Chapter 1

Cross-Layer Resource Allocation in Energy Harvesting Sensor Networks

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1. Introduction

Recent advances in the field of wireless communications and data acquisition have enabled us with the unique capability to remotely sense our environment. Data acquisition networks can be used to sense natural as well as human-created phenomena. Several experimental networks are already in place for studying earthquakes, fire, and glacial movements, which are all critical to human existence. As these applications require deployment in remote and hard-to-reach areas, it is to ensure that such networks can operate unattended for long durations. The lack of easy access to a continuous power source in most scenarios and the limited lifetime of batteries have hindered the deployment of such networks. To that end, new and exciting developments in the areas of renewable sources of energy[1–3] suggest that this is feasible. These renewable sources of energy could be attached to new nodes and would typically provide energy replenishment at a slow rate (compared to the rate at which energy is consumed by a continuous stream of packet transmissions) that could be variable and dependent on the surroundings. For example, self-powered sensors have been developed that rely on harvesting strain and vibration energies from their working environment,[1] as well as other types of energy sources including solar cells.[2,3] Further, sensor networks could be comprised of highly heterogeneous nodes in which some nodes may have more efficient sources of energy replenishment than others (including some nodes that may have no renewable sources of energy), thus making it imperative that energy efficient mechanisms be put in place to manage these networks. The design and control of networks with the added dimension of renewable energy makes the problem of managing these networks substantially different from their non-replenishment counterparts. For example, in the case of networks without replenishment, having a battery that is close to being full is a desirable feature. However, in the case of a network with replenishment, a full battery means that there can be no replenishment. Hence this is a missed opportunity to utilize the replenishment energy, resulting in a lower overall performance. This is further exacerbated by the fact that, in practice, battery replenishment rates are the highest when battery levels are low, which means that determining the right balance between being disconnected and achieving high throughput is a complex issue. Moreover, while there has been significant prior work on sensor networks that has explicitly or implicitly optimized the network lifetime (often defined as the time when the first node runs out of energy) new metrics of performance are required for networks with replenishment. For example, with appropriately chosen data sampling rate and routes, the lifetime of network with replenishment could be made infinite, and metrics, such as the throughput or some function of the throughput are more relevant in such scenarios. Thus, a new resource management paradigm needs to be developed to optimize the performance of sensor networks with energy replenishment. For networks with replenishment, conservative energy expenditure may lead to missed recharging opportunities due to battery capacity limitations, and aggressive usage of energy may lead to lack of coverage or connectivity for certain time periods, which could hurt the applications' requirements. Thus, new techniques and protocols must be developed to balance these seemingly contradictory goals to maximize sensor network performance by jointly allocating energy and other resources such as bandwidth, rate and routes.

There are numerous works in both theoretical and systems oriented research on rechargeable networks.[4–24] In this chapter, we focus on some of these key papers.[4–8] Many fundamental wