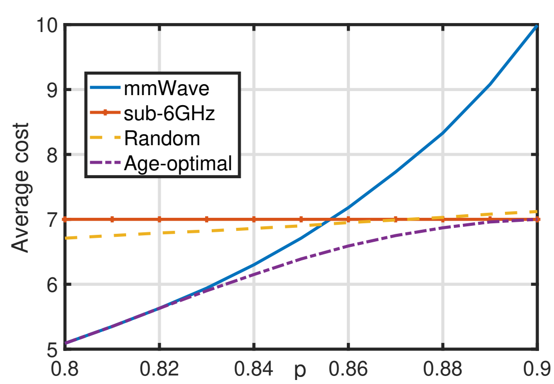

Figure 11. Time-average expected age vs. the parameter p of the mmWave channel, where $d = 5$ and $q = 0.1$. The considered regions are $\mathbf{B}_1$, $\mathbf{B}_4$, and $\mathbf{B}_2$ with boundary $p = 1 - 1/d$. In $\mathbf{B}_1$, the optimal decision $\mu^*(\delta, l_1, 0) = 1$. In $\mathbf{B}_3$, both $\mu^*(\delta, 1, 0)$ and $\mu^*(\delta, 0, 0)$ are non-decreasing in age.

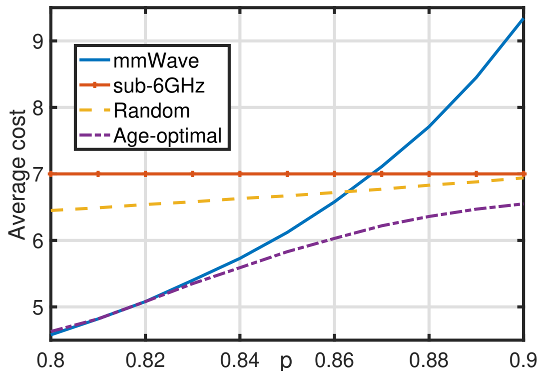

Figure 12. Time-average expected age vs. the parameter p of the mmWave channel, where $d = 5$ and $q = 0.5$. The considered regions are $\mathbf{B}_1$ and $\mathbf{B}_2$ with boundary $p = 1 - 1/d$. In $\mathbf{B}_1$, the optimal decision $\mu^*(\delta, l_1, 0) = 1$. In $\mathbf{B}_2$, $\mu^*(\delta, 1, 0) = 1$, and $\mu^*(\delta, 0, 0)$ is non-decreasing in age.

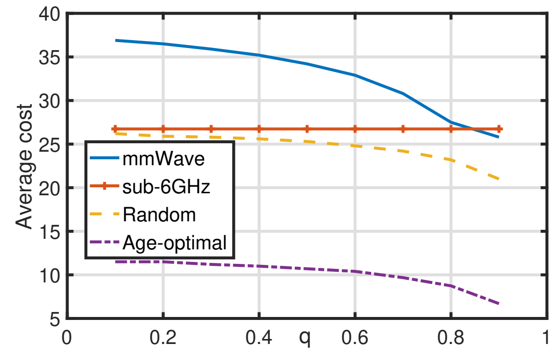

Figure 13. Time-average expected age penalty vs. the parameter q of the mmWave channel, where $p = 0.9$, $d = 20$, and the age penalty function is $f(\Delta) = (\frac{1}{p-0.003})^{\Delta}$.

parameters of the markovian mmWave channel p, q and the transmission delay of the sub-6GHz channel d. We first study a special case with i.i.d. mmWave channel. We provide the optimal threshold λ_0^* with the change of p for $d = 5, 10, 20$, respectively, where λ_0^* is the optimal threshold in i.i.d. channel described in Corollary 1. From Fig. 7, the optimal threshold diverges to infinity when p is approaching $p^* = 0.8, 0.9, 0.95$, respectively. Note that $p^* = 0.8, 0.9, 0.95$ is the boundary between $\mathbf{B}_1$ and $\mathbf{B}_3$ for $d = 5, 10, 20$, respectively. As p enlarges, the mmWave channel has worse connectivity, thus the threshold goes down and the optimal solution converges to always choosing the sub-6GHz channel. For non i.i.d. case, we then provide the optimal thresholds with $q = 0.5$. When $q = 0.5$, the mmWave channel gets positively correlated and the covered regions are $\mathbf{B}_1$ and $\mathbf{B}_2$. In region $\mathbf{B}_1$ and $\mathbf{B}_2$, we have observed that $\mu^*(\delta, 1, 0) = 1$ for all $\delta \geq 1$ ($\lambda_1^* = 1$), so we only list the optimal threshold λ_0^*. From Fig. 8, we observe that the evolution of the optimal threshold is similar to that of i.i.d. channel case (Fig. 7). The optimal threshold is smaller than that of i.i.d. channel, but the difference vanishes as d enlarges.

Further, we compare our optimal scheduling policy (called *Age-optimal*) with three other policies, including (i) always choosing the mmWave channel (called *mmWave*), (ii) always choosing the sub-6GHz channel (called *sub-6GHz*), and (iii) randomly choosing the mmWave and sub-6GHz channels with equal probability (called *Random*). We first provide the performance of these policies and their optimal decisions for different q in Fig. 9 and Fig 10. We observe that our optimal policy outperforms other policies. If the two channels have a similar age performance, we can observe the benefit of the optimal policy, and the benefit enlarges as the mmWave channel becomes positively correlated (q is larger). If the two channels have a large age performance disparity, the optimal policy is close to always choosing a single channel, and thus the benefit is obviously low. Further, we compare the optimal policy with the other three policies for different p and list the structure of optimal decisions in Fig. 11 and Fig. 12. In Fig. 11, the parameters (p, q, d) range in regions $\mathbf{B}_1$, $\mathbf{B}_4$, and $\mathbf{B}_3$. In Fig. 12, the parameters (p, q, d) range in regions $\mathbf{B}_1$ and $\mathbf{B}_2$. As p increases, mmWave channel has worse connectivity and gets outperformed by sub-6GHz channel. From Fig. 11 and Fig. 12, our optimal policy is close to mmWave when p is lower, and close to sub-6GHz when p is larger.

Finally, although our theoretical results consider linear age, we also provide numerical results when the cost function is nonlinear on age by using value iteration [38]. For exponential age in Fig. 13, the gain is significantly larger: for all values of q, the other policies have more than 2 times of average cost than the optimal policy. The numerical simulation for nonlinear age penalty function indicates the importance of exploring optimal policy for nonlinear age cost function, which is our future research direction.

6 Proofs of Main Results

In this section, we prove our main results: Theorem 1 (Section 6.2) and Theorems 2—5 (Section 6.3). In Section 6.1, we describe a discounted problem that helps to solve average problem (2). In Section 6.2, we introduce Proposition 1 which plays an important role in proving Theorem 1. Section 6.3 provides the proofs of Theorems 2—5.

6.1 Preliminaries

To solve Problem (2), we introduce a discounted problem below. The objective is to solve the discounted sum of expected cost given an initial state $\mathbf{s}$:

$$J^{\alpha}(\mathbf{s}) = \inf_{\pi \in \Pi'} \lim_{T \to \infty} \sum_{t=0}^{T} \mathbb{E}[\alpha^t \Delta^{\pi}(t) | \mathbf{s}(0) = \mathbf{s}], \tag{50}$$

where $\alpha \in (0, 1)$ is the discount factor. We call $J^{\alpha}(\mathbf{s})$ the *value function* given the initial state $\mathbf{s}$. Recall that we use $\mathbf{s} = (\delta, l_1, l_2)$ to denote the system state, where δ is the age value and l_1, l_2 are the state of Channel 1 and Channel 2. From Lemma 1, we only need to consider $\pi \in \Pi'$ instead of $\pi \in \Pi$.

The value function $J^{\alpha}(\mathbf{s})$ satisfies a following property:

Lemma 4. *For any given α and $\mathbf{s}$, $J^{\alpha}(\mathbf{s}) < \infty$.*

Proof. See Appendix C in our supplementary material. □

A policy π is deterministic stationary if $\pi(t) = Z(\mathbf{s}(t))$ at any time t, where $Z : \mathbf{S} \to \Pi'$ is a deterministic function. According to [46], and Lemma 4, there is a direct result for Problem (50):

Lemma 5. *(a) The value function $J^\alpha(\boldsymbol{s})$ satisfies the Bellman equation*

$$Q^\alpha(\boldsymbol{s}, u) \triangleq \delta + \alpha \sum_{\boldsymbol{s}' \in S} P_{\boldsymbol{s}\boldsymbol{s}'}(u) J^\alpha(\boldsymbol{s}'),$$
$$J^\alpha(\boldsymbol{s}) = \min_{u \in \Pi'} Q^\alpha(\boldsymbol{s}, u). \tag{51}$$

(b) There exists a deterministic stationary policy $\mu^{\alpha,*}$ that satisfies Bellman equation (51)*. The policy $\mu^{\alpha,*}$ solves Problem* (50) *for all initial state $\boldsymbol{s}$.*

(c) Assume that $J_0^\alpha(\boldsymbol{s}) = 0$ for all $\boldsymbol{s}$. For $n \geq 1$, J_n^α is defined as

$$Q_n^\alpha(\boldsymbol{s}, u) \triangleq \delta + \alpha \sum_{\boldsymbol{s}' \in S} P_{\boldsymbol{s}\boldsymbol{s}'}(u) J_{n-1}^\alpha(\boldsymbol{s}'),$$
$$J_n^\alpha(\boldsymbol{s}) = \min_{u \in \Pi'} Q_n^\alpha(\boldsymbol{s}, u), \tag{52}$$

then $\lim_{n\to\infty} J_n^\alpha(\boldsymbol{s}) = J^\alpha(\boldsymbol{s})$ for every $\boldsymbol{s}$.

Also, since the cost function is linearly increasing in age, utilizing Lemma 5(c), we also have

Lemma 6. *For all given l_1 and l_2, $J^\alpha(\delta, l_1, l_2)$ is increasing in δ.*

Proof. See Appendix D in our supplementary material. □

Since Problem (50) satisfies the properties in Lemma 5, utilizing Lemma 5 and Lemma 6, the following Lemma gives the connection between Problem (2) and Problem (50).

Lemma 7. *(a) There exists a stationary deterministic policy that is optimal for Problem* (2)*.*

(b) There exists a value J^* for all initial state $\boldsymbol{s}$ such that

$$\lim_{\alpha \to 1^-} (1 - \alpha) J^\alpha(\boldsymbol{s}) = J^*.$$

Moreover, J^* is the optimal average cost for Problem (2)*.*

(c) For any sequence $(\alpha_n)_n$ of discount factors that converges to 1, there exists a subsequence $(\beta_n)_n$ such that $\lim_{n\to\infty} \mu^{\beta_n,*} = \mu^*$. Also, μ^* is the optimal policy for Problem 2.

Proof. See Appendix E in our supplementary material. □

Lemma 7 provides the fact that: We can solve Problem (50) to achieve Problem (2). The reason is that the optimal policy of Problem (50) converges to the optimal policy of Problem (2) in a limiting scenario (as $\alpha \to 1$).

6.2 Proof of Theorem 1

We begin with providing an optimal structural result of discounted policy $\mu^{\alpha,*}$. Then, we achieve the average optimal policy μ^* by letting $\alpha \to 1$.

Definition 3. *For any discount factor $\alpha \in (0, 1)$, the channel parameters $p, q \in (0, 1)$ and $d \in \{2, 3, ...\}$, we define*

$$\boldsymbol{B}_1(\alpha) = \{(p, q, d) : F(p, q, d, \alpha) \leq 0, H(p, q, d, \alpha) \leq 0\},$$
$$\boldsymbol{B}_2(\alpha) = \{(p, q, d) : F(p, q, d, \alpha) > 0, G(p, q, d, \alpha) \leq 0\},$$
$$\boldsymbol{B}_3(\alpha) = \{(p, q, d) : F(p, q, d, \alpha) > 0, G(p, q, d, \alpha) > 0\},$$
$$\boldsymbol{B}_4(\alpha) = \{(p, q, d) : F(p, q, d, \alpha) \leq 0, H(p, q, d, \alpha) > 0\}, \tag{53}$$

where functions $F(\cdot), G(\cdot), H(\cdot) : \Theta \times (0, 1) \to \mathbb{R}$ are defined as:

$$F(p, q, d, \alpha) = \sum_{i=0}^{\infty} (\alpha p)^i - \sum_{i=0}^{d-1} \alpha^i,$$
$$G(p, q, d, \alpha) = 1 + \alpha(1 - q) \sum_{i=0}^{d-1} \alpha^i - \sum_{i=0}^{d-1} \alpha^i, \tag{54}$$
$$H(p, q, d, \alpha) = 1 + \alpha(1 - q) \sum_{i=0}^{\infty} (\alpha p)^i - \sum_{i=0}^{d-1} \alpha^i.$$

Observe that all four regions $\mathbf{B}_i(\alpha)$ converge to $\mathbf{B}_i$ as the discount factor $\alpha \to 1$, where the regions $\mathbf{B}_i$ are described in Definition 6.

The optimal structural result of Problem (50) with a discount factor α is provided in the following proposition:

Proposition 1. *There exists a threshold type policy $\mu^{\alpha,*}(\delta, l_1, 0)$ on age δ that is the solution to Problem* (50) *such that:*

(a) If $l_1 = 0$ and $(p, q, d) \in \boldsymbol{B}_1(\alpha) \cup \boldsymbol{B}_4(\alpha)$, then $\mu^{\alpha,*}(\delta, l_1, 0)$ is non-increasing in the age δ.

(b) If $l_1 = 0$ and $(p, q, d) \in \boldsymbol{B}_2(\alpha) \cup \boldsymbol{B}_3(\alpha)$, then $\mu^{\alpha,*}(\delta, l_1, 0)$ is non-decreasing in the age δ.

(c) If $l_1 = 1$ and $(p, q, d) \in \boldsymbol{B}_1(\alpha) \cup \boldsymbol{B}_2(\alpha)$, then $\mu^{\alpha,*}(\delta, l_1, 0)$ is non-increasing in the age δ.

(d) If $l_1 = 1$ and $(p, q, d) \in \boldsymbol{B}_3(\alpha) \cup \boldsymbol{B}_4(\alpha)$, then $\mu^{\alpha,*}(\delta, l_1, 0)$ is non-decreasing in the age δ.

Note that Theorem 1 can be immediately shown from Proposition 1, Lemma 7 and the convergence of the regions $\mathbf{B}_i(\alpha)$ to $\mathbf{B}_i$ (for $i = 1, 2, 3, 4$) as $\alpha \to 1$. *The rest of Section 6.2 provides the proof for Proposition 1.*

Since Channel 1 and Channel 2 have different delays, we are not able to show that the optimal policy is threshold type by directly observing the Bellman equation like [23]. Thus, we will use the concept of *super-modularity* [36, Theorem 2.8.2]. The domain of age set and decision set in the Q-function is $\{1, 2, ...\} \times \{1, 2\}$, which is a *lattice*. Given a positive s, the subset $\{s, s + 1, ...\} \times \{1, 2\}$ is a *sublattice* of $\{1, 2, ...\} \times \{1, 2\}$. Thus, if the following holds for all $\delta > s$:

$$Q^\alpha(\delta, l_1, 0, 1) - Q^\alpha(\delta - 1, l_1, 0, 1)$$
$$\leq Q^\alpha(\delta, l_1, 0, 2) - Q^\alpha(\delta - 1, l_1, 0, 2), \tag{55}$$

then the Q-function $Q^\alpha(\delta, l_1, 0, u)$ is super-modular in (δ, u) for $\delta > s$, which means the optimal decision

$$\mu^{\alpha,*}(\delta, l_1, 0) = \text{argmin}_{u \in \{1,2\}} Q^\alpha(\delta, l_1, 0, u) \tag{56}$$

is non-increasing in δ for $\delta \geq s$. If the inequality of (55) is inversed, then we call $Q^\alpha(\delta, l_1, 0)$ is sub-modular in (δ, u) for $\delta > s$, and $\mu^{\alpha,*}(\delta, l_1, 0)$ is non-decreasing in δ for $\delta \geq s$.

