Minimizing Age of Information via Scheduling over Heterogeneous Channels

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ABSTRACT

In this paper, we study the problem of minimizing the age of information when a source can transmit status updates over two heterogeneous channels. Our work is motivated by recent developments in 5G mmWave technology, where transmissions may occur over an unreliable but fast (e.g., mmWave) channel or a slow reliable (e.g., sub-6GHz) channel. The unreliable channel is modeled as a time-correlated Gilbert-Elliot channel, where information can be transmitted at a high rate when the channel is in the "ON" state. The reliable channel provides a deterministic but lower data rate. The scheduling strategy determines the channel to be used for transmission with the aim to minimize the time-average age of information (AoI). The optimal scheduling problem is formulated as a Markov Decision Process (MDP), which in our setting poses some significant challenges because e.g., supermodularity does not hold for part of the state space. We show that there exists a multi-dimensional threshold-based scheduling policy that is optimal for minimizing the age. A low-complexity bisection algorithm is further devised to compute the optimal thresholds. Numerical simulations are provided to compare different scheduling policies.

CCS CONCEPTS

• **Networks → Network performance evaluation**; *Network performance analysis*;

KEYWORDS

Age of information; Heterogeneous channels; Scheduling policy

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1 INTRODUCTION

The timely update of the system state is of great significance in cyber-physical systems such as vehicular networks, sensor networks, and UAV navigations. In these applications, newly generated data is more desirable than outdated data. *Age of information* (AoI), or simply *age*, was introduced as an end-to-end application-layer metric to evaluate data freshness [1, 3–6, 8, 9, 12, 14–18, 20, 21, 24, 29, 35–40, 43]. The age at time t is defined as $\Delta(t) = t - U_t$, where U_t is the generation time of the freshest packet that has been received by time t. The difference between age and classical metrics like delay and throughput is evident even in elementary queuing systems [18]. High throughput implies frequent status updates, but tends to cause a higher queuing delay that worsens timeliness. On the other hand, delay can be greatly reduced by decreasing the update frequency, which, however, may increase the age because the status is updated infrequently.

In future wireless networks, the sub-6GHz frequency spectrum is insufficient for fulfilling the high throughput demand of emerging real-time applications such as VR/AR applications, where contents must be delivered within 5-20 ms of latency, requiring a high throughput of 400-600 Mbps [30]. To address this challenge, 5G technology utilizes high-frequency millimeter wave (mmWave) bands such as 28/38 GHz, which provide a much higher data rate than sub-6GHz [31]. Recently, Verizon and Samsung demonstrated that a throughput of nearly 4Gbps was achieved in their mmWave demo system, using a 28GHz frequency band with 800MHz bandwidth [32]. However, unlike sub-6GHz spectrum bands, mmWave channels are highly unreliable due to blocking susceptibility, strong atmospheric absorption, and low penetration. Real-world smartphone experiments have shown that even obstructions by hands could significantly degrade the mmWave throughput [22]. One solution to mitigate this effect is to let sub-6GHz coexist with mmWave to form two *heterogeneous* channels, so that the user equipment can offload data to sub-6GHz when mmWave communications are unfeasible [2, 26, 27, 33]. Some work has already been done based on mmWave/sub-6GHz heterogeneous networks [10, 13]. However, how to improve information freshness in such hybrid networks has remained largely unexplored.

In this study, we consider a hybrid status updating system where a source can transmit the update packets over either an unreliable but fast mmWave channel or a slow reliable sub-6GHz channel. Our objective is to find a dynamic channel scheduling policy that minimizes the long-term average expected age. The main contributions of this paper are stated as follows:

- The optimal scheduling problem for minimizing the age over heterogeneous channels is formulated as a Markov Decision Process (MDP). The state transition of this MDP is complicated for two reasons: (i) the two channels have different data rates and packet transmission times, and (ii) the state of the unreliable mmWave channel is correlated over time. We prove that there exists a multi-dimensional threshold-based scheduling policy that is optimal. This optimality result holds for all possible values of the channel parameters. Supermodularity [41] has been one of the tools used to prove this result. Because of the complicated state transitions, the supermodular property only holds in a part of the state space, which is a key difference from the scheduling problems considered earlier in, e.g., [1, 19, 23, 28, 35, 38, 42]. We have developed additional techniques to show that the threshold-based scheduling policy is optimal in the rest part of the state space where supermodularity does not hold.
- Further, we show that the thresholds of the optimal scheduling policy can be evaluated efficiently, by using closed-form expressions or a low-complexity bisection search algorithm. Compared with the algorithms for calculating the thresholds and optimal scheduling policies in, e.g., [1, 19, 23, 28, 35, 38, 42], our solution algorithms have much lower computational complexities.
- In the special case that the state of the unreliable mmWave channel is independent and identically distributed (i.i.d.) over time, the optimal scheduling policy is shown to possess an even simpler form.
- Finally, numerical results show that the optimal policy can reduce the age compared with several other policies.

2 RELATED WORKS

Age of information has become a popular research topic in recent years, e.g., [1, 3–6, 8, 9, 12, 14–18, 20, 21, 24, 29, 35–40, 43]. A comprehensive survey of the area was recently provided in [43]. First, there has been substantial work on age performance analysis in queuing disciplines [4, 5, 8, 9, 15, 18]. Average age and peak age in elementary queuing systems were analyzed in [9, 15, 18]. A similar setting with Gilbert-Elliot sampler or Gilbert-Elliot server was considered in [8]. A Last-Generated, First-Served (LGFS) policy was shown (near) optimal in general single source, multiple servers, and multihop networks with arbitrary generation and arbitrary arrival process [4, 5]. These results were extended to the multi-source multi-server regime in [37]. Next, there has been a significant effort in age-optimal sampling [3, 24, 35, 36, 38]. The optimal sampling policy was provided for minimizing a monotonic age function in [24, 35, 38]. Sampling and scheduling in multi-source systems were analyzed where the optimal joint problem could be decoupled into maximum age first (MAF) scheduling [37] and an optimal sampling problem in [3]. Finally, age in wireless networks has been substantially explored in [14, 16, 17, 20, 21, 29, 40]. Scheduling in a broadcast network with random arrival was provided where whittle index policy can achieve (near) age optimality [14]. Some other age-optimal scheduling works for cellular networks were considered in [16, 17, 21, 39, 40]. A class of age-optimal scheduling policies was analyzed in the asymptotic regime when the number of sources and channels both grow to infinity [29]. An age minimization multi-path routing strategy was introduced in [20].

However, the age-optimal problem via heterogeneous channels has been largely unexplored yet. To the best of our knowledge, technical models similar to ours were reported in [1, 12]. Their study assumed that the first channel is unreliable but consumes a lower cost, and the second channel has a steady connection with the same delay but consumes a higher cost. They derived the scheduling policy for the trade-off between age performance and cost. Our study is significantly different from theirs in two aspects: (i) The study in [1, 12] shows the optimality of the threshold type policy and efficiently solves the optimal threshold when the first channel is i.i.d. [1], but our work allows a Markovian channel which is a generalization of the i.i.d. case. (ii) In addition to allowing mmWave to be unreliable, our study assumes that sub-6GHz has a larger delay than mmWave since this assumption complies with the property of dual mmWave/sub-6GHz channels in real applications. These two differences between mmWave and sub-6GHz make the MDP formulation more complex. Thus, most of the well-known techniques that show a nice structure of the optimal policy or even solve the optimal policy with low complexity (e.g., [1, 19, 23, 28, 35, 38, 42]) do not apply to our model.

3 SYSTEM MODEL AND PROBLEM FORMULATION

3.1 System Models

Consider a single-hop network as illustrated in Fig. 1, where a source sends status update packets to the destination. We assume that time is slotted with slot index $t \in \{0, 1, 2...\}$. The source can generate a fresh status update packet at the beginning of each time slot. The packets can be transmitted either over the mmWave channel or over the sub-6GHz channel. The packet transmission time of the mmWave channel is 1 time slot, whereas the packet transmission time of the sub-6GHz channel is d time slots ($d \geq 2$)[1] because of its lower data rate.

The mmWave channel, called *Channel 1*, follows a two-state Gilbert-Elliot model that is shown in Fig. 2. We say that Channel 1 is *ON* in time slot t, denoted by $l_1(t) = 1$, if the packet is successfully transmitted to the destination in time slot t; otherwise Channel 1 is said to be *OFF*, denoted by $l_1(t) = 0$. If a packet is not successfully transmitted, then it is dropped, and a new status update packet is generated at the beginning of the next time slot. The self transition probability of the *ON* state is q, and the self transition probability of the *OFF* state is p, where $0 < q < 1$ and $0 < p < 1$. We assume that the source has access to the state of Channel 1, but with one

[1] If $d = 1$, one can readily see that it is better to choose sub-6GHz than mmWave. Thus, in this paper we study the nontrivial case of $d \geq 2$.

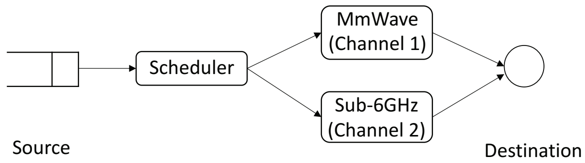

Figure 1: The system model for status updates in heterogeneous channels. The scheduler chooses mmWave (Channel 1) or sub-6GHz (Channel 2) for transmission over time.

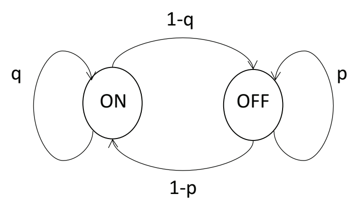

Figure 2: The Gilbert-Elliot model for Channel 1.

time slot of feedback delay. That is, at the beginning of time slot t, the source knows $l_1(t-1)$ perfectly.

The sub-6GHz channel, called *Channel 2*, has a steady connection. As mentioned above, the packet transmission time of Channel 2 is d time slots. Define $l_2(t) \in \{0, 1, ..., d-1\}$ as the state of Channel 2 in time slot t, where $l_2(t)$ is the remaining transmission time of the packet being sent over Channel 2 at the beginning of time slot t, and $l_2(t) = 0$ means that Channel 2 is currently idle and ready for sending the next packet. In time slot t, the source has immediate knowledge about the state $l_2(t)$ of Channel 2. On the other hand, because the packet transmission time of Channel 1 is 1 time slot, Channel 1 is always ready for transmission at the beginning of each time slot.

Following the application settings in [2, 10, 26, 27, 33], a packet can be transmitted using only one channel at a time, i.e., the two channels cannot be used simultaneously. The scheduler decides which channel to be used to transmit a packet at each time slot. We also assume that the scheduler can choose idle (neither channel) since it has been shown that channel idling could reduce the average age in the system [3, 35, 38]. The scheduling decision at the beginning of time slot t is denoted by $u(t) \in \{1, 2, none\}$. The action $u(t) = 1$ or 2 means that the source generates a packet and assigns it to Channel 1 or Channel 2, respectively. The action $u(t) = none$ means that no new packet is assigned to any channel at time slot t. Hence, u(t) = *none* can occur if (i) a packet is was assigned to Channel 2 earlier and has not completed its transmission, i.e., $l_2(t) \in \{1, 2, \ldots, d-1\}$ such that no packet can be assigned for transmission, or (ii) $l_2(t) = 0$, but both channels are kept idle on purpose.

The age of information (AoI) $\Delta(t)$ is the time difference between the current time slot t and the generation time of the freshest delivered packet [18]. By this definition, when a packet is delivered, the age drops to the transmission time of the delivered packet. Specifically, if Channel 1 is selected in time slot t and Channel 1

Table 1: Value of State Transition Probability

$P_{\mathbf{ss'}}(u)$	Action and State Transition
p	$u=1, \mathbf{s}=(\delta,0,0), \mathbf{s}'=(\delta+1,0,0)$ $u=2, \mathbf{s}=(\delta,0,0), \mathbf{s}'=(\delta+1,0,d-1)$ $u=none, \mathbf{s}=(\delta,0,0), \mathbf{s}'=(\delta+1,0,0)$ $u=none, \mathbf{s}=(\delta,0,m), \mathbf{s}'=(\delta+1,0,m-1), m\geq 2$ $u=none, \mathbf{s}=(\delta,0,1), \mathbf{s}'=(d,0,0)$
$1-p$	$u=1, \mathbf{s}=(\delta,0,0), \mathbf{s}'=(1,1,0)$ $u=2, \mathbf{s}=(\delta,0,0), \mathbf{s}'=(\delta+1,1,d-1)$ $u=none, \mathbf{s}=(\delta,0,0), \mathbf{s}'=(\delta+1,1,0)$ $u=none, \mathbf{s}=(\delta,0,m), \mathbf{s}'=(\delta+1,1,m-1), m\geq 2$ $u=none, \mathbf{s}=(\delta,0,1), \mathbf{s}'=(d,1,0)$
q	$u=1, \mathbf{s}=(\delta,1,0), \mathbf{s}'=(1,1,0)$ $u=2, \mathbf{s}=(\delta,1,0), \mathbf{s}'=(\delta+1,1,d-1)$ $u=none, \mathbf{s}=(\delta,1,0), \mathbf{s}'=(\delta+1,1,0)$ $u=none, \mathbf{s}=(\delta,1,m), \mathbf{s}'=(\delta+1,1,m-1), m\geq 2$ $u=none, \mathbf{s}=(\delta,1,1), \mathbf{s}'=(d,1,0)$
$1-q$	$u=1, \mathbf{s}=(\delta,1,0), \mathbf{s}'=(\delta+1,0,0)$ $u=2, \mathbf{s}=(\delta,1,0), \mathbf{s}'=(\delta+1,0,d-1)$ $u=none, \mathbf{s}=(\delta,1,0), \mathbf{s}'=(\delta+1,0,0)$ $u=none, \mathbf{s}=(\delta,1,m), \mathbf{s}'=(\delta+1,0,m-1), m\geq 2$ $u=none, \mathbf{s}=(\delta,1,1), \mathbf{s}'=(d,0,0)$
0	Otherwise

is *ON*, then the age drops to 1 at time slot $t+1$. If the remaining service time of Channel 2 at time slot t is 1, then age drops to d at time slot $t+1$. When there is no packet delivery at time slot t, the age increases by one time slot. Hence, the time-evolution of the age is given by

$$\Delta(t+1) = \begin{cases} 1 & \text{if } u(t) = 1 \text{ and } l_1(t) = 1, \\ d & \text{if } l_2(t) = 1, \\ \Delta(t) + 1 & \text{Otherwise.} \end{cases} \tag{1}$$

3.2 Problem Formulations

We use $\pi = \{u(0), u(1)...\}$ to denote a scheduling policy. A scheduling policy is said to be *admissible* if (i) $u(t) = none$ whenever $l_2(t) \geq 1$ and (ii) $u(t)$ is determined by the current and history information that is available at the scheduler. Let $\Delta_\pi(t)$ denote the AoI induced by policy π. The expected time-average age of policy π is

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[\Delta_\pi(t)].$$

Our objective in this paper is to solve the following optimal scheduling problem for minimizing the expected time-average age:

$$\bar{\Delta}_{\text{opt}} = \inf_{\pi\in\Pi} \limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[\Delta_\pi(t)], \tag{2}$$

where Π is the set of all admissible policies. Problem (2) can be equivalently expressed as an average-cost MDP problem [7, 28], which is illustrated below.

- **Markov State:** The system state in time slot t is defined as

$$\mathbf{s}(t) = (\Delta(t), l_1(t-1), l_2(t)), \tag{3}$$

where $\Delta(t) \in \{1, 2, 3, ...\}$ is the AoI in time slot t, $l_1(t-1) \in \{0, 1\}$ is the $ON-OFF$ state of Channel 1 in time slot $t-1$, and $l_2(t) \in \{0, 1, ..., d-1\}$ is the remaining transmission time of Channel 2 at the beginning of time slot t. Let $\mathbf{S}$ denote the state space which is countably infinite. The time-evolution of $\Delta(t)$ is determined by the state and action in time slot $t-1$.

- **Action:** As mentioned before, if Channel 2 is busy (i.e., $l_2(t) > 0$), the scheduler always chooses an idle action, i.e., $u(t) = none$. Otherwise, the action $u(t) \in \{1, 2, none\}$.
- **Cost function:** Suppose that a decision $u(t)$ is applied at a time slot t, we encounter a cost $C(\mathbf{s}(t), u(t)) = \Delta(t)$.
- **Transition probability:** We use $P_{\mathbf{ss}'}(u)$ to denote the transition probability from state $\mathbf{s}$ to $\mathbf{s}'$ for action u. The value of $P_{\mathbf{ss}'}(u)$ is summarized in Table 1. See our technical report [25] for the explanation of Table 1.

4 MAIN RESULTS

In this section, we show that there exists a threshold-type policy that solves Problem (2). We then provide a low-complexity algorithm to obtain the optimal policy and optimal average age.

4.1 Optimality of threshold-type policies

As mentioned in Section 3.2, the action space of the MDP allows $u(t) = none$ even if Channel 2 is idle, i.e., $l_2(t) = 0$. In the following lemma, we show that the action $u(t) = none$ can be abandoned when $l_2(t) = 0$. Define

$$\Pi' = \{\pi \in \Pi : u(t) \neq none, \text{ if } l_2(t) = 0\}. \quad (4)$$

Lemma 1. *For any $\pi \in \Pi$, there exists a policy $\pi' \in \Pi'$ that is no worse than π.*

Proof. See our technical report [25]. □

By Lemma 1, the scheduler only needs to choose from the actions $u(t) = 1$ or 2 when $l_2(t) = 0$. This lemma simplifies the MDP problem.

Remark 1. *In [3, 35, 38], the authors showed that in certain scenarios, the zero wait policy (transmitting immediately after the previous update has been received) might not be optimal. However, in our model, the zero wait policy is indeed optimal. The reason is that in our model, the minimum non-zero waiting time is one time slot which is the same as the delay of Channel 1. If $l_2(t) = 0$, it is better to choose Channel 1 than keeping both channels idle, because, by choosing Channel 1, fresh packets could be delivered over Channel 1.*

For the ease of description, we divide the possible values of channel parameters (p, q, d) into four complementary regions $\mathbf{B}_1, \ldots, \mathbf{B}_4$.

Definition 1. *The regions $\boldsymbol{B}_1, \ldots, \boldsymbol{B}_4$ are defined as*

$$\begin{aligned} \boldsymbol{B}_1 &= \{(p, q, d) : F(p, q, d) \leq 0, H(p, q, d) \leq 0\}, \\ \boldsymbol{B}_2 &= \{(p, q, d) : F(p, q, d) > 0, G(p, q, d) \leq 0\}, \\ \boldsymbol{B}_3 &= \{(p, q, d) : F(p, q, d) > 0, G(p, q, d) > 0\}, \\ \boldsymbol{B}_4 &= \{(p, q, d) : F(p, q, d) \leq 0, H(p, q, d) > 0\}, \end{aligned} \quad (5)$$

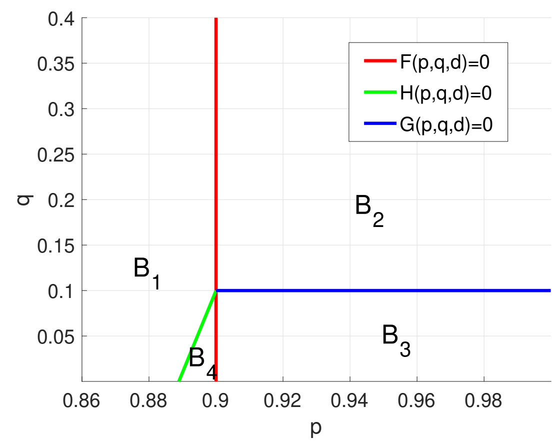

Figure 3: The Diagram of the regions $\mathbf{B}_1, \ldots, \mathbf{B}_4$ with an example of $d = 10$. In the diagram, each function F, G, H divides the whole plane ($(p, q) \in (0, 1)\times(0, 1)$) into two half-planes respectively. Each region $\mathbf{B}_1, \ldots, \mathbf{B}_4$ is the intersection of some two half-plane areas. Since we emphasize the differences of the four regions, we provide the partial but enlarged diagram.

where

$$\begin{aligned} F(p, q, d) &= \frac{1}{1-p} - d, \\ G(p, q, d) &= 1 - dq, \\ H(p, q, d) &= \frac{1-q}{1-p} + 1 - d. \end{aligned} \quad (6)$$

In this study, the regions $\mathbf{B}_1 - \mathbf{B}_4$ will serve as the sufficient conditions for optimality of threshold type policy. Note that the inequality $1/(1-p) > d$ also represents a comparison between the channel delay d and the average length of OFF period given that the last channel state is OFF. Similarly, $1 - dq > 0$ represents a comparison between d and the average length of OFF period given that the last channel state is ON. Finally, $(1-q)/(1-p) + 1 - d > 0$ represents a comparison between d and the average delay of Channel 1. These comparisons are the interpretations of all the boundary functions F, G, H of the regions $\mathbf{B}_1 - \mathbf{B}_4$. The four regions $\mathbf{B}_1, \ldots, \mathbf{B}_4$ are depicted in Fig. 3, where $d = 10$.

Consider a stationary policy $\mu(\delta, l_1, l_2)$. As mentioned in Lemma 1, when $l_2 = 0$, the decision $\mu(\delta, l_1, 0)$ can be 1 (Channel 1) or 2 (Channel 2). Given the value of l_1, $\mu(\delta, l_1, 0)$ is said to be *non-decreasing in the age δ*, if

$$\mu(\delta, l_1, 0) = \begin{cases} 1 & \text{if } \delta < \lambda; \\ 2 & \text{if } \delta \geq \lambda. \end{cases} \quad (7)$$

Conversely, $\mu(\delta, l_1, 0)$ is said to be *non-increasing in the age δ*, if

$$\mu(\delta, l_1, 0) = \begin{cases} 2 & \text{if } \delta < \lambda; \\ 1 & \text{if } \delta \geq \lambda. \end{cases} \quad (8)$$

One can observe that scheduling policies in the form of (7) and (8) are both with a *threshold-type*, where λ is the threshold on the age δ at which the value of $\mu(\delta, l_1, 0)$ changes.

One optimal solution to Problem (2) is of a special threshold-type structure, as stated in the following theorem:

Theorem 1. *There exists an optimal solution $\mu^*(\delta, l_1, 0)$ to Problem* (2)*, which satisfies the following properties:*

(a) *if $(p, q, d) \in \boldsymbol{B}_1$, then $\mu^*(\delta, 0, 0)$ is non-increasing in the age δ and $\mu^*(\delta, 1, 0)$ is non-increasing in the age δ;*
(b) *if $(p, q, d) \in \boldsymbol{B}_2$, then $\mu^*(\delta, 0, 0)$ is non-decreasing in the age δ and $\mu^*(\delta, 1, 0)$ is non-increasing in the age δ;*
(c) *if $(p, q, d) \in \boldsymbol{B}_3$, then $\mu^*(\delta, 0, 0)$ is non-decreasing in the age δ and $\mu^*(\delta, 1, 0)$ is non-decreasing in the age δ;*
(d) *if $(p, q, d) \in \boldsymbol{B}_4$, then $\mu^*(\delta, 0, 0)$ is non-increasing in the age δ and $\mu^*(\delta, 1, 0)$ is non-decreasing in the age δ.*

Proof. Please see Section 7.2 for the proof. □

As is shown in Theorem 1, for all possible parameters p, q, d of the two channels, the optimal action $\mu^*(\delta, l_1, 0)$ of channel selection is a monotonic function of the age δ. Whether $\mu^*(\delta, l_1, 0)$ is non-decreasing or non-increasing in δ depends on the channel parameters (p, q, d) and the previous state l_1 of Channel 1.

