Minimizing Age of Information in Multi-channel Time-sensitive Information Update Systems

Zenzhi Qian*, Fei Wu*, Jiayu Pan†, Kannan Srinivasan*, Ness B. Shroff*†
*Department of Computer Science and Engineering, The Ohio State University, Columbus 43210, OH
†Department of Electrical and Computer Engineering, The Ohio State University, Columbus 43210, OH

***Abstract*—Age of information, as a metric measuring the data freshness, has drawn increasing attention due to its importance in many data update applications. Most existing studies have assumed that there is one single channel in the system. In this work, we are motivated by the plethora of multi-channel systems that are being developed, and investigate the following question: how can one exploit multi-channel resources to improve the age performance? We first derive a policy-independent lower bound of the expected long-term average age in a multi-channel system. The lower bound is jointly characterized by the external arrival process and the channel statistics. Since direct analysis of age in multi-channel systems is very difficult, we focus on the asymptotic regime, when the number of users and number of channels both go to infinity. In the many-channel asymptotic regime, we propose a class of Maximum Weighted Matching policies that converge to the lower bound near exponentially fast. In the many-user asymptotic regime, we design a class of Randomized Maximum Weighted Matching policies that achieve a constant competitive ratio compared to the lower bound. Finally, we use simulations to validate the aforementioned results.**

***Index Terms*—Age of information, Multi-channel, Scheduling**

I. Introduction

Age of information is a new performance metric that has attracted significant recent attention [1–17]. This concept is motivated by various data update applications that deal with time-sensitive data, such as stock price, traffic information and news updates. In such applications, the data with the latest generation time is usually the most valuable one to users, and thus, users want to keep their data as fresh as possible. Age of information, defined as the the elapsed time of the last served packet since it was generated, is a good measure of the data freshness from the user side. In an age minimization problem, the goal is to minimize the user's age to keep data fresh.

Age of information and delay share the same feature that measures the elapsed time of a packet since its generation. However, they are fundamentally different in that delay is defined for a certain packet whereas *age of information* captures the data freshness from the user (flow) side. To see this, consider an $M/M/1$ queue with low arrival rate and high service rate [7]. The queue is often empty and the packet delay is very low. However, high *age of information* is still observed due to the long inter-arrival time. In general, good delay performance does not necessarily guarantee good age performance which requires that packets with low delay are served regularly. Compared to the extensively-studied delay metric, *age of information* is a new performance metric that calls for new designs of scheduling policies to minimize age.

Multi-channel communications have become commonplace in modern cellular systems, e.g., WiMax [18], 4G/LTE [19] and 5G NR [20]. In these systems, the wide bandwidth at the Base Station is divided into hundreds or thousands of orthogonal sub-carriers (channels), which can be dynamically allocated to serve users. The availability of multiple channels introduces flexible user-channel allocation and thus, provides diversity and multiplexing gain compared to the single-channel system. In the literature [21–23], it has been shown that near optimal delay performance can be achieved in a multi-channel system. Given the close relationship between *age of information* and delay, we can ask and answer the following natural question in this paper: *Can we exploit the flexibility from multi-channel systems to improve the age performance?*

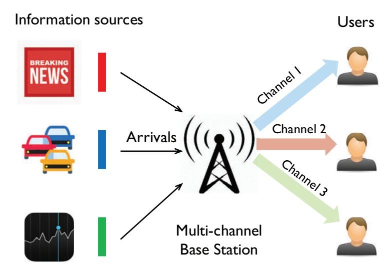

Fig. 1. A multi-user multi-channel information update system.

We focus on the age minimization problem over a multi-channel system. An example network is shown in Fig. 1. The Base Station keeps track of the most updated information of each user's interested flow (such as news or stock price updates). Users are able to use a channel to download a packet from the Base Station and update its information and age at the user terminal. Due to channel fading, the channel conditions are time-varying across both users and channels. A scheduling policy decides the allocation of multi-channel resources to serve the time-sensitive data flows. Now the following important question remains: *How to design a scheduling policy that achieves provably good age performance for a multi-channel time-sensitive information update system with time-varying channels?*

In this paper, we answer this question by proposing two classes of scheduling policies. The key contributions of this paper are summarized as follows:

- We first derive a policy-independent lower bound of expected age in multi-channel systems. The lower bound is jointly characterized by the arrival process of applications and the channel statistics.

- In the many-channel asymptotic regime, we propose a class of Maximum Weighted Matching (MWM) policies that converge to the lower bound near exponentially.
- In the many-user asymptotic regime, we design a class of Randomized Maximum Weighted Matching (RMWM) policies that achieve a constant competitive ratio compared to the lower bound.

To the best of our knowledge, this is the first work that develops scheduling policies whose age performance is provably near-optimal for time-varying multi-channel systems.

The rest of the paper is organized as follows. In Section II, we summarize the results of related works. In Section III, we introduce the system model and formulate the age minimization problem. In Section IV, a fundamental lower bound is derived with respect to inter-arrival and inter-service time. In Section V, we propose a class of MWM policies whose age performance converges to the lower bound in the many-channel asymptotic regime. In Section VI, we design a class of randomized RMWM policies which achieve a constant competitive ratio compared to the lower bound in the many-user asymptotics. We use numerical simulations to validate our theoretical results in Section VII and make concluding remarks in Section VIII.

II. Related Work

Recently, the optimization of age performance for multiple sources has become a hot topic, e.g., [1–17]. In [7], the authors considered the problem of minimizing weighted expectation of long term average age in broadcast networks with an unreliable ON/OFF channel and periodic arrivals. Randomized policy, Maximum Age First policy and Whittle's index policy have been shown to achieve a constant competitive ratio. Time-sensitive information update system is considered in [3, 4] where no queue is used or the buffer only stores the latest information and any outdated packets will be discarded. An MDP-based online scheduling algorithm and an index-based online scheduling algorithm are proposed in [4] to minimize the average age. Most of the existing works assume that there is one single channel which is shared by all the users/sources. It is not clear how to extend these results to the case when multiple channels are available. The paper [12] considered multi-server systems and proposed two near-optimal policies following the maximum age first and last generated first served disciplines. Nevertheless, in multi-channel wireless networks, the channel conditions are time-varying across both channels and users, which marks a fundamental difference from the multi-server systems.

III. System Model

We consider a time-sensitive information update system which consists of one Base Station and n users. For ease of presentation, we assume each user has one flow that takes new data from one of the information sources[1]. Assume that time is slotted, and all arrivals occur at the beginning of each time-slot. New packets are generated from time to time and arrive in the system based on the arrival process $\mathbf{A}(t)$. Since time is slotted, the update data packets in each time-slot for the same user flow is considered identical. Therefore, for each time-slot t, there is at most one packet arrival for each user flow i, i.e., $A_i(t) \leq 1$. We consider Bernoulli arrival processes for any $1 \leq i \leq n$:

$$A_i(t) = \begin{cases} 1, & \text{with probability } p_i, \\ 0, & \text{with probability } 1 - p_i. \end{cases} \quad (1)$$

The Base Station maintains a separate buffer Q_i to store the latest generated packet for each user i. Let $D_i(t)$ denote the delay of the packet in Q_i at the beginning of time-slot t, i.e., the time difference between the packet generation time[2] and current time t. $D_i(t)$ is updated by $A_i(t)$ as follows:

$$D_i(t) = \begin{cases} 0, & A_i(t) = 1, \\ D_i(t-1) + 1, & \text{otherwise}. \end{cases} \quad (2)$$

Note that each buffer Q_i cannot have more than one packet. If there is a new arrival in time-slot t, i.e., $A_i(t) = 1$, then any existing packet in Q_i will be replaced by the new packet. If there is no new arrival, the delay of the packet in the buffer grows linearly with t[3].