For ease of notations, we give Definition 4:

Definition 4. *Given $l_1 \in \{0,1\}$, $u \in \{1,2\}$,*

$$L^\alpha(\delta, l_1, u) \triangleq Q^\alpha(\delta, l_1, 0, u) - Q^\alpha(\delta-1, l_1, 0, u). \quad (57)$$

Note that $L^\alpha(\delta, l_1, 1)$ is the left hand side of (55), and $L^\alpha(\delta, l_1, 2)$ is the right hand side of (55).

Our high-level idea to show Proposition 1 is as follows: First, we show that $L^\alpha(\delta, l_1, 2)$ is a constant (see Lemma 8 below), then we compare $L^\alpha(\delta, l_1, 1)$ with the constant to check super-modularity (see the proofs of Lemma 9 and Lemma 10 below).

Suppose that $m \triangleq \sum_{i=0}^{d-1} \alpha^i$, and we have:

Lemma 8. *For all $\delta \geq 2$ and $l_1 \in \{0,1\}$, $L^\alpha(\delta, l_1, 2) = m$.*

Proof. See Appendix F in our supplementary material. □

Also, we have

Lemma 9. *(a) If $l_1 = 0$ and $(p,q,d) \in \boldsymbol{B}_1(\alpha) \cup \boldsymbol{B}_4(\alpha)$, then $Q^\alpha(\delta, l_1, 0, u)$ is super-modular in (δ, u) for $\delta \geq 2$.*

(b) If $l_1 = 0$ and $(p,q,d) \in \boldsymbol{B}_2(\alpha) \cup \boldsymbol{B}_3(\alpha)$, then $Q^\alpha(\delta, l_1, 0, u)$ is sub-modular in (δ, u) for $\delta \geq 2$.

Proof. See Appendix H in our supplementary material. □

Lemma 9(a) implies that $\mu^{\alpha,*}(\delta, 0, 0)$ is non-increasing in δ if $(p,q,d) \in \mathbf{B}_1(\alpha) \cup \mathbf{B}_4(\alpha)$. Lemma 9(b) implies that $\mu^{\alpha,*}(\delta, 0, 0)$ is non-decreasing in δ if $(p,q,d) \in \mathbf{B}_2(\alpha) \cup \mathbf{B}_3(\alpha)$. Thus, Proposition 1(a),(b) hold.

Lemma 9 gives the result when the previous state of Channel 1 is 0. We then need to solve when the previous state of Channel 1 is 1. Different from $Q^\alpha(\delta, 0, 0, u)$, the Q-function $Q^\alpha(\delta, 1, 0, u)$ does not satisfy super-modular (or sub-modular) in (δ, u) for all the age value δ. Thus, we give a weakened condition: we can find out a value s, such that the Q-function $Q^\alpha(\delta, 1, 0, u)$ is super-modular (or sub-modular) for *a partial age set* $s, s+1, ...$ and $\mu^{\alpha,*}(\delta, 1, 0)$ is a constant on the set $1, 2, ..., s$. Then, $\mu^{\alpha,*}(\delta, 1, 0)$ is still non-increasing (or non-decreasing). Note that super-/sub-modularity is the sufficient but not necessary condition to the monotonicity of $\mu^{\alpha,*}(\delta, l_1, 0)$ in δ.

Thus, to solve Proposition 1(c),(d), we provide the following lemma:

Lemma 10. *(a) If $l_1 = 1$ and $(p,q,d) \in \boldsymbol{B}_1(\alpha) \cup \boldsymbol{B}_2(\alpha)$, then there exists a positive integer s, such that $Q^\alpha(\delta, l_1, 0, u)$ is super-modular in (δ, u) for $\delta > s$, and $\mu^{\alpha,*}(\delta, l_1, 0)$ is always 1 or always 2 for all $\delta \leq s$.*

(b) If $l_1 = 1$ and $(p,q,d) \in \boldsymbol{B}_3(\alpha) \cup \boldsymbol{B}_4(\alpha)$, then there exists a positive integer s, such that $Q^\alpha(\delta, l_1, 0, u)$ is sub-modular in (δ, u) for $\delta > s$, and $\mu^{\alpha,*}(\delta, l_1, 0)$ is always 1 or always 2 for all $\delta \leq s$.

Proof. See Appendix J in our supplementary material. □

Lemma 10(a) implies that $\mu^{\alpha,*}(\delta, 1, 0)$ is non-increasing for $\delta \geq s$ and is constant for for $\delta \leq s$. Thus, $\mu^{\alpha,*}(\delta, 1, 0)$ is non-increasing in δ. Similarly, Lemma 10(b) implies that $\mu^{\alpha,*}(\delta, 1, 0)$ is non-decreasing for $\delta > 0$. Thus, we have shown Proposition 1(c),(d). Showing the threshold structure of $\mu^{\alpha,*}(\delta, l_1, 0)$ even if super-modularity does not hold is one of the key technical contributions in this paper.

Overall, Lemma 8 and Lemma 9 shows Proposition 1(a),(b). Lemma 8 and Lemma 10 shows Proposition 1(c),(d). Thus we have completed the proof of Proposition 1.

To summarize Section 6.2, Proposition 1, Lemma 7 and the convergence of $\mathbf{B}_1(\alpha), \ldots, \mathbf{B}_4(\alpha)$ to $\mathbf{B}_1, \ldots, \mathbf{B}_4$ show Theorem 1.

6.3 Proofs of Theorems 2—5

In this section, we prove Theorems 2—5 with $(p,q,d) \in \mathbf{B}_1$—$(p,q,d) \in \mathbf{B}_4$, respectively for efficiently deriving an optimal threshold-type solution.

6.3.1 Proof of Theorem 2

For $(p,q,d) \in \mathbf{B}_1$, we firstly prove that $\mu^*(\delta, 0, 0) = 1$ and then show that $\mu^*(\delta, 1, 0) = 1$.

Lemma 11. *If $(p,q,d) \in \boldsymbol{B}_1 \cup \boldsymbol{B}_4$, then the optimal decisions at states $(\delta, 0, 0)$ for all δ are 1.*

Proof. See Appendix M in our supplementary material. □

In addition, when $l_1 = 1$, we have the following:

Lemma 12. *If $(p,q,d) \in \boldsymbol{B}_1$, then the optimal decisions at states $(\delta, 1, 0)$ for all δ are 1.*

Proof. See Appendix O in our supplementary material. □

Since $\mu^*(\delta, 1, 0)$ is non-increasing in the region $\mathbf{B}_1$ by Theorem 1, Lemma 12 implies that $\mu^*(\delta, 1, 0) = 1$ for all δ. Besides, Lemma 11 implies that $\mu^*(\delta, 0, 0) = 1$ for all δ. Thus, Theorem 2 follows directly from Lemma 11 and Lemma 12. The optimal policy for $(p,q,d) \in \mathbf{B}_1$ is always choosing Channel 1.

6.3.2 Proof of Theorem 3

In (9), we have stated that the MDP problem (2) is reduced to deriving the steady-state distributions of the DTMCs. Note that Channel 1 is Markovian ($l_1 = 0$ or 1). When $l_1 = 1$, we observe that only the states $(1,1,0)$ and $(d,1,0)$ can be reached with positive probability for any policy in Π'. As a result, (9) can be reduced to a number of the steady-state distributions of the DTMCs with different actions at $(1,1,0)$ and $(d,1,0)$. In addition, we observe that the state transition matrices of the DTMCs in (9) are significantly different depending on the action at $(d,0,0)$. Thus, we conclude that there are at most 2^3 different steady-state distributions of DTMCs based on the actions at three system states: $(1,1,0), (d,1,0)$ with $l_1 = 1$ and $(d,0,0)$ with $l_1 = 0$. Despite that there are totally 2^3 cases to enumerate, we manage to reduce to only 4 cases as in (27) (for $(p,q,d) \in \mathbf{B}_2$). The reason is that the remaining cases are impossible to occur due to the two following restrictions: (1) the monotonicity is known by Theorem 1, and (2) the following lemma:

Lemma 13. *If Channel 1 is positive-correlated, i.e., $p+q \geq 1$, and $\mu^*(\delta, 0, 0) = 1$, then $\mu^*(\delta, 1, 0) = 1$. Conversely, if Channel*

1 is negative-correlated, i.e. $p+q \le 1$, *and* $\mu^*(\delta,0,0)=2$, *then* $\mu^*(\delta,1,0)=2$.

Proof. See Appendix K in our supplementary material. □

Since our optimal policy is of threshold-type, the action at $(d,0,0)$ is equivalent to whether the threshold of $\mu^*(\delta,0,0)$ is larger or smaller than d. Thus, we use s to denote the possible threshold of $\mu^*(\delta,0,0)$.

For $(p,q,d) \in \mathbf{B}_2$, $\mu^*(\delta,1,0)$ is non-increasing, and $\mu^*(\delta,0,0)$ is non-decreasing. Note that $(p,q,d)\in\mathbf{B}_2$ implies $p+q\ge 1$. According to Lemma 13, if $\mu^*(1,1,0)=2$, then $\mu^*(1,0,0)=2$, hence $\mu^*(\delta,0,0)=2$ for all δ. Thus, there are two possible types of DTMCs regarding $\mu^*(d,1,0)=1$ or $\mu^*(d,1,0)=2$. If $\mu^*(1,1,0)=1$, then $\mu^*(\delta,1,0)=1$ for all δ, there are thus two possible types of DTMCs regarding the threshold $s>d$ or $s\le d$. Thus, for $(p,q,d)\in\mathbf{B}_2$, there are four possible ways to represent the DTMC diagram of the threshold policy based on the value of the threshold s and the actions at states $(d,l_1,0)$ and $(1,1,0)$ (see Appendix P in our supplementary material for the corresponding DTMCs and derivations):

- The threshold $s>d$ and $\mu^*(1,1,0)=\mu^*(d,1,0)=1$ ($\lambda_1^*=1$). Note that we have mentioned $\bar{\Delta}_2(\lambda_0,\lambda_1)$ as the average age of the DTMC with thresholds (λ_0,λ_1) when $(p,q,d)\in\mathbf{B}_2$. Then, the average age is derived as $\bar{\Delta}_2(s,1)=f_1(s)/g_1(s)$, which is shown in Appendix P.1 in our supplementary material. The functions $f_1(s), g_1(s)$ are described in Table III of our supplementary material. As is shown later, β_1 described in Definition 2 is the minimum of $f_1(s)/g_1(s)$.
- The threshold $s\le d$ and $\mu^*(1,1,0)=\mu^*(d,1,0)=1$ ($\lambda_1^*=1$). Then the average age is $\bar{\Delta}_2(s,1)=f_2(s)/g_2(s)$, which is shown in Appendix P.2 in our supplementary material. The functions $f_2(s), g_2(s)$ are described in Table III in our supplementary material. As is shown later, β_2 described in Definition 2 is the minimum of $f_2(s)/g_2(s)$.
- The threshold $s=1$, $\mu^*(1,1,0)=2$ and $\mu^*(d,1,0)=1$ ($\lambda_1^*\in\{2,3,...,d\}$). The average age is the constant f_0/g_0, which is shown in Appendix P.3 in our supplementary material. Note that f_0/g_0 is described in (20). Theorem 3.
- The threshold $s=1$ and $\mu^*(1,1,0)=\mu^*(d,1,0)=2$ ($\lambda_1^*\in\{d+1,d+2,...\}$). This policy means that we always choose Channel 2. So the average age is $(3/2)d-1/2$.

The listed statements illustrated above directly provides the following property:

Proposition 2. *If* $(p,q,d)\in\boldsymbol{B}_2$, *then the optimal scheduling policy is*

$$\mu^*(\delta,0,0)=\begin{cases}1 & \text{if } \delta<\lambda_0^*;\\ 2 & \text{if } \delta\ge\lambda_0^*,\end{cases} \tag{58}$$

$$\mu^*(\delta,1,0)=\begin{cases}2 & \text{if } \delta<\lambda_1^*;\\ 1 & \text{if } \delta\ge\lambda_1^*,\end{cases} \tag{59}$$

where λ_0^* *and* λ_1^* *are given by*

$$\begin{cases}\lambda_0^*=\underset{s\in\{d+1,\dots\}}{\operatorname{argmin}}\ f_1(s)/g_1(s), \lambda_1^*=1 & \text{if } \bar{\Delta}_{opt}=\beta_1',\\ \lambda_0^*=\underset{s\in\{1,\dots,d\}}{\operatorname{argmin}}\ f_2(s)/g_2(s),\ \lambda_1^*=1 & \text{if } \bar{\Delta}_{opt}=\beta_2',\\ \lambda_0^*=1,\quad \lambda_1^*\in\{2,3,\dots,d\} & \text{if } \bar{\Delta}_{opt}=f_0/g_0,\\ \lambda_0^*=1,\quad \lambda_1^*\in\{d+1,\dots\} & \text{if } \bar{\Delta}_{opt}=(3/2)d-1/2,\end{cases} \tag{60}$$

$\bar{\Delta}_{opt}$ *is the optimal objective value of* (2), *determined by*

$$\bar{\Delta}_{opt}=\min\Big\{\beta_1',\beta_2',\frac{f_0}{g_0},\frac{3}{2}d-\frac{1}{2}\Big\}, \tag{61}$$

f_0, g_0 *are given by Theorem 3, and* β_1', β_2' *are given in* (28), (29), *respectively.*

By using Dinkelbach's method [47], we can change the minimization problem (28), (29) into a two-layer problem. The inner-layer problem is shown to be unimodal and we derive an exact solution. Thus, we only need a bisection algorithm for the outer-layer, i.e., solving the roots of the equations $h_1(\beta)=0, h_2(\beta)=0$ in (21). To show this, we introduce the following lemma:

Lemma 14. *Suppose that* $i\in\{1,2,3,4\}$. *Define*

$$h_i'(c)=\min_{s\in\{d+1,\dots\}} f_i(s)-cg_i(s),\ i\in\{1,3,4\}, \tag{62}$$

$$h_2'(c)=\min_{s\in\{2,\dots d\}} f_2(s)-cg_2(s), \tag{63}$$

then for all $i\in\{1,2,3,4\}$, $h_i'(c)\lesseqqgtr 0$ *if and only if* $c\gtreqqless\beta_i'$.

Proof. See Appendix Q in our supplementary material. □

The solution to $h_i'(c)$ in Lemma 14 is shown in the following lemma:

Lemma 15. *Suppose that* $i\in\{1,2,3,4\}$. *If* $(p,q,d)\in\boldsymbol{B}_2\cup\boldsymbol{B}_3$, *then the threshold* $s_i(c)$ *defined in* (22) *is the solution to* (62) *and* (63), *i.e.,* $h_i(c)=h_i'(c)$.

Proof. See Appendix R in our supplementary material. □

Therefore, we can immediately conclude that for all $i\in\{1,2,3,4\}$:

$$\beta_i'=\beta_i, \tag{64}$$

where β_i' is defined in (28), (29) and β_i is derived in Definition 2 with low complexity algorithm. In addition,

$$s_i(\beta_i)=\underset{s\in\{d+1,\dots\}}{\operatorname{argmin}}\ f_i(s)/g_i(s), i\in\{1,3,4\}, \tag{65}$$

$$s_2(\beta_2)=\underset{s\in\{2,\dots,d\}}{\operatorname{argmin}}\ f_2(s)/g_2(s). \tag{66}$$

The studies in [10], [11], [21] also derive an exact solution to their inner-layer problem. However, their technique is using optimal stopping rules [11], [21] or stochastic convex optimization [10], which is different from our study. In conclusion, (64) and Proposition 2 shows Theorem 3.

6.3.3 Proof of Theorem 4

When $(p,q,d) \in \mathbf{B}_3$, $\mu^*(\delta,0,0)$ and $\mu^*(\delta,1,0)$ are non-decreasing. Then, the two cases are removed: $\mu^*(\delta,0,0)=2$, $\mu^*(\delta,1,0)=1$, $s \le d$ or $s > d$. Since $(p,q,d) \in \mathbf{B}_3$ does not imply $p+q \le 1$ or $p+q \ge 1$, we will enumerate all of the five possible ways to represent the DTMCs of the threshold policy based on the value of the threshold s and the optimal decision at states $(d,1,0)$ and $(1,1,0)$ (see Appendix P in our supplementary material for the corresponding DTMCs):

- The threshold $s > d$ and $\mu^*(1,1,0)=\mu^*(d,1,0)=1$ ($\lambda_1^* \in \{d+1, d+2, ...\}$). The average age is derived as $f_1(s)/g_1(s)$.
- The threshold $s > d$, $\mu^*(1,1,0)=1$ and $\mu^*(d,1,0)=2$ ($\lambda_1^* \in \{2,...,d\}$). Then, the average age is $f_3(s)/g_3(s)$, which is shown in Appendix P.4 in our supplementary material.
- The threshold $s > d$ and $\mu^*(1,1,0)=\mu^*(d,1,0)=2$ ($\lambda_1^* \in \{2,...,d\}$) with average age $f_4(s)/g_4(s)$, which is shown in Appendix P.5 in our supplementary material.
- The threshold $s \le d$ and $\mu^*(1,1,0)=\mu^*(d,1,0)=1$ ($\lambda_1^* \in \{d+1, d+2, ...\}$), with average age $f_2(s)/g_2(s)$.
- The threshold $s \le d$ and $\mu^*(d,1,0)=2$. Then, regardless of $\mu^*(1,1,0)$ ($\lambda_1^* \in \{1,2,...,d\}$), the DTMC corresponds to always choosing 2, with average age $(3/2)d - 1/2$.

Then, we directly have the following result:

Proposition 3. *If $(p,q,d) \in \boldsymbol{B}_3$, then the optimal scheduling policy is*

$$\mu^*(\delta,0,0) = \begin{cases} 1 & \text{if } \delta < \lambda_0^*; \\ 2 & \text{if } \delta \ge \lambda_0^*, \end{cases} \tag{67}$$

$$\mu^*(\delta,1,0) = \begin{cases} 1 & \text{if } \delta < \lambda_1^*; \\ 2 & \text{if } \delta \ge \lambda_1^*, \end{cases} \tag{68}$$

where λ_0^* and λ_1^* are given by

$$\begin{cases} \lambda_0^* = \underset{s\in\{d+1,\dots\}}{\operatorname{argmin}} f_1(s)/g_1(s), \lambda_1^* \in \{d+1,\dots\} \text{ if } \bar{\Delta}_{opt} = \beta_1', \\ \lambda_0^* = \underset{s\in\{2,\dots,d\}}{\operatorname{argmin}} f_2(s)/g_2(s),\ \lambda_1^* \in \{d+1,\dots\} \text{ if } \bar{\Delta}_{opt} = \beta_2', \\ \lambda_0^* = \underset{s\in\{d+1,\dots\}}{\operatorname{argmin}} f_3(s)/g_3(s), \lambda_1^* \in \{2,\dots,d\} \quad \text{if } \bar{\Delta}_{opt} = \beta_3', \\ \lambda_0^* = \underset{s\in\{d+1,\dots\}}{\operatorname{argmin}} f_4(s)/g_4(s), \lambda_1^* \in \{2,\dots,d\} \quad \text{if } \bar{\Delta}_{opt} = \beta_4', \\ \lambda_0^* = 1, \quad \lambda_1^* \in \{1,2,\dots,d\} \quad \text{if } \bar{\Delta}_{opt} = (3/2)d - 1/2, \end{cases} \tag{69}$$

$\bar{\Delta}_{opt}$ is the optimal objective value of (2)*, determined by*

$$\bar{\Delta}_{opt} = \min\left\{\beta_1', \beta_2', \beta_3', \beta_4', \frac{3}{2}d - \frac{1}{2}\right\}. \tag{70}$$

According to (64), (65) and (66), Theorem 4 is shown directly from Proposition 3.

6.3.4 Proof of Theorem 5

For $(p,q,d) \in \mathbf{B}_4$, $\mu^*(\delta,1,0)$ is non-decreasing in δ from Theorem 1. Also, $\mu^*(\delta,0,0)=1$ by Lemma 11.

If $\mu^*(1,1,0)=1$, the policy becomes always choosing Channel 1 (since $(d,1,0)$ is not reached at any time slot with probability 1). If $\mu^*(1,1,0)=2$, then $\mu^*(\delta,1,0)=2$ for all δ. Thus, the solution to the optimal threshold-type policy when $(p,q,d) \in \mathbf{B}_4$ may contain two possible steady-state DTMCs which directly gives Theorem 5:

- The optimal decision $\mu^*(\delta,0,0)=1$ for all $\delta \ge 1$ and $\mu^*(1,1,0)=1$. Then, the optimal policy is always choosing Channel 1. The average age of always choosing Channel 1 is $((1-q)(2-p)+(1-p)^2)/((2-q-p)(1-p))$ as in (12).
- The optimal decision $\mu^*(\delta,0,0)=1$ and $\mu^*(\delta,1,0)=2$ for all $\delta \ge 1$. See Appendix P.6 in our supplementary material for the corresponding DTMC. The average age by analyzing the steady-state distribution of this DTMC is f_0'/g_0' which is shown in Appendix P.6 in our supplementary material.