The study in [1] assumed that the first channel is unreliable and consumes a lower cost, and the second channel has a steady connection with the same delay but a higher cost. They studied the scheduling policy for the trade-off between age performance and cost. The optimal scheduling policy in Theorem 1 is quite different from that in [1]: The study in [1] assumes the first channel to be i.i.d., but our result allows a Markovian Channel 1, which is a generalization of the i.i.d. case. The optimal scheduling policy in [1] is non-decreasing in age since the first channel is inferior to the second channel. However, the optimal policy in our study can be non-decreasing or non-increasing since the two channels (i.e., Channel 1 and 2) have their own advantages. In conclusion, our study allows for general channel parameters and applies to all types of comparisons between Channel 1 and Channel 2, and our policy can be non-increasing in some regions and non-decreasing in other regions.

4.2 Insights Behind the Regions $\mathbf{B}_1 - \mathbf{B}_4$

The regions $\mathbf{B}_1 - \mathbf{B}_4$ were introduced in Theorem 1 for proving that the action value function $Q(\mathbf{s}, u)$ is *supermodular* or *submodular*, where $\mathbf{s} = (\delta, l_1, 0)$ denotes the state of the MDP and u is the action. For example, in the case of $l_1 = 0$, if $1/(1-p) > d$ and $1/q \leq d$ (i.e., $(p, q, d) \in \mathbf{B}_2$), Lemma 8 in our technical report showed that $Q(\delta, 0, 0, u)$ is submodular in (δ, u) (in the discounted case). As a result, the optimal action $\mu^*(\delta, 0, 0)$ is increasing in δ.

However, in the case $l_1 = 1$ of Theorem 1, there are additional technical challenges: For example, if $(p, q, d) \in \mathbf{B}_2$, we were unable to use $1/(1-p) > d$ and $1/q \leq d$ (where are provided in the definition of $\mathbf{B}_2$) to prove that $Q(\delta, 1, 0, u)$ is super-modular or submodular. A new method was developed in Lemma 9 in our Appendices to conquer this challenge. Technically, super-/sub-modularity is a sufficient but not necessary condition for the monotonicity of $\mu^*(\delta, l_1, 0)$. In some part of the state space ($l_1 = 0$), we proved super-/sub-modularity. In the rest part of the state space ($l_1 = 1$), neither super-modularity nor sub-modularity may hold, but we were able to show that the optimal decision $\mu^*(\delta, 1, 0)$ does not change with δ when $l_1 = 1$. By this, we proved the monotonicity of $\mu^*(\delta, 1, 0)$ for all cases, without requiring $Q(\mathbf{s}, u)$ to be supermodular or submodular over the entire state space.

Thus, one technical contribution of the paper is: we proved that the optimal action $\mu^*(\delta, 1, 0)$ is monotonic in δ even if super-/sub-modularity does not hold. This is a key difference from prior studies, e.g., [1, 19, 23, 28, 35, 38], where super-modularity (or sub-modularity) holds for the entire state space.

4.3 Optimal Scheduling Policy

According to Theorem 1, $\mu^*(\delta, 0, 0)$ and $\mu^*(\delta, 1, 0)$ are both threshold-type, so there are two thresholds. We use λ_0^* and λ_1^* to denote the thresholds of $\mu^*(\delta, 0, 0)$ and $\mu^*(\delta, 1, 0)$, respectively.

Theorem 2. *An optimal solution to* (2) *is presented below for the 2 regions $\boldsymbol{B}_1, \boldsymbol{B}_2$ of the channel parameters:*

(a) If $(p, q, d) \in \boldsymbol{B}_1$, then the optimal scheduling policy is

$$\mu^*(\delta, 0, 0) = 1, \delta \geq 1; \tag{9}$$

$$\mu^*(\delta, 1, 0) = 1, \delta \geq 1. \tag{10}$$

In this case, the optimal objective value of (2) *is*

$$\bar{\Delta}_{opt} = \frac{(1-q)(2-p) + (1-p)^2}{(2-q-p)(1-p)}. \tag{11}$$

(b) If $(p, q, d) \in \boldsymbol{B}_2$, then the optimal scheduling policy is

$$\mu^*(\delta, 0, 0) = \begin{cases} 1 & \text{if } \delta < \lambda_0^*; \\ 2 & \text{if } \delta \geq \lambda_0^*, \end{cases} \tag{12}$$

$$\mu^*(\delta, 1, 0) = \begin{cases} 2 & \text{if } \delta < \lambda_1^*; \\ 1 & \text{if } \delta \geq \lambda_1^*, \end{cases} \tag{13}$$

where λ_0^* is unique, but λ_1^* may take multiple values, given by

$$\begin{cases} \lambda_0^* = s_1(\beta_1), & \lambda_1^* = 1 & \text{if } \bar{\Delta}_{opt} = \beta_1, \\ \lambda_0^* = s_2(\beta_2), & \lambda_1^* = 1 & \text{if } \bar{\Delta}_{opt} = \beta_2, \\ \lambda_0^* = 1, & \lambda_1^* \in \{2, 3, \ldots, d\} & \text{if } \bar{\Delta}_{opt} = f_0/g_0, \\ \lambda_0^* = 1, & \lambda_1^* \in \{d+1, \ldots\} & \text{if } \bar{\Delta}_{opt} = (3/2)d - 1/2, \end{cases} \tag{14}$$

$\bar{\Delta}_{opt}$ is the optimal objective value of (2)*, determined by*

$$\bar{\Delta}_{opt} = \min\left\{\beta_1, \beta_2, \frac{f_0}{g_0}, \frac{3}{2}d - \frac{1}{2}\right\}, \tag{15}$$

$s_1(\cdot)$, $s_2(\cdot)$, β_1, and β_2 are given in Definition 2 below, and

$$f_0 = q\sum_{i=1}^{d} i + (1-q)\sum_{i=d+1}^{2d} i + \left(\frac{b'q+b}{1-b} + 1\right)\sum_{i=d+1}^{2d} i, \tag{16}$$

$$g_0 = \frac{b'q+b}{1-b}d + d + 1, \tag{17}$$

$$\begin{bmatrix} b' \\ b \end{bmatrix} = \begin{bmatrix} q & 1-q \\ 1-p & p \end{bmatrix}^d \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{18}$$

Proof. Please see Section 7.3 for the details. □

The results for the regions $\mathbf{B}_3$ and $\mathbf{B}_4$ are of similar forms and are relegated to our technical report [25]. The result of Theorem 2(a) is simple: if $(p, q, d) \in \mathbf{B}_1$, the optimal policy always chooses Channel 1. However, Theorem 2(b) contains a number of cases. For each case, the optimal thresholds λ_0^* and λ_1^* can be either expressed in closed-form, or computed by using a low-complexity bisection search method to compute the root of (19) given in below.

Algorithm 1: Bisection method for solving (19)

Given function h_i. $l = 0$, l' sufficiently large, tolerance ϵ small. The value $i \in \{1, 2\}$.
repeat
  $\beta = \frac{1}{2}(l + l')$
  if $h_i(\beta) < 0$: $l' = \beta$. **else** $l = \beta$
until $l' - l < \epsilon$
return $\beta_i = \beta$

Definition 2. *The values β_1, β_2 used in Theorem 2(b) are the root of*

$$f_i(s_i(\beta_i)) - \beta_i g_i(s_i(\beta_i)) = 0, \ i \in \{1, 2\}, \tag{19}$$

where

$$s_1(\beta_1) = \max\left\{\left\lceil \frac{-k_1(\beta_1)}{1 - d(1-p)} \right\rceil, d\right\}, \tag{20}$$

$$s_2(\beta_2) = \max\left\{\min\left\{\left\lceil \frac{-k_2(\beta_2)}{1 - d(1-p)} \right\rceil, d\right\}, 1\right\}, \tag{21}$$

$$k_i(\beta_i) = l_i - \beta_i o_i, \ i \in \{1, 2\}, \tag{22}$$

and $\lceil x \rceil$ is the smallest integer that is greater or equal to x. For the ease of presentation, 8 closed-form expressions of $f_i(\cdot)$, $g_i(\cdot)$, l_i, and o_i for $i = 1, 2$ are relegated to our technical report [25].

For notational simplicity, we define

$$h_i(\beta) = f_i(s_i(\beta)) - \beta g_i(s_i(\beta)), \ i \in \{1, 2\}. \tag{23}$$

The functions $h_1(\beta), h_2(\beta)$ have the following nice property:

Lemma 2. *For all $i \in \{1, 2\}$, the function $h_i(\beta)$ satisfies the following properties:*
(1) $h_i(\beta)$ is continuous, concave, and strictly decreasing on β;
(2) $h_i(0) > 0$ and $\lim_{\beta\to\infty} h_i(\beta) = -\infty$.

Proof. See our technical report [25]. □

Lemma 2 implies that (19) has a unique root on $[0, \infty)$. Therefore, we can use a low-complexity bisection method to compute β_1, β_2, as illustrated in Algorithm 1.

Note that the structure of Lemma 2 is motivated by Lemma 2 in [24] and Lemma 2 in [36]. In [24] and [36], since the channel is error free, the age state at the end of each transmission is independent with history information. Thus, Lemma 2 in [24] and Lemma 2 in [36] are related with a per-sample (single transmission) control. However, our study does not have such a property and thus Lemma 2 arises from solving the long term average cost of the threshold type policy.

The advantage of Theorem 2 is that the solution is easy to implement. In Theorem 2(a), we showed that the optimal policy is a constant policy that always chooses Channel 1. In Theorem 2(b), $\bar{\Delta}_{\text{opt}}$ is expressed as the minimization of only a few precomputed values, and the optimal policy (or the thresholds) are then provided based on which value that $\bar{\Delta}_{\text{opt}}$ is equal to.

Since we can use a low complexity algorithm such as bisection method to obtain β_1, β_2 in Theorem 2(b), Theorem 2 provides a solution that has much lower complexity than the state-of-the-art solution such as value iteration or Monte Carlo simulation.

4.4 Insights behind Theorem 2

In Theorem 1, we have successfully characterized the threshold structure for an optimal policy in region $\mathbf{B}_1, \ldots, \mathbf{B}_4$. A threshold type policy is fully identified by its thresholds λ_0, λ_1, where λ_0 is the threshold given that previous state of Channel 1 is *OFF* (i.e., $l_1 = 0$) and λ_1 is the threshold given that previous state of Channel 1 is *ON* (i.e., $l_1 = 1$). Thus, for a given region $\mathbf{B}_i$ $(i = 1, \ldots, 4)$, the MDP problem (2) reduces to

$$\bar{\Delta}_{\text{opt}} = \min_{\lambda_0 \in \mathbb{N}^+, \lambda_1 \in \mathbb{N}^+} \bar{\Delta}_i(\lambda_0, \lambda_1), \tag{24}$$

where $\bar{\Delta}_i(\lambda_0, \lambda_1)$ is the long term average cost of the threshold type policy such that: (1) the threshold (monotone) structure is determined by Theorem 1 and $\mathbf{B}_i$; (2) the thresholds are λ_0, λ_1. Note that a threshold type policy is stationary and thus can be modeled as a discrete time Markov chain (DTMC). Then, $\bar{\Delta}_i(\lambda_0, \lambda_1)$ can be solved by deriving the stationary distribution of the DTMC.

From Lemma 10 and Lemma 11 in Section 7.3, if $(p, q, d) \in \mathbf{B}_1$, then $\mu^*(1, 0, 0) = 1$ and $\mu^*(1, 1, 0) = 1$. According to Theorem 1 (a), if $(p, q, d) \in \mathbf{B}_1$, then the optimal policy $\mu^*(\delta, 0, 0)$ is non-increasing and $\mu^*(\delta, 1, 0)$ is non-increasing. Thus, $\mu^*(\delta, 0, 0) = 1$ and $\mu^*(\delta, 1, 0) = 1$ for all δ. That is, the optimal policy is always choosing Channel 1. Since the DTMC for always choosing Channel 1 is easy to analyze, we omit the derivation steps and get

$$\bar{\Delta}_{\text{opt}} = \bar{\Delta}_1(1, 1) = \frac{(1-q)(2-p) + (1-p)^2}{(2-q-p)(1-p)}. \tag{25}$$

This result directly implies Theorem 2(a).

While the result of case $(p, q, d) \in \mathbf{B}_1$ seems easy to describe, the result of the case $(p, q, d) \in \mathbf{B}_2$ is not, because the optimal policy is no longer constant in δ. We now provide the sketch of the proof idea when $(p, q, d) \in \mathbf{B}_2$.

First, by deriving the stationary distributions of some DTMCs with different thresholds, we have found that

$$\bar{\Delta}_2(\lambda_0, 1) = \begin{cases} f_1(\lambda_0)/g_1(\lambda_0) & \lambda_0 \in \{d+1, \ldots\}, \\ f_2(\lambda_0)/g_2(\lambda_0) & \lambda_0 \in \{2, \ldots d\}, \end{cases} \tag{26}$$

$$\bar{\Delta}_2(1, \lambda_1) = \begin{cases} (3/2)d - 1/2 & \lambda_1 \in \{d+1, \ldots\}, \\ f_0/g_0 & \lambda_1 \in \{1, \ldots d\}. \end{cases} \tag{27}$$

Note that (24) is a two-dimensional optimization problem in (λ_0, λ_1). However, (24) can be reduced to a couple of one-dimensional optimization problem in λ_0. The reason is that the threshold type policies with different λ_1 may have the same DTMC; see (27) for an example.

Then, the optimal average age $\bar{\Delta}_{\text{opt}}$ satisfies[2]

$$\bar{\Delta}_{\text{opt}} = \min\left\{\beta_1', \beta_2', \frac{f_0}{g_0}, \frac{3}{2}d - \frac{1}{2}\right\}, \tag{28}$$

where β_1', β_2' are defined as the solution to two one-dimensional problems:

$$\beta_1' = \min_{\lambda_0 \in \{d+1, \ldots\}} \frac{f_1(\lambda_0)}{g_1(\lambda_0)}, \tag{29}$$

$$\beta_2' = \min_{\lambda_0 \in \{1, \ldots d\}} \frac{f_2(\lambda_0)}{g_2(\lambda_0)}. \tag{30}$$

[2]Note that (26), (27) do not cover all of the values set of λ_1, λ_0. However, only considering the 4 types of DTMC described in (26), (27) is sufficient to solve (24) for $i = 2$. The proof of this statement is relegated to our technical report [25].

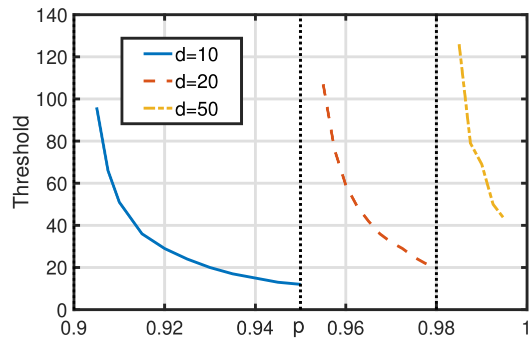

Figure 4: Thresholds of the optimal scheduling policy for i.i.d. mmWave channel state, where the packet transmission time of the sub-6GHz channel is $d = 10, 20, 50$.

Since $f_i(\lambda_0)$ and $g_i(\lambda_0)$ have a complicated structure, optimizing $f_i(\lambda_0)/g_i(\lambda_0)$ in (29) and (30) is challenging. However, using Dinkelbach's method [11], we can change the problem into a two-layer problem. One of our technical contributions is that the inner-layer problem is shown to be unimodal, thus we can derive an exact solution. Thus, we only need a bisection algorithm for the outer-layer, i.e., solving the roots of the equations $h_1(\beta) = 0, h_2(\beta) = 0$ in (19). We have

$$\beta_i' = \beta_i, \ i \in \{1, 2\}. \tag{31}$$

The studies in [24, 35, 36] also derive an exact solution to their inner-layer problem. However, their technique is using optimal stopping rules [24, 36] or stochastic convex optimization [35], which is different with our study.

Thus, Theorem 2(b) is solved by (26) – (31).

4.5 Optimal Scheduling policy for i.i.d. Channel

We finally consider a special case in which Channel 1 is i.i.d., i.e., $p + q = 1$. In i.i.d. case, Theorem 2 reduces to the following:

Corollary 1. *Suppose $p + q = 1$, i.e., Channel 1 is i.i.d., then*

(a) If $1 - p \geq 1/d$, then the optimal policy is always choosing Channel 1. In this case, the optimal objective value of (2) is $\bar{\Delta}_{opt} = 1/(1 - p)$.

(b) If $1 - p < 1/d$, then the optimal policy is non-decreasing in age and the optimal thresholds $\lambda_0^* = \lambda_1^*$. The threshold λ_0^* may take multiple values, given by

$$\begin{cases} \lambda_0^* \in \{1, 2, \ldots, d\} & \text{if } \bar{\Delta}_{opt} = (3/2)d - 1/2, \\ \lambda_0^* = s_1(\beta_1) & \text{if } \bar{\Delta}_{opt} = \beta_1, \end{cases} \tag{32}$$

$\bar{\Delta}_{opt}$ is the optimal objective value of (2), determined by

$$\bar{\Delta}_{opt} = \min\left\{\beta_1, \frac{3}{2}d - \frac{1}{2}\right\}. \tag{33}$$

Proof. See our technical report [25]. □

If Channel 1 is i.i.d., then the state information of Channel 1 in the previous time slot should not affect the scheduling decision in the current time slot. Thus, we have only one threshold, i.e., $\lambda_0^* = \lambda_1^*$.

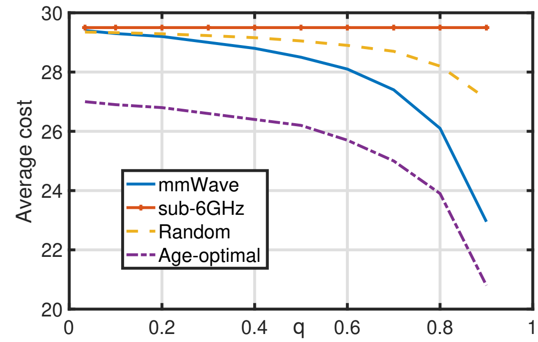

Figure 5: Time-average expected age vs. the parameter q of the mmWave channel, where $d = 20$ and $p = 0.966$.

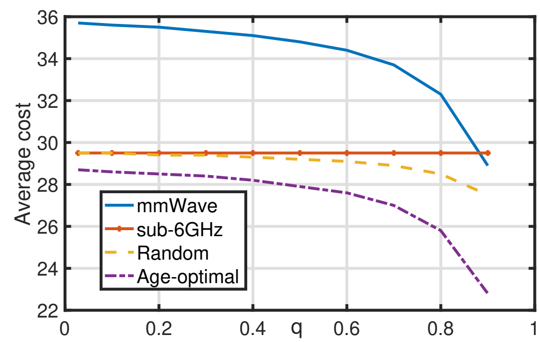

Figure 6: Time-average expected age vs. the parameter q of the mmWave channel, where $d = 20$ and $p = 0.972$.

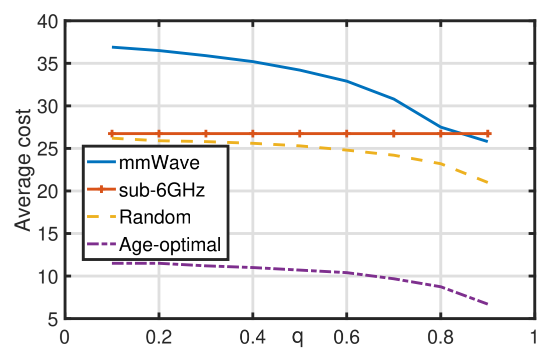

Figure 7: Time-average expected age penalty vs. the parameter q of the mmWave channel, where $p = 0.9$, $d = 20$, and the age penalty function is $f(\Delta) = (\frac{1}{p - 0.003})^{\Delta}$.

Corollary 1(a) suggests that if the transmission rate of Channel 1 is larger than the rate of Channel 2 (which is $1/d$), then the age-optimal policy always chooses Channel 1. Corollary 1(b) implies that if the transmission rate of Channel 1 is smaller than the rate of Channel 2, then the age-optimal policy is non-decreasing threshold-type on age.

5 NUMERICAL RESULTS

According to Corollary 1, λ_0^* is the optimal threshold in i.i.d. channel. We provide λ_0^* with the change of p for $d = 10, 20, 50$ respectively. From Fig. 4, the optimal threshold diverges to boundary $p^* = 0.9, 0.95, 0.98$ respectively. As p enlarges, the mmWave channel has worse connectivity, thus the thresholds goes down and converges to always choosing the sub-6GHz channel.

Then we compare our optimal scheduling policy (called *Age-optimal*) with three other policies, including (i) always choosing the mmWave channel (called *mmWave*), (ii) always choosing the sub-6GHz channel (called *sub-6GHz*), and (iii) randomly choosing the mmWave and sub-6GHz channels with equal probability (called *Random*). We provide the performance of these policies for different q in Fig. 5 and Fig. 6. Our optimal policy outperforms other policies. If the two channels have a similar age performance, the benefit of the optimal policy enlarges as the mmWave channel becomes positively correlated (q is larger). If the two channels have a large age performance disparity, the optimal policy is close to always choosing a single channel, and thus the benefit is obviously low. Although our theoretical results consider linear age, we also provide numerical results when the cost function is nonlinear on age by using value iteration [28]. For exponential age in Fig. 7, the gain is significantly large for all q: other policies have more than 2 times of average cost than the optimal policy. The numerical simulation indicates the importance of exploring optimal policy for nonlinear age cost function, which is our future research direction.

6 CONCLUSION

In this paper, we have studied age-optimal transmission scheduling for hybrid mmWave/sub-6GHz channels. For all possibly values of the channel parameters and the ON-OFF state of the mmWave channel, the optimal scheduling policy have been proven to be of a threshold-type on the age. Low complexity algorithms have been developed for finding the optimal scheduling policy. Finally, our numerical results show that the optimal policy can reduce age compared with other policies.

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7 APPENDICES

In this section we prove our main results: Theorem 1 (Section 7.2) and Theorem 2 (Section 7.3). In Section 7.1, we describe a discounted problem that helps to solve average problem (2). In Section 7.2, we introduce Proposition 1 which plays an important role in proving Theorem 1. Section 7.3 provides the proof of Theorem 2.