We consider the downlink phase of a single-cell OFDM system. There are $m = \alpha n$ sub-carriers (linear number of channels) that can be used to download new packets and update information, where $\alpha \triangleq m/n$ denotes the ratio between the number of channels and the number of users. As shown in Fig. 2, each wireless channel between a user and a subcarrier (channel) has unit capacity and varies from time to time due to channel fading.

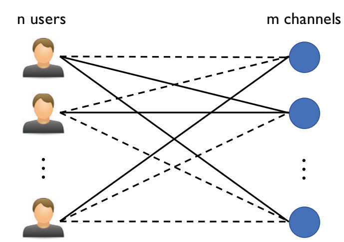

Fig. 2. Stochastic connectivity in wireless multi-channel systems. The connectivity between user i and channel j is "ON" if they are connected by a solid line, and "OFF" otherwise (connected by a dashed line).

In time-slot t, the user-channel connectivity is given by the $n \times m$ binary matrix $\mathbf{C}(t)$, where $C_{i,j}(t) = 1$ means user i is connected to channel j with rate 1 in time-slot t. We assume the channel process is given by:

$$C_{i,j}(t) = \begin{cases} 1, & \text{with probability } q, \\ 0, & \text{with probability } 1 - q. \end{cases} \quad (3)$$

[1]This assumption can be generalized to arbitrary number of flows per user.

[2]Assume packets arrive at the Base Station immediately after they are generated. The technique in this paper can also be applied when the propagation delay before reaching the Base Station is not negligible.

[3]Packets will remain in the buffer until the next arrival comes in (replacement), even if they are served.

for all $1 \le i \le n$ and $1 \le j \le m$. We assume the scheduler knows the perfect channel state information (CSI), i.e., binary matrix $\mathbf{C}(t)$ is known to the Base Station in each time-slot t.

Let $\mathbf{S}(t)$ denote the decision matrix in time-slot t, where:

$$S_{i,j}(t) = \begin{cases} 1, & \text{channel } j \text{ serves user } i \text{ in time-slot } t, \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

Due to the interference constraint, a single channel cannot serve two or more users at the same time, i.e.,

$$\sum_{i=1}^{n} S_{i,j}(t) \le 1. \quad (5)$$

for all $1 \le j \le m$. Let $\mathcal{S}$ denote the set of all feasible decision matrix $\mathbf{S}(t)$. For each feasible decision matrix $\mathbf{S}(t) \in \mathcal{S}$, define $X_i(t)$ to be the service indicator for user i in time-slot t:

$$X_i(t) = \sum_{j=1}^{m} S_{i,j}(t) C_{i,j}(t). \quad (6)$$

since there is at most one packet in buffer Q_i in time-slot t, we have

$$X_i(t) \le 1. \quad (7)$$

$X_i(t)$ is a binary variable and $X_i(t) = 1$ indicates that user i is updated in time-slot t.

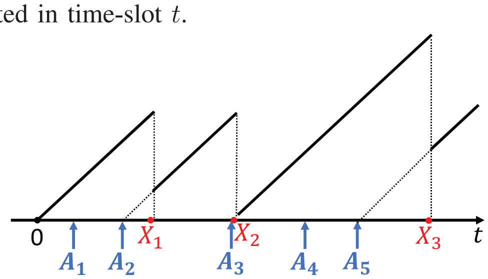

Fig. 3. Age evolution for user i. For the example shown in the figure, we have two arrivals A_1 and A_2 before the first service X_1. Packet A_1 is replaced by A_2 since A_2 becomes the latest arrival. The age grows linearly before X_1, and drops to $D_i(X_1)+1 = X_1 - A_2 + 1$ after serving packet A_2. The age again grows linearly until the next service X_2, which happens in the same time-slot as the third arrival A_3, so the age reduces to 1 after X_2.

Next, we introduce *age of information* from user's perspective. In this information update system, *age of information* measures the time difference between the current time and the generation time of the packet from the last service (update). Once a new packet p has been downloaded by user i, user i's *age of information* will be updated by the current packet delay of p. If there is no such packet delivery, *age of information* grows linearly in time. Consider the example shown in Fig. 3, the age of user i between $X_1 + 1$ and $A_3 - 1$ is equal to the packet delay of A_2. Let $H_i(t)$ denote the *age of information* of user i at the beginning of time-slot t, it evolves as follows:

$$H_i(t+1) = \begin{cases} D_i(t) + 1, & \text{if } X_i(t) = 1, D_i(t) < H_i(t), \\ H_i(t) + 1, & \text{otherwise.} \end{cases} \quad (8)$$

Note that $H_i(t+1)$ depends on the state in time-slot t. For a given time-slot t, let $\mathcal{X}(t) = \{\mathbf{A}(t), \mathbf{C}(t), \mathbf{D}(t), \mathbf{H}(t)\}$ denote the system state.

Define $U_i(t) = \mathbf{1}_{\{D_i(t) < H_i(t)\}}$ to be the update indicator of user i. $H_i(t+1)$ can be reduced only if $U_i(t) = 1$. Assume the last service of user i before t happens in time-slot t_0, i.e., $t_0 = \max_{\tau < t}\{\tau | X_i(\tau) = 1\}$. According to Equation (2), $U_i(t) = 1$ if and only if there exists at least one packet arrival of user i during time interval $[t_0 + 1, t]$ and the packet delay $D_i(t)$ has been updated to a smaller number. If there is no such arrival, then the age and packet delay are always the same as they both continue to grow with the same rate of 1.

For each time-slot t, a scheduling policy π needs to determine a feasible decision matrix $\mathbf{S}^\pi(t) \in \mathcal{S}$ based on the system state $\mathcal{X}(t)$. In this paper, we aim to minimize the expectation of long-term average of the user's age:

$$\mathbb{E}[J^\pi] = \mathbb{E}\left[\lim_{T\to\infty} J_T^\pi\right] = \mathbb{E}\left[\lim_{T\to\infty} \frac{1}{nT} \sum_{i=1}^{n} \sum_{t=0}^{T-1} H_i^\pi(t)\right]. \quad (9)$$

We use Π to denote the set of all feasible policies such that the limit in (9) exists. Define J^* to be the minimum value of $\mathbb{E}[J^\pi]$ for all $\pi \in \Pi$, i.e., $J^* = \min_{\pi\in\Pi} \mathbb{E}[J^\pi]$.