Therefore, the two listed items above directly prove Theorem 5.

From our analysis in Section 6.3, we have the following conclusion for the proof of Theorems 2-5: (i) If $(p,q,d) \in \mathbf{B}_1$, the optimal decision is always choosing Channel 1; (ii) If $(p,q,d) \in \mathbf{B}_2$, $\mathbf{B}_3$ or $\mathbf{B}_4$, there are a couple of possible cases (4 cases for $(p,q,d) \in \mathbf{B}_2$, 5 cases for $(p,q,d) \in \mathbf{B}_3$ and 2 cases for $(p,q,d) \in \mathbf{B}_4$, respectively). Each case corresponds to analyzing the steady-state distribution of a single DTMC or a collection of DTMCs over the threshold s; in the latter case, the optimal threshold can be computed efficiently using bisection search. The optimal objective value in (2) is the minimum of the derived ages in each cases and the optimal thresholds are determined by the case that achieves the minimum.

7 CONCLUSION

In this paper, we have studied age-optimal transmission scheduling for hybrid mmWave/sub-6GHz channels. For all possibly values of the channel parameters and the ON-OFF state of the mmWave channel, the optimal scheduling policy have been proven to be of threshold-type on the age. Low complexity algorithms have been developed for finding the optimal scheduling policy. Finally, our numerical results show that the optimal policy can reduce age compared with other policies.

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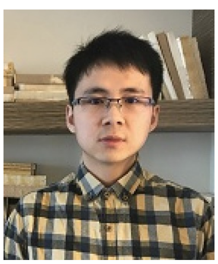

Jiayu Pan received the B.S. degree in Physics from Shanghai Jiao Tong University, Shanghai, China in 2017. He is currently pursuing the Ph.D. degree with the Electrical and Computer Engineering Department, at the Ohio State University, OH, USA. His research interests include wireless communication and age of information. He received the First Prize Scholarship from Shanghai Jiao Tong University in 2015.

Ahmed M. Bedewy received the B.S. and M.S. degrees in electrical and electronics engineering from Alexandria University, Alexandria, Egypt, in 2011 and 2015, respectively, and the Ph.D. degree in electrical and computer engineering from The Ohio State University, OH, USA, in 2021. His research interests include wireless communication, cognitive radios, resource allocation, communication networks, information freshness, optimization, and scheduling algorithms. He received the Awarded Certificate of Merit and First Class Honors for being one of the top ten undergraduate students, for the period 2006–2008, and the First, for the period 2008–2011, in electrical and electronics engineering. His article received the runner-up for the Best Paper Award of ACM MobiHoc 2020.

Yin Sun (Senior Member, IEEE) is currently an Assistant Professor with the Department of Electrical and Computer Engineering, Auburn University. He received his B.Eng. and Ph.D. degrees in electronic engineering from Tsinghua University in 2006 and 2011, respectively. From 2011 to 2017, he was a Post-Doctoral Scholar and a Research Associate with The Ohio State University. His research interests include wireless networks, age of information, machine learning, robotic control, and information theory. He co-authored a monograph *Age of Information: A New Metric for Information Freshness* (Morgan and Claypool Publishers, 2019). He has been a Guest Editor of the *IEEE Journal on Selected Areas in Communications* for the special issue on "Age of Information in Real-time Systems and Networks," a Guest Editor of *Entropy* for the special issue on "Age of Information: Concept, Metric and Tool for Network Control," a Guest Editor of *Frontiers in Communications and Networks* for the special issue on "Age of Information," an Associate Editor of the *IEEE Transactions on Network Science and Engineering*, and an Editor of the *Journal of Communications and Networks.* His articles received the Best Student Paper Award from the IEEE/IFIP WiOpt 2013, the Best Paper Award from the IEEE/IFIP WiOpt 2019, runner-up for the Best Paper Award of ACM MobiHoc 2020, and the 2021 *Journal of Communications and Networks (JCN)* Best Paper Award. He received the Auburn Author Award in 2020. He co-founded the Age of Information Workshop in 2018. He is a member of the ACM.

Ness B. Shroff (S'91–M'93–SM'01–F'07) received the Ph.D. degree in electrical engineering from Columbia University in 1994. He joined Purdue University immediately thereafter as an Assistant Professor with the School of Electrical and Computer Engineering. At Purdue, he became a Full Professor of ECE and the director of a university-wide center on wireless systems and applications in 2004. In 2007, he joined The Ohio State University, where he holds the Ohio Eminent Scholar Endowed Chair in networking and communications, in the departments of ECE and CSE. He holds or has held visiting (chaired) professor positions at Tsinghua University, Beijing, China, Shanghai Jiaotong University, Shanghai, China, and IIT Bombay, Mumbai, India. He has received numerous best paper awards for his research and is listed in Thomson Reuters' on The World's Most Influential Scientific Minds, and is noted as a Highly Cited Researcher by Thomson Reuters. He currently serves as the steering committee chair for ACM Mobihoc and the Editor in Chief of the IEEE/ACM Transactions on Networking. He received the IEEE INFOCOM Achievement Award for seminal contributions to scheduling and resource allocation in wireless networks.