7.1 Preliminaries

To solve Problem (2), we introduce a discounted problem below. The objective is to solve the discounted sum of expected cost given an initial state $\mathbf{s}$:

$$J^{\alpha}(\mathbf{s}) = \inf_{\pi \in \Pi'} \lim_{T \to \infty} \sum_{t=0}^{T} \mathbb{E}[\alpha^t \Delta^{\pi}(t) | \mathbf{s}(0) = \mathbf{s}], \tag{34}$$

where $\alpha \in (0,1)$ is the discount factor. We call $J^{\alpha}(\mathbf{s})$ the *value function* given the initial state $\mathbf{s}$. Recall that we use $\mathbf{s}= (\delta, l_1, l_2)$ to denote the system state, where δ is the age value and l_1, l_2 are the state of Channel 1 and Channel 2. From Lemma 1, we only need to consider $\pi \in \Pi'$ instead of $\pi \in \Pi$.

The value function $J^{\alpha}(\mathbf{s})$ satisfies a following property:

Lemma 3. *For any given α and $\mathbf{s}$, $J^{\alpha}(\mathbf{s}) < \infty$.*

Proof. See our technical report [25]. □

A policy π is deterministic stationary if $\pi(t) = Z(\mathbf{s}(t))$ at any time t, where $Z : \mathbf{S} \to \Pi'$ is a deterministic function. According to [34], and Lemma 3, there is a direct result for Problem (34):

Lemma 4. *(a) The value function $J^{\alpha}(\mathbf{s})$ satisfies the Bellman equation*

$$\begin{aligned} Q^{\alpha}(\mathbf{s}, u) &\triangleq \delta + \alpha \sum_{\mathbf{s}' \in \mathbf{S}} P_{\mathbf{s}\mathbf{s}'}(u) J^{\alpha}(\mathbf{s}'), \\ J^{\alpha}(\mathbf{s}) &= \min_{u \in \Pi'} Q^{\alpha}(\mathbf{s}, u). \end{aligned} \tag{35}$$

(b) There exists a deterministic stationary policy $\mu^{\alpha,*}$ that satisfies Bellman equation (35). *The policy $\mu^{\alpha,*}$ solves Problem* (34) *for all initial state $\mathbf{s}$.*

(c) Assume that $J_0^{\alpha}(\mathbf{s}) = 0$ for all $\mathbf{s}$. For $n \geq 1$, J_n^{α} is defined as

$$\begin{aligned} Q_n^{\alpha}(\mathbf{s}, u) &\triangleq \delta + \alpha \sum_{\mathbf{s}' \in \mathbf{S}} P_{\mathbf{s}\mathbf{s}'}(u) J_{n-1}^{\alpha}(\mathbf{s}'), \\ J_n^{\alpha}(\mathbf{s}) &= \min_{u \in \Pi'} Q_n^{\alpha}(\mathbf{s}, u), \end{aligned} \tag{36}$$

then $\lim_{n\to\infty} J_n^{\alpha}(\mathbf{s}) = J^{\alpha}(\mathbf{s})$ for every $\mathbf{s}$.

Also, since the cost function is linearly increasing in age, utilizing Lemma 4(c), we also have

Lemma 5. *For all given l_1 and l_2, $J^{\alpha}(\delta, l_1, l_2)$ is increasing in δ.*

Proof. See our technical report [25]. □

Since Problem (34) satisfies the properties in Lemma 4, utilizing Lemma 4 and Lemma 5, the following Lemma gives the connection between Problem (2) and Problem (34).

Lemma 6. *(a) There exists a stationary deterministic policy that is optimal for Problem* (2).

(b) There exists a value J^* for all initial state $\mathbf{s}$ such that

$$\lim_{\alpha \to 1^-} (1 - \alpha) J^{\alpha}(\mathbf{s}) = J^*.$$

Moreover, J^* is the optimal average cost for Problem (2).

(c) For any sequence $(\alpha_n)_n$ of discount factors that converges to 1, *there exists a subsequence $(\beta_n)_n$ such that $\lim_{n\to\infty} \mu^{\beta_n,*} = \mu^*$. Also, μ^* is the optimal policy for Problem 2.*

Proof. See our technical report [25]. □

Lemma 6 provides the fact that: We can solve Problem (34) to achieve Problem (2). The reason is that the optimal policy of Problem (34) converges to the optimal policy of Problem (2) in a limiting scenario (as $\alpha \to 1$).

7.2 Proof of Theorem 1

We begin with providing an optimal structural result of discounted policy $\mu^{\alpha,*}$. Then, we achieve the average optimal policy μ^* by letting $\alpha \to 1$.

Definition 3. *For any discount factor $\alpha \in (0,1)$, the channel parameters $p, q \in (0,1)$ and $d \in \{2, 3, ...\}$, we define*

$$\begin{aligned} \mathbf{B}_1(\alpha) &= \{(p,q,d) : F(p,q,d,\alpha) \leq 0, H(p,q,d,\alpha) \leq 0\}, \\ \mathbf{B}_2(\alpha) &= \{(p,q,d) : F(p,q,d,\alpha) > 0, G(p,q,d,\alpha) \leq 0\}, \\ \mathbf{B}_3(\alpha) &= \{(p,q,d) : F(p,q,d,\alpha) > 0, G(p,q,d,\alpha) > 0\}, \\ \mathbf{B}_4(\alpha) &= \{(p,q,d) : F(p,q,d,\alpha) \leq 0, H(p,q,d,\alpha) > 0\}, \end{aligned} \tag{37}$$

where functions $F(\cdot), G(\cdot), H(\cdot) : \Theta \times (0,1) \to \mathbb{R}$ are defined as:

$$\begin{aligned} F(p,q,d,\alpha) &= \sum_{i=0}^{\infty} (\alpha p)^i - \sum_{i=0}^{d-1} \alpha^i, \\ G(p,q,d,\alpha) &= 1 + \alpha(1-q) \sum_{i=0}^{d-1} \alpha^i - \sum_{i=0}^{d-1} \alpha^i, \\ H(p,q,d,\alpha) &= 1 + \alpha(1-q) \sum_{i=0}^{\infty} (\alpha p)^i - \sum_{i=0}^{d-1} \alpha^i. \end{aligned} \tag{38}$$

Observe that all four regions $\mathbf{B}_i(\alpha)$ converge to $\mathbf{B}_i$ as the discount factor $\alpha \to 1$, where the regions $\mathbf{B}_i$ are described in Definition 6.

The optimal structural result of Problem (34) with a discount factor α is provided in the following proposition (Note that Theorem 1 can be immediately shown from Proposition 1, Lemma 6 and the convergence of the regions $\mathbf{B}_i(\alpha)$ to $\mathbf{B}_i$ (for $i = 1, 2, 3, 4$) as $\alpha \to 1$):

Proposition 1. *There exists a threshold type policy $\mu^{\alpha,*}(\delta, l_1, 0)$ on age δ that is the solution to Problem (34) such that:*

(a) If $l_1 = 0$ and $(p, q, d) \in \boldsymbol{B}_1(\alpha) \cup \boldsymbol{B}_4(\alpha)$, then $\mu^{\alpha,*}(\delta, l_1, 0)$ is non-increasing in the age δ.

(b) If $l_1 = 0$ and $(p, q, d) \in \boldsymbol{B}_2(\alpha) \cup \boldsymbol{B}_3(\alpha)$, then $\mu^{\alpha,*}(\delta, l_1, 0)$ is non-decreasing in the age δ.

(c) If $l_1 = 1$ and $(p, q, d) \in \boldsymbol{B}_1(\alpha) \cup \boldsymbol{B}_2(\alpha)$, then $\mu^{\alpha,*}(\delta, l_1, 0)$ is non-increasing in the age δ.

(d) If $l_1 = 1$ and $(p, q, d) \in \boldsymbol{B}_3(\alpha) \cup \boldsymbol{B}_4(\alpha)$, then $\mu^{\alpha,*}(\delta, l_1, 0)$ is non-decreasing in the age δ.