IV. A Fundamental Lower Bound

In this section, we derive a policy-independent lower bound $L \le \mathbb{E}[J^\pi]$ for any scheduling policy $\pi \in \Pi$. The lower bound could be intuitively interpreted from two extreme cases. In the case of less-frequent arrivals, the inter-arrival time could be the dominating factor regardless of the channel condition. On the other hand, in the case of bursty arrivals, the inter-service time dominates since there is always a new arrival coming in. In the next two subsections, we derive two lower bounds based on inter-arrival and inter-service time respectively.

A. Inter-arrival Based Lower Bound

We consider the lower bound determined by the inter-arrival time. Note that a service can happen only if a new packet arrival has arrived. As a result, the age performance is closely related to the inter-arrival time. If the arrivals are not frequent, then the age performance of any scheduling policy will suffer from the long inter-arrival time. We first consider a virtual-perfect policy π^{vp} that relies purely on the inter-arrival time.

Definition 1: π^{vp} is a virtual-perfect policy if it is able to instantly serve any new packet within its arrival time-slot.

Lemma 1: Let $\mathbb{E}[J^{\pi^{vp}}]$ denote the expectation of the long-term average of the user's age under virtual-perfect policy π^{vp} (assume full-connectivity virtual channels), then we have $\mathbb{E}[J^{\pi^{vp}}] \le \mathbb{E}[J^\pi]$ for any feasible policy $\pi \in \Pi$.

Proof: We use a per-user sample path dominance argument to prove Lemma 1. Due to the space limit, we provide a proof sketch using Fig. 4. Assume system S_1 is using virtual-perfect policy and S_2 is using an arbitrary policy. In addition, assume in system S_2, user i has received N_i^T services before T. We can divide the timeline $[0, T-1]$ into $N_i^T + 1$ inter-service intervals based on the service timestamps. For instance, in Fig. 4, we have intervals $[0, X_1], [X_1 + 1, X_2], \cdots, [X_4 + 1, T-1]$. We then show that for each interval, the starting age is the same for both systems, while the growing rate in system

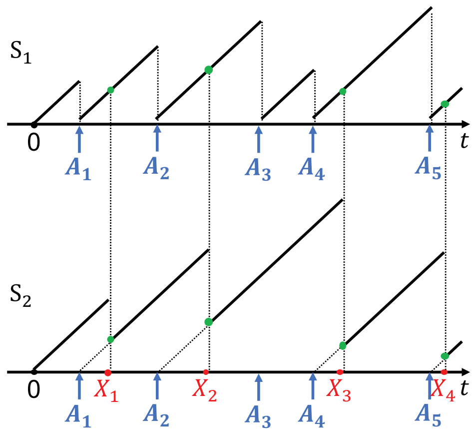

Fig. 4. The age comparison of a certain user between the virtual-perfect policy π^{vp} (system S_1) and an arbitrary policy π (system S_2). Within each inter-service interval $[X_i+1, X_{i+1}]$ of policy π, the age under policy π^{vp} is always smaller than or equal to that under policy π. The starting age for each interval is the same (green markers) for two policies, and the growing rate under policy v^{vp} is smaller than or equal to the growing rate under policy π.

1 (at most linear, may have services in between) is no more than that in system 2 (linear between consecutive services). ■

Remark 1: Since the channels are not always perfect, π^{vp} is usually not feasible. However, it is still an important baseline policy to derive the lower bound based on inter-arrival.

Lemma 2: There exists a lower bound L_A such that $L_A \leq J^*$, where

$$L_A = \frac{1}{2n}\sum_{i=1}^{n}\frac{2-p_i}{p_i} + \frac{1}{2}. \tag{10}$$

Proof: From Lemma 1 we know $\mathbb{E}[J^{\pi^{vp}}] \leq \mathbb{E}[J^{\pi}]$ for any feasible policy $\pi \in \Pi$. Hence, there must exist a lower bound for the optimal value J^*. Next, we focus on deriving a lower bound for $\mathbb{E}[J^{\pi^{vp}}]$.

Consider the stochastic process $\tilde{\mathcal{X}}(t)$ under policy π^{vp} and a finite horizon T, where $\tilde{\mathcal{X}}(t) = \{\mathbf{A}(t), \tilde{\mathbf{C}}(t), \mathbf{D}(t), \mathbf{H}(t)\}$. To make sure channels are always perfect, we use the following channel process $\tilde{\mathbf{C}}(t)$:

$$\tilde{\mathbf{C}}_{i,j}(t) = 1, \quad \forall i, j, t. \tag{11}$$

Let A_T^i be the total number of arrivals from time-slot 0 to time-slot $T-1$ for user i. In this case, a service event happens immediately after each arrival event, thus, we do not differentiate these two events within this proof. Let $I_i(\tau)$ be the number of time-slots between $(\tau-1)^{\text{th}}$ and τ^{th} service (arrival) event to user i (0^{th} service happens right before time-slot 0). Let R_i denote the number of remaining time-slots after the last service (arrival) event before T. We have the following equation for any user $i \in \{1, 2, \cdots, n\}$:

$$T = \sum_{\tau=1}^{A_T^i} I_i(\tau) + R_i. \tag{12}$$

Note that the inter-service time $\{I_i(i)\}_{\tau=1}^{A_T^i}$ is a series of *i.i.d.* random variables and the arrival (service) process $\mathbf{A}(t)$ is a renewal process. Similar to the proof of Theorem 6 in [7], we have with probability one,

$$J_T^{\pi^{vp}} \geq \frac{1}{2n}\sum_{i=1}^{n}\frac{1}{T}\sum_{\tau=1}^{A_T^i} I_i^2(\tau) + \frac{1}{2}. \tag{13}$$

and with probability one,

$$J_T^{\pi^{vp}} \geq \frac{1}{2n}\sum_{i=1}^{n}\frac{\mathbb{E}[I_i^2(1)]}{\mathbb{E}[I_i(1)]} + \frac{1}{2}. \tag{14}$$

Furthermore, from (1) we know $I_i(1)$ is geometric distributed with support $\{1, 2, \cdots\}$, (14) can be rewritten as:

$$J^{\pi^{vp}} \geq \frac{1}{2n}\sum_{i=1}^{n}\frac{2-p_i}{p_i} + \frac{1}{2} \qquad w.p.1. \tag{15}$$

Taking the expectation of (15), we have $\mathbb{E}[J^{\pi^{vp}}] \geq \frac{1}{2n}\sum_{i=1}^{n}\frac{2-p_i}{p_i} + \frac{1}{2} = L_A$. L_A is a lower bound for the optimal value J^* based on the inter-arrival time. ■

B. Inter-service Based Lower Bound

We consider the lower bound determined by the inter-service time. This lower bound is non-trivial especially when the number of users is greater than the number of channels, i.e., $n > m$. In this case, it is not possible to find a perfect matching that covers all users regardless of the channel condition. A scheduling policy has to select a subset of users to serve, hence, the inter-service time is not negligible when the number of users becomes large. Even when arrivals are very frequent ($p_i \to 1$), the age performance is still bounded by the inter-service time due to user-selection, especially when $\alpha = m/n$ becomes small. We have the following lemma:

Lemma 3: There exists a lower bound L_S such that $L_S \leq J^*$, where

$$L_S = \frac{1}{2\alpha} + \frac{1}{2}. \tag{16}$$

Proof: Consider an arbitrary policy π and a sample path $\omega \in \Omega$, assume user i has N_i^T services before T. For each user i, let $I_i(\tau)$ be the inter-service time between $(\tau-1)^{th}$ service and τ^{th} service. We use $R_i = T - \sum_{\tau=1}^{N_i^T} I_i(\tau)$ to denote the remaining time before T. The age evolves as $1, 2, \cdots$, within each inter-service interval and we can bound the objective function as follows:

$$\begin{aligned} J^{\pi}(\omega) &\geq \lim_{T\to\infty}\frac{1}{nT}\sum_{i=1}^{n}\sum_{\tau=1}^{N_i^T}\frac{I_i(\tau)\left(I_i(\tau)+1\right)}{2} + \frac{R_i(R_i+1)}{2} \\ &\geq \lim_{T\to\infty}\frac{1}{2nT}\sum_{i=1}^{n}\left[\sum_{\tau=1}^{N_i^T} I_i^2(\tau) + R_i^2\right] + \frac{1}{2}. \end{aligned} \tag{17}$$

The set of intervals $\{I_i(1), I_i(2), \cdots, I_i(N_i^T), R_i\}$ is a partition of interval $[0, T-1]$. Applying Cauchy-Schwarz inequality to (17), we have for all sample path $\omega \in \Omega$

$$J^{\pi}(\omega) \geq \lim_{T\to\infty}\frac{T}{2n}\sum_{i=1}^{n}\frac{1}{N_i^T} + \frac{1}{2}. \tag{18}$$

Note that there are at most m services in each time-slot, we have $\sum_{i=1}^{n} N_i^T \leq mT$. Applying Cauchy-Schwarz inequality again to (18), we have for all $\omega \in \Omega$:

$$J^{\pi}(\omega) \geq \frac{n}{2m} + \frac{1}{2}. \tag{19}$$

Therefore, we have

$$\mathbb{E}\left[J^{\pi}\right] \geq \frac{1}{2\alpha} + \frac{1}{2}. \tag{20}$$

for any feasible policy $\pi \in \Pi$, and the result holds. ■

Finally, combining the inter-arrival based lower bound L_A and the inter-service based lower bound L_S leads to a lower bound of age which is jointly characterized by the external arrival process and the channel statistics.

Theorem 1: There exists a lower bound L such that $L \leq J^*$, where L is given by:

$$L = \max\left\{\frac{1}{2n}\sum_{i=1}^{n}\frac{2-p_i}{p_i}, \frac{1}{2\alpha}\right\} + \frac{1}{2}. \tag{21}$$

Proof: Applying Lemma 2, Lemma 3 and taking $L = \max\{L_A, L_S\}$, the result follows. ■

Theorem 1 provides a benchmark for evaluating the age performance of scheduling policies. Next, we consider two asymptotic regimes and propose scheduling policies that achieve near-optimal age performance.

V. Many-channel Asymptotic Regime

In this regime, we assume that the number of channels m is greater than or equal to the number of users n and fix $\alpha \geq 1$. The inter-service based bound becomes trivial as $L_A \geq L_S$ and $L = L_A \leq J^*$.

A. Scheduling Policies

In this section, we propose two kinds of scheduling policies that achieve close to optimal objective function J^* in the many-channel asymptotic regime, i.e, the performance gap $J^{\pi} - J^*$ vanishes when $\alpha \geq 1$ and $n \to \infty$.

Consider a random bipartite graph $G[X \cup Y, E]$ where X is the user set and Y is the channel set. There is an edge (i, j) connecting user i and channel j if and only if $\mathbf{C}_{i,j}(t) = 1$. A matching $\mathcal{M}$ is a set of edges such that no two edges share an endpoint. Based on constraints (5) and (7), the scheduling problem boils down to find a matching $\mathcal{M}$ in each time-slot. The corresponding decision matrix $\mathbf{S}(t)$ is determined by

$$S_{i,j}(t) = \mathbf{1}_{\{(i,j)\in\mathcal{M}\}}. \tag{22}$$

In this section, we use bipartite graph G to model the user-channel connectivity and develop matching policies.

PM (Perfect Matching) Policy. If G has a perfect matching $\mathcal{M}$ (every user vertex x is incident to exactly one edge $(x, y) \in \mathcal{M}$), then for any edge $(i, j) \in \mathcal{M}$, set $S_{i,j}(t) = 1$. Otherwise, set $X_i(t) = 0$ for all user i. In other words, the PM policy updates any user i who has the desire to update ($U_i(t) = 1$) if there exists a perfect matching. Otherwise, no packet is served even though some users can still get service.

This policy is sub-optimal when the number of channels is small, since some channel resources may be wasted due to its lazy behavior. However, as we will soon see, this policy achieves close-to-optimal age performance when the number of channels becomes large.

In the following lemma, we show that in each time-slot, with high probability (close to 1) that the random bipartite graph G has a perfect matching.

Lemma 4: In each time-slot t, assume bipartite graph $G[X \cup Y, E]$ is generated by the binary matrix $\mathbf{C}(t)$. There exists a constant $N_1 > 0$, such that the probability that G has a perfect matching is lower bounded by:

$$\mathbb{P}\left(G \text{ has a PM}\right) \geq 1 - 3ne^{-C_1 n}. \tag{23}$$

where constant $C_1 = \log_{1/(1-q)}$ for all $n > N_1$.

Proof: The result follows from Lemma 1 in [24]. ■

As $n \to \infty$, it is highly possible that G has a perfect matching in every time-slot. In other words, with high probability, each user is offered the transmission opportunity. In this case, the PM policy approximates the virtual-perfect policy in the asymptotic regime. As we know $\mathbb{E}[J^{\pi^{vp}}] \leq \mathbb{E}[J^{\pi}]$ for any $\pi \in \Pi$, one can expect that the PM policy achieves near-optimal age performance in the asymptotic regime.

Theorem 2: Consider a many-channel information update system described by $\mathcal{X}(t)$. Let $J^{\pi^{\text{PM}}}$ denote the expectation of the long-term average of user's age under PM policy, the gap between $\mathbb{E}[J^{\pi^{\text{PM}}}]$ and J^* can be bounded by

$$\mathbb{E}[J^{\pi^{\text{PM}}}] - J^* < 6ne^{-C_1 n}. \tag{24}$$

for all $n > M$.

Proof: Please refer to Section V-B. ■

Remark 2: Although the PM policy achieves near-optimal age performance in the many-channel asymptotic regime, i.e., $\alpha \geq 1$ and $n \to \infty$, the transmission opportunity of all channels would be lost if a perfect matching could not be found (even by one or a few disconnected users).

Next, we develop a policy which shares the nice asymptotic near-optimality of the PM policy, but overcomes its limitation in non-asymptotic regimes.

Class of MWM (Maximum Weighted Matching) Policies. Consider the random bipartite graph $G = [X \cup Y, E]$, now we associate each user vertex $i \in X$ with a non-negative weight $W_i(t) \geq 0$ in each time-slot t. Based on G and the weight vector $\mathbf{W}(t)$, the users to update is determined by the Maximum Weighted Matching $\mathcal{M}$. A user is served in time-slot t if it is covered by an edge from $\mathcal{M}$, i.e., $X_i(t) = 1$ only if there exists channel j, such that edge $(i, j) \in \mathcal{M}$, and $X_i(t) = \sum_{j=1}^{m} \mathbf{1}_{\{(i,j)\in\mathcal{M}\}}$.

Note that if $\mathbf{W}(t)$ is age-independent, the policy may still be inefficient. For example, assume user 1 has the desire to update, i.e., $U_i(t) = 1$. If we assign $W_1(t) = 0$, then user 1 may not be covered by the Maximum Weighted Matching even a perfect matching does exist. On the other hand, assume user 2's information is up to date and there is no new pending packet. If we assign $W_2(t)$ by a large weight that dominates all other weights in $\mathbf{W}(t)$. User 2 has the highest priority for the maximum weighted matching, however, it could waste the channel resources since it contributes 0 to age reduction. To address this issue, we define an age-aware weight vector $\mathbf{W}(t)$ based on system state $\mathcal{X}(t)$ to indicate the desire for update.

Definition 2: Given the system state $\mathcal{X}(t)$, $\mathbf{W}(t)$ is an age-aware weight vector if the following property holds

$$\begin{cases} W_i(t) > 0, & \text{if } U_i(t) = 1, \\ W_i(t) = 0, & \text{if } U_i(t) = 0. \end{cases} \tag{25}$$

Remark 3: With an age-aware weight vector $\mathbf{W}(t)$, no user with 0 age-difference ($H_i(t) = D_i(t)$) is updated. Any strictly positive number can be assigned to the user with strictly positive age-difference ($H_i(t) - D_i(t) > 0$, i.e., $U_i(t) = 1$).

Definition 3: A policy π belongs to the class of MWM policies (Π^{MWM}) if there exists an age-aware weight vector $\mathbf{W}(t)$, such that the users to update is determined by the Maximum Weighted Matching with weight $\mathbf{W}(t)$.

It is obvious that any policy $\pi \in \Pi^{MWM}$ is work-conserving even with finite number of users or channels. Now we show that the class of MWM policies are also near-optimal in the asymptotic regime. The following lemmas help us build the relationship between the class of MWM policies and the PM policy.

Lemma 5: Let π be an arbitrary policy from Π^{MWM}. For any given sample path ω and time-slot t, by the end of time-slot t, policy π has served every packet that π^{PM} has served.

Proof: Consider two information update systems $\mathbf{S}_1$ and $\mathbf{S}_2$, each consists of n users and m channels. Both systems have the same arrivals and channel state realization. $\mathbf{S}_1$ uses policy π and $\mathbf{S}_2$ uses policy π^{PM}. We only need to show that for any given sample path ω, if a packet x for user i is served by π^{PM} by the end of time-slot t, then the same packet x must be served in the same time-slot or it has already been served by the policy π.

First of all, packet x is not replaced by a new packet without being served in system $\mathbf{S}_1$. Assume there is a new packet y for the same user i, packet y arrives in time-slot $t' \le t$. If packet x has not been served by policy π, packet x is no longer the latest packet and will be replaced. Since $\mathbf{S}_1$ and $\mathbf{S}_2$ share the same arrivals realization, packet y also arrives in system $\mathbf{S}_2$ in time-slot t'. At this time, packet x has not been served by policy π^{PM} since $t' \le t$, it should be replaced by packet y as well. This fact contradicts with the assumption that packet x is still in system $\mathbf{S}_2$ and is served exactly in time-slot t. We know that packet x is not replaced in system $\mathbf{S}_1$.

Now we consider two cases:

Case 1: packet x has already been served in system $\mathbf{S}_1$. This is what we want and there is nothing to prove.

Case 2: packet x is the latest pending packet for user i in system $\mathbf{S}_1$. Since packet x is served in system $\mathbf{S}_2$, we know that the bipartite graph G has a perfect matching that covers all user vertices. Note that even a perfect matching exists, policy π^{PM} only serve packets for users that have the desire to update ($U(t) = 1$). Due to the existence of perfect matching, the maximum weighted matching can serve any user i that satisfies $U_i(t) = 1$ as the total weight increases by adding up strictly positive values. Therefore, packet x is also served in time-slot t in system $\mathbf{S}_1$. ■

Lemma 6: Let π be an arbitrary policy from Π^{MWM}, we have $\mathbb{E}[J^\pi] \le \mathbb{E}[J^{\pi^{PM}}]$.

Proof: We only need to show that with probability one, $J_T^\pi \le J_T^{\pi^{PM}}$ for any $T \ge 0$. Similar to the proof of lemma 5, we consider two information update systems $\mathbf{S}_1$ and $\mathbf{S}_2$, $\mathbf{S}_1$ uses policy π and $\mathbf{S}_2$ uses policy π^{PM}. Consider time period $[0, T-1]$, assume system $\mathbf{S}_2$ served p packets for user i, namely $x_1, x_2, \cdots, x_p$. According to Lemma 5, $x_1, x_2, \cdots, x_p$ have also been served in system $\mathbf{S}_1$. Let $t_k(x)$ denote the service time of packet x in system $\mathbf{S}_k$, we have $t_1(x) \le t_2(x)$ for all $x \in \{x_1, x_2, \cdots, x_p\}$. We divide time interval $[0, T-1]$ into subintervals, $[0, t_2(x_1)]$, $[t_2(x_1)+1, t_2(x_2)], \cdots, [t_2(x_{p-1})+1, t_2(x_p)], [t_2(x_p)+1, T-1]$. As shown in Fig. 5, we can apply the same argument from Lemma 1 to prove the dominating result. ■

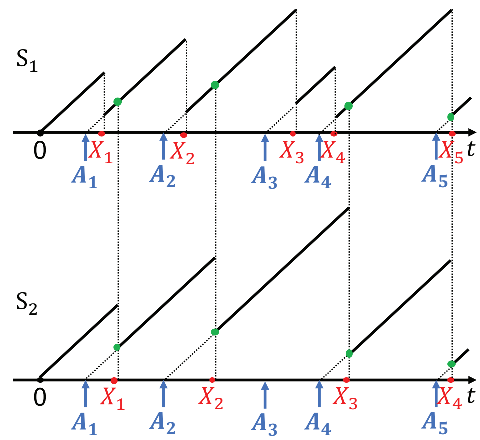

Fig. 5. The age comparison of a certain user between policy π from the class of MWM policies ($\mathbf{S}_1$) and the perfect matching policy π^{PM} ($\mathbf{S}_2$) for a certain user. For any time-slot t, policy π has served every packet that π^{PM} has served. In this example, policy π has one more service to serve packet A_3. The green markers from two systems have the same age, the age in system $\mathbf{S}_1$ is always no larger than that in system $\mathbf{S}_2$ between two consecutive markers.

Based on Lemma 6 and Theorem 2, we have the following theorem.