Since Channel 1 and Channel 2 have different delays, we are not able to show that the optimal policy is threshold type by directly observing the Bellman equation like [1]. Thus, we will use the concept of *supermodularity* [41, Theorem 2.8.2]. The domain of age set and decision set in the Q-function is $\{1, 2, ...\} \times \{1, 2\}$, which is a *lattice*. Given a positive s, the subset $\{s, s+1, ...\} \times \{1, 2\}$ is a *sublattice* of $\{1, 2, ...\} \times \{1, 2\}$. Thus, if the following holds for all $\delta > s$:

$$\begin{aligned} &Q^{\alpha}(\delta, l_1, 0, 1) - Q^{\alpha}(\delta-1, l_1, 0, 1) \\ \le &Q^{\alpha}(\delta, l_1, 0, 2) - Q^{\alpha}(\delta-1, l_1, 0, 2), \end{aligned} \tag{39}$$

then the Q-function $Q^{\alpha}(\delta, l_1, 0, u)$ is supermodular in (δ, u) for $\delta > s$, which means the optimal decision

$$\mu^{\alpha,*}(\delta, l_1, 0) = \text{argmin}_u Q^{\alpha}(\delta, l_1, 0, u) \tag{40}$$

is non-increasing in δ for $\delta \ge s$. If the inequality of (39) is inversed, then we call $Q^{\alpha}(\delta, l_1, 0)$ is submodular in (δ, u) for $\delta > s$, and $\mu^{\alpha,*}(\delta, l_1, 0)$ is non-decreasing in δ for $\delta \ge s$.

For ease of notations, we give Definition 4:

Definition 4. *Given $l_1 \in \{0, 1\}$, $u \in \{1, 2\}$,*

$$L^{\alpha}(\delta, l_1, u) \triangleq Q^{\alpha}(\delta, l_1, 0, u) - Q^{\alpha}(\delta-1, l_1, 0, u). \tag{41}$$

Note that $L^{\alpha}(\delta, l_1, 1)$ is the left hand side of (39), and $L^{\alpha}(\delta, l_1, 2)$ is the right hand side of (39).

However, because of the mismatch of delays in our problem, most of the well-known techniques to show supermodularity (e.g., [28],[23],[19] etc) do not apply in our setting. Thus, we need a new approach to show the supermodularity. Our key idea is as follows: First, we show that $L^{\alpha}(\delta, l_1, 2)$ is a constant (see Lemma 7 below), then we compare $L^{\alpha}(\delta, l_1, 1)$ with the constant to check supermodularity and get Lemma 8 and Lemma 9 below. The proof of comparing $L^{\alpha}(\delta, l_1, 1)$ with the constant is relegated to the proofs of Lemma 8 and Lemma 9 in our technical report [25].

Suppose that $m \triangleq \sum_{i=0}^{d-1} \alpha^i$, and we have:

Lemma 7. *For all $\delta \ge 2$ and $l_1 \in \{0, 1\}$, $L^{\alpha}(\delta, l_1, 2) = m$.*

Proof. See our technical report [25]. □

Also, we have

Lemma 8. *(a) If $l_1 = 0$ and $(p, q, d) \in \boldsymbol{B}_1(\alpha) \cup \boldsymbol{B}_4(\alpha)$, then $Q^{\alpha}(\delta, l_1, 0, u)$ is supermodular in (δ, u) for $\delta \ge 2$.*

(b) If $l_1 = 0$ and $(p, q, d) \in \boldsymbol{B}_2(\alpha) \cup \boldsymbol{B}_3(\alpha)$, then $Q^{\alpha}(\delta, l_1, 0, u)$ is submodular in (δ, u) for $\delta \ge 2$.

Proof. See our technical report [25]. □

Lemma 8(a) implies that $\mu^{\alpha,*}(\delta, 0, 0)$ is non-increasing in δ if $(p, q, d) \in \mathbf{B}_1(\alpha) \cup \mathbf{B}_4(\alpha)$. Lemma 8(b) implies that $\mu^{\alpha,*}(\delta, 0, 0)$ is non-increasing in δ if $(p, q, d) \in \mathbf{B}_1(\alpha) \cup \mathbf{B}_4(\alpha)$. Thus, Proposition 1(a),(b) hold.

Lemma 8 gives the result when the state of Channel 1 is 0. We then need to solve when the state of Channel 1 is 1. Different from $Q^{\alpha}(\delta, 0, 0, u)$, the Q-function $Q^{\alpha}(\delta, 1, 0, u)$ does not satisfy supermodular (or submodular) in (δ, u) for all the age value. Thus, we give a weakened condition: we can find out a value s, such that the Q-function $Q^{\alpha}(\delta, 1, 0, u)$ is supermodular (or submodular) for a partial set $s, s+1, ...$ and $\mu^{\alpha,*}(\delta, 1, 0)$ is a constant on the set $1, 2, ..., s$. Then, $\mu^{\alpha,*}(\delta, 1, 0)$ is non-increasing (or non-decreasing).

Thus, to solve Proposition 1(c),(d), we provide the following lemma:

Lemma 9. *(a) If $l_1 = 1$ and $(p, q, d) \in \boldsymbol{B}_1(\alpha) \cup \boldsymbol{B}_2(\alpha)$, then there exists a positive integer s, such that $Q^{\alpha}(\delta, l_1, 0, u)$ is supermodular in (δ, u) for $\delta > s$, and $\mu^{\alpha,*}(\delta, l_1, 0)$ is always 1 or always 2 for all $\delta \le s$.*

(b) If $l_1 = 1$ and $(p, q, d) \in \boldsymbol{B}_3(\alpha) \cup \boldsymbol{B}_4(\alpha)$, then there exists a positive integer s, such that $Q^{\alpha}(\delta, l_1, 0, u)$ is submodular in (δ, u) for $\delta > s$, and $\mu^{\alpha,*}(\delta, l_1, 0)$ is always 1 or always 2 for all $\delta \le s$.

Proof. See our technical report [25]. □

Lemma 9(a) implies that $\mu^{\alpha,*}(\delta, 1, 0)$ is non-increasing for $\delta \ge s$ and is constant for for $\delta \le s$. Thus, $\mu^{\alpha,*}(\delta, 1, 0)$ is non-increasing in δ. Similarly, Lemma 9 (b) implies that $\mu^{\alpha,*}(\delta, 1, 0)$ is non-decreasing for $\delta > 0$. Thus, we have shown Proposition 1(c),(d).

Overall, Lemma 7 and Lemma 8 shows Proposition 1(a),(b). Lemma 7 and Lemma 9 shows Proposition 1(c),(d). Thus we have completed the proof of Proposition 1.

Finally, Proposition 1, Lemma 6 and the convergence of $\mathbf{B}_1(\alpha)$, ... $\mathbf{B}_4(\alpha)$ to $\mathbf{B}_1, ..., \mathbf{B}_4$ show Theorem 1.

7.3 Proof of Theorem 2

For $(p, q, d) \in \mathbf{B}_1$, we firstly prove that $\mu^*(\delta, 0, 0) = 1$ and then show $\mu^*(\delta, 1, 0) = 1$.

Lemma 10. *If $(p, q, d) \in \boldsymbol{B}_1 \cup \boldsymbol{B}_4$, then the optimal decisions at states $(\delta, 0, 0)$ for all δ are 1.*

Proof. See our technical report [25]. □

On the other hand, when $l_1 = 1$, we have the following:

Lemma 11. *If $(p, q, d) \in \boldsymbol{B}_1$, then the optimal decision $\mu^*(1, 1, 0) = 1$.*

Proof. See our technical report [25]. □

Theorem 2(a) follows directly from Lemma 10 and Lemma 11. Due to the space limit, the proof of Theorem 2(b) when $(p, q, d) \in \mathbf{B}_2$ is relegated to our technical report [25].