Theorem 3: Consider a many-channel information update system described by $\mathcal{X}(t)$. Let π be an arbitrary policy from the class of MWM policies, then π is work-conserving and the gap between J^π and J^* can be bounded by

$$\mathbb{E}[J^\pi] - J^* < 6ne^{-C_1 n}. \tag{26}$$

for all $n > M$.

There are a few weight vectors $\mathbf{W}(t)$ that satisfy (25). In the Age Difference Weighted Matching (ADWM) policy, we choose $W_i(t) = H_i(t) - D_i(t)$ to be the age difference between the user's age and packet's age (*a.k.a.* delay). The ADWM policy minimizes the total age sum in each time-slot, hence, it is expected to achieve good age performance. We can also use an update-indicator weight vector $W_i(t) = U_i(t)$ to develop an Update-indicator Weighted Matching (UWM) policy. UWM is also a work-conserving policy but treats all users that have the desire for update equally regardless of the age-difference contribution.

However, Delay Weighted Matching (DWM) with $W_i(t) = D_i(t)$, which achieves good delay performance [22] in multi-user multi-channel systems, is sub-optimal in terms of age performance. Since the weight vector violates the requirement (25). Assume a new packet arrival for user i comes in time-slot t with $D_i(t) = 0$, it is associated with 0 weight. If the current age $H_i(t)$ is very high, then DWM loses the opportunity to reduce the age to 1.

B. Analysis of the PM Policy

In this section, we focus on the age performance analysis of the PM policy. The same argument also works for Theorem 3 using Lemma 5 and 6.

We first consider a finite horizon T and a user i, assume there are N_i^T services to user i from 0 to time-slot $T-1$ with τ^{th} service happens in time-slot X_τ^i and set $X_0^i = 0$. Let $I_i(\tau)$ be the number of time-slots between $(\tau-1)^{\text{th}}$ and τ^{th} service to user i, i.e., $I_i(\tau) = X_\tau^i - X_{\tau-1}^i$. Let R_i denote the number of remaining time-slots after the last service event before T. In τ^{th} interval, the age evolves as $D_i(X_{\tau-1}^i)+1, D_i(X_{\tau-1}^i)+2, \cdots, D_i(X_{\tau-1}^i)+I_i(\tau)$. Hence, the total age sum in τ^{th} interval is given by $D_i(X_{\tau-1}^i)I_i(\tau) + \frac{I_i^2(\tau)+I_i(\tau)}{2}$. We use (9) and (13) to characterize the objective function under PM policy:

$$\begin{aligned}J^{\pi^{PM}} &= \lim_{T\to\infty}\frac{1}{nT}\sum_{i=1}^{n}\sum_{t=0}^{T-1} H_i^{\pi^{PM}}(t)\\ &= \sum_{i=1}^{n}\Bigg\{\lim_{T\to\infty}\frac{1}{nT}\sum_{\tau=1}^{N_i^T}\left(D_i(X_{\tau-1}^i)I_i(\tau)+\frac{I_i^2(\tau)+I_i(\tau)}{2}\right)\\ &+\lim_{T\to\infty}\frac{1}{nT}\left[D_i(X_{N_i^T}^i)R_i+\frac{R_i^2+R_i}{2}\right]\Bigg\} \quad \textit{w.p.1.} \qquad (27)\end{aligned}$$

Before evaluating the age performance, we introduce the following lemma which helps to simplify (27).

Lemma 7: Let $\mathcal{R}_i$ denote the remaining part of user i in (27) after $N_i^{T\,th}$ service, i.e.,

$$\mathcal{R}_i = \lim_{T\to\infty}\frac{1}{nT}\left[D_i(X_{N_i(T)}^i)R_i + \frac{R_i^2+R_i}{2}\right], \qquad (28)$$

then $\mathcal{R}_i = 0$ with probability one.

We omit the proof of Lemma 7 due to the limited space.

Remark 4: From Lemma 7, we know that the remaining part in (27) is negligible. We only need to focus on the first N_i^T complete intervals.

Now (27) can be rewritten as

$$J^{\pi^{PM}} = \sum_{i=1}^{n}\left[RHS1_i + RHS2_i\right] \quad \textit{w.p.1.} \qquad (29)$$

where

$$RHS1_i = \lim_{T\to\infty}\frac{1}{nT}\sum_{\tau=1}^{N_i^T}\frac{I_i^2(\tau)+I_i(\tau)}{2}. \qquad (30)$$

$$RHS2_i = \lim_{T\to\infty}\frac{1}{nT}\sum_{\tau=1}^{N_i^T} D_i(X_{\tau-1}^i)I_i(\tau). \qquad (31)$$

We divide the rest of the analysis into part 1 and part 2 to evaluate $RHS1_i$ and $RHS2_i$ respectively.

Part 1: Evaluating $RHS1_i$ in (30)

If we consider a service event as a renewal, then $\{X^i\}$ is a renewal process, $\{I_i\}$ is a series of *i.i.d.* inter-renewal time. Note that a service (renewal) event can happen only if there is at least one packet arrival coming in since the last service. Therefore, after the last service, we need to wait until the first packet arrival occurs, and then wait until there is a good channel opportunity, i.e., there exists a perfect matching. We have the following lemma to derive the first and second moment of the inter-renewal time I_i.

Lemma 8: The first and second moments of the inter-renewal time I_i are given by:

$$\mathbb{E}\left[I_i\right] = \frac{p_i+p_s-p_ip_s}{p_ip_s}. \qquad (32)$$

$$\mathbb{E}\left[I_i^2\right] = \frac{p_ip_s}{p_s-p_i}\left[\frac{(1-p_i)(2-p_i)}{p_i^3} - \frac{(1-p_s)(2-p_s)}{p_s^3}\right]. \qquad (33)$$

where p_s is the probability that the bipartite graph G has a perfect matching.

Applying the elementary reward renewal theorem, we have:

$$\begin{aligned}RHS1_i &= \lim_{T\to\infty}\frac{1}{2nT}\sum_{\tau=1}^{N_i^T} I_i^2(\tau) + \lim_{T\to\infty}\frac{1}{2nT}\sum_{\tau=1}^{N_i^T} I_i(\tau)\\ &= \frac{1}{2n}\left(\frac{\mathbb{E}[I_i^2]}{\mathbb{E}[I_i]}+1\right) \quad \textit{w.p.1.} \qquad (34)\end{aligned}$$

Part 2: Evaluating $RHS2_i$ in (31)

The distribution of $D_i(X_{\tau-1}^i)$, i.e., the packet delay at $(\tau-1)^{th}$ service of user i, is given by the following lemma:

Lemma 9: The PMF of $D_i(X_{\tau-1}^i)$ is given by:

$$\mathbb{P}\left(D_i(X_{\tau-1}^i) = d\right) = (1-p_s)^d(1-p_i)^d\left(p_s+p_i-p_ip_s\right). \qquad (35)$$

for all $\tau > 1$ and $d \geq 0$. The initial packet delay $D_i(X_0^i) = 0$ with probability one.

Let us consider the reverse process $\{\hat{X}^i\}$ of the renewal process $\{X^i\}$, and a sequence of *i.i.d.* random variables $\{\hat{D}_i(X_\tau^i)\}$, that follows the distribution (35) for all $\tau \geq 0$. The only difference between $\{\hat{D}_i(X_\tau^i)\}$ and $\{D_i(X_\tau^i)\}$ is that $\hat{D}_i(X_0^i)$ may not be zero and is treated as if the system is running from $-\infty$. The process $\{\hat{X}^i\}$ starts with the first interval $I_i(N_i^T)$, a random number $\hat{D}_i(X_{N_i^T-1}^i)$ is then generated based on (35). We have $I_i(N_i^T-1) \geq \hat{D}_i(X_{N_i^T-1}^i)$, and hence, the next interval length depends on the value of $\hat{D}_i(X_{N_i^T-1}^i)$. Therefore, the sequence $\left\{(X_\tau^i, \hat{D}_i(X_{\tau-1}^i))\right\}_{\tau=N_i^T}^{1}$ is a Markov renewal process. Let $Y_i(\tau) = \hat{D}_i(X_{\tau-1}^i)I_i(\tau)$ denote the reward of τ^{th} interval.

The expectation of the reward function is finite[4].

$$
\begin{aligned}
\mathbb{E}[Y_i(\tau)] &= \mathbb{E}\left[\hat{D}_i(X^i_{\tau-1}) I_i(\tau)\right] \\
&= \mathbb{E}\left[\hat{D}_i(X^i_{\tau-1})\right]\mathbb{E}\left[I_i(\tau)\right] < \infty. \qquad (36)
\end{aligned}
$$

Applying the renewal theorem for Markov renewal process [25], we have:

$$
\begin{aligned}
&\lim_{T\to\infty} \frac{1}{T} \sum_{\tau=N_i^T}^{1} Y_i(\tau) \\
&= \frac{\mathbb{E}\left[\mathbb{E}\left[\hat{D}_i(X^i_{\tau-1}) I_i(\tau) | \hat{D}_i(X^i_\tau)\right]\right]}{\mathbb{E}\left[\mathbb{E}\left[I_i(\tau)|\hat{D}_i(X^i_\tau)\right]\right]} \\
&= \frac{\mathbb{E}\left[\mathbb{E}\left[\hat{D}_i(X^i_{\tau-1})\right]\mathbb{E}\left[I_i(\tau)|\hat{D}_i(X^i_\tau)\right]\right]}{\mathbb{E}\left[I_i(\tau)\right]} \\
&= \frac{\mathbb{E}\left[\hat{D}_i(X^i_{\tau-1})\right]\mathbb{E}\left[\mathbb{E}\left[I_i(\tau)|\hat{D}_i(X^i_\tau)\right]\right]}{\mathbb{E}\left[I_i(\tau)\right]} \\
&= \mathbb{E}\left[\hat{D}_i(X^i_{\tau-1})\right]. \qquad (37)
\end{aligned}
$$

with probability one.

Since $\hat{D}_i(X_0^i) \geq 0$ and $D_i(X_0^i) = 0$ with probability one,

$$
RHS2_i \leq \lim_{T\to\infty} \frac{1}{nT} \sum_{\tau=N_i^T}^{1} Y_i(\tau). \qquad (38)
$$

with probability one.

Substituting (34), (37) and (38) into (29), we have

$$
J^{\pi^{PM}} \leq \frac{1}{n}\sum_{i=1}^{n}\left[\mathbb{E}\left[\hat{D}_i(X^i_{\tau-1})\right] + \frac{1}{2}\left(\frac{\mathbb{E}[I_i^2]}{\mathbb{E}[I_i]}+1\right)\right]. \qquad (39)
$$

with probability one, which leads to

$$
\mathbb{E}\left[J^{\pi^{PM}}\right] \leq \frac{1}{n}\sum_{i=1}^{n}\left[\mathbb{E}\left[\hat{D}_i(X^i_{\tau-1})\right] + \frac{1}{2}\left(\frac{\mathbb{E}[I_i^2]}{\mathbb{E}[I_i]}+1\right)\right]. \qquad (40)
$$

The performance gap $\mathbb{E}\left[J^{\pi^{PM}}\right] - J^*$ can be bounded by

$$
\begin{aligned}
&\mathbb{E}\left[J^{\pi^{PM}}\right] - J^* \\
&\leq \frac{1}{n}\sum_{i=1}^{n}\left\{\mathbb{E}\left[\hat{D}_i(X^i_{\tau-1})\right] + \frac{1}{2}\left(\frac{\mathbb{E}[I_i^2]}{\mathbb{E}[I_i]} - \frac{2-p_i}{p_i}\right)\right\} \\
&= \frac{1}{n}\sum_{i=1}^{n}\frac{1}{p_s}(1-p_s) = \frac{1}{p_s}(1-p_s). \qquad (41)
\end{aligned}
$$

Recall that $p_s \geq 1 - 3ne^{-C_1 n}$ for all $n > N_1$. We can choose a large enough number $N_2 > 0$, such that for all $n > \max\{N_1, N_2\}$, $\frac{1}{p_s} \leq \frac{1}{1-3ne^{-C_1 n}} < 2$. Let $M \triangleq \max\{N_1, N_2\}$, we have

$$
\mathbb{E}[J^{\pi^{PM}}] - J^* < 6ne^{-C_1 n}. \qquad (42)
$$

for all $n > M$.

[4]Note that $I_i(\tau)$ depends on $\hat{D}_i(X^i_\tau)$ but is independent from $\hat{D}_i(X^i_{\tau-1})$.

As $n \to \infty$, the gap between $\mathbb{E}[J^{\pi^{PM}}]$ and the optimal objective value J^* vanishes near-exponentially fast.

VI. Many-user Asymptotic Regime

In this regime, we fix $\alpha < 1$, i.e., the number of channels m is smaller than the number of users n and let the number of channels $m \to \infty$. Lower bounds L_A and L_S are both non-trivial, whichever becomes larger depends on the arrival probabilities $\{p_i\}_{i=1}^n$ and channel-user ratio α.

Next, we propose a class of RMWM (Randomized Maximum Weighted Matching) policies that achieve good age performance when the number of channels $m \to \infty$.

Class of RMWM Policies. The scheduler randomly generates m different users with equal probability, let X' denote the set of generated users. Now construct a subgraph $G' = [X' \cup Y, E']$ from the random bipartite graph $G = [X \cup Y, E]$, where $E' \subseteq E$ is the set of all edges that have an endpoint in X'. In subgraph G', there are equal number of users and channels, i.e., $|X'| = |Y| = m$. The scheduler then uses an age-aware weight vector $\mathbf{W}(t)$ to determine the Maximum Weighted Matching $\mathcal{M}$ on G'. A user is updated in time-slot t if it is covered by an edge from $\mathcal{M}$, i.e., $X_i(t) = 1$ only if there exists channel j, such that edge $(i,j) \in \mathcal{M}$, and $X_i(t) = \sum_{j=1}^m \mathbf{1}_{\{(i,j)\in\mathcal{M}\}}$.

The analysis of the class of RMWM policies is difficult since the distribution of the total number of updates per time-slot is intractable. As in Section V, we propose a simple policy RPM (Randomized Perfect Matching) for ease of analysis.

RPM Policy. The scheduler randomly generates m different users with equal probability, and the subgraph G' is constructed in the same way. If G' has a perfect matching, then any user i with $U_i(t) > 0$ will be updated by the packet in the buffer. Otherwise, no user is updated.

Remark 5: The same argument in Lemma 5 does not apply directly since two policies may randomly generate two different user sets. The same result still holds if they select the same user set $X'(t)$ in each time-slot t.

Lemma 10: Let π be an arbitrary policy from the class of RMWM policies and π^{RPM} denote the RPM policy, assume they select the same user set $X'(t)$ in each time-slot t. For any given sample path ω and time-slot t, by the end of time-slot t, policy π has served every packet that π^{RPM} has served.

Similarly, we have the following dominance property.

Lemma 11: $\mathbb{E}\left[J^\pi\right] \leq \mathbb{E}\left[J^{\pi^{RPM}}\right]$.

Proof: Based on the result of Lemma 10, applying the same argument from Lemma 6, we have:

$$
\mathbb{E}\left[J^\pi | \mathbf{X'}_\pi = \mathbf{X'}\right] \leq \mathbb{E}\left[J^{\pi^{RPM}} | \mathbf{X'}_{\pi^{RPM}} = \mathbf{X'}\right]. \qquad (43)
$$

Note that $\mathbf{X'}_\pi$ and $\mathbf{X'}_{\pi^{RPM}}$ have the same distribution, taking the expectation for both sides the result follows. ■

Next, we focus on the analysis of the RPM policy. We can apply the same argument from Section V-B, we only need to replace p_s by αp_s due to the randomness (each user is selected with probability α). Based on (41), we have:

$$\mathbb{E}\left[J^{\pi^{RPM}}\right] \leq \frac{1}{2n}\sum_{i=1}^{n}\frac{2-p_i}{p_i}+\frac{1}{\alpha p_s}-\frac{1}{2}$$
$$\leq L_A+\frac{2}{p_s}L_S \leq (1+\frac{2}{p_s})J^*. \quad (44)$$

From Lemma 4, we have $p_s \geq 1-3me^{-C_1 m}$ for all $m > N_1$. Take $m \to \infty$ and apply Lemma 11, we have:

Theorem 4: Consider a many-user information update system described by $\mathcal{X}(t)$. Let π be an arbitrary policy from the class of RMWM policies, the competitive ratio can be upper bounded by:

$$\lim_{m\to\infty}\frac{\mathbb{E}\left[J^{\pi}\right]}{J^*} \leq 3. \quad (45)$$

Remark 6: Competitive ratio (45) is a very general result that holds for all channel-user ratio α, arrival probabilities $\{p_i\}_{i=1}^{n}$ and any connectivity probability $q > 0$.

VII. Numerical Simulations

In this section, we use numerical simulations to validate the theoretical results under two regimes and compare the age performance between different policies. In both regimes, we set arrival probabilities $p_i = 0.5$ for any user $1 \leq i \leq n$ and set connectivity probability $q = 0.2$.

A. Many-channel Asymptotic Regime

In this regime, the number of channels m is greater than or equal to the number of users n. We assume $m = n$ and run simulations for different m. Based on (21), the lower bound $L = 2$ is independent from n and q. We use DWM (Delay Weighted Matching), UWM (Update-indicator Weighted Matching) and ADWM (Age Difference Weighted Matching) policies and compare the age performance in terms of the age expectation $\mathbb{E}\left[J^{\pi}\right]$.

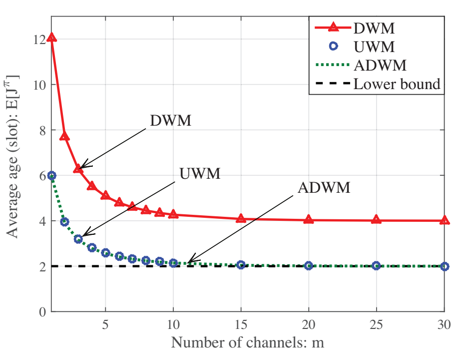

Fig. 6. Simulation results when $m = n$.

From Fig. 6, we have the following observations

- UWM and ADWM policy both converge to the lower bound very fast. Even for $m = 10$, the gap to the lower bound is already negligible. On the other hand, DWM is sub-optimal, the gap $\mathbb{E}\left[J^{\pi}\right] - J^* \geq 2$ even in the asymptotic regime.
- The age performance of UWM policy is very close to ADWM policy. They are both from the class of MWM policies and only differ from the choice of weight vectors. As long as (25) is satisfied, any policy from Π^{MWM} should converge to the lower bound asymptotically.

B. Many-user Asymptotic Regime

In this regime, the number of channels m is less than the number of users n. We assume $n = 5m$, i.e., $\alpha = 0.2$, and run simulations for different m. We use RUWM (Randomized Update-indicator Weighted Matching), RADWM (Randomized Age Difference Weighted Matching) to compare the age performance in terms of the age expectation $\mathbb{E}\left[J^{\pi}\right]$. We also use MADWM (Maximum Age Difference Weighted Matching) as a benchmark policy where in each time-slot m users with the largest age difference are selected in set $\hat{X}$ and the schedule is determined by the maximum weighted matching with age difference weight in the subgraph $[\hat{X}, Y, \hat{E}]$.

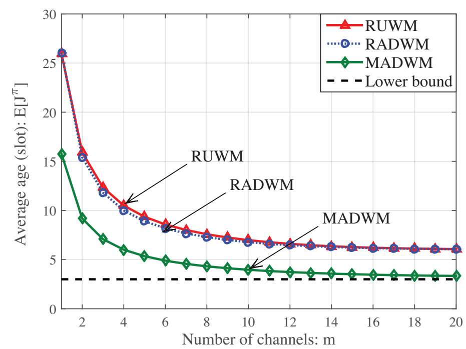

Fig. 7. Simulation results when $n = 5m$.

From Fig. 7, we have the following observations

- The objective value $\mathbb{E}\left[J^{\pi}\right]$ of RUWM and RADWM policy is less than 3 times of lower bound when $m \geq 6$. The competitive ratio $\frac{\mathbb{E}[J^{\pi}]}{J^*} \to 2$ when $m \to \infty$.
- All three policies have very similar convergence rate, for all m the objective values of RUWM and RADWM are within 2 times of that in MADWM.

Note that the scheduler in MADWM needs to track the up-to-date age information of all flows, while RUWM and RADWM only require 1-bit information of the update-indicator from each flow.

VIII. Conclusion

In this paper, we investigate the age minimization problem in multi-user multi-channel systems. *Age of information* measures the elapsed time of the last served packet since its generation, hence, it depends on the inter-arrival and inter-service time. Based on this observation, we derive a policy-independent lower bound for the age minimization problem. Then we focus on two asymptotic regimes, i.e., many-channel asymptotic regime and many-user asymptotic regime. In both regimes, we propose classes of policies which achieve provably good age performance. This paper demonstrates how to exploit multi-channel flexibility to improve the age performance in information updating systems.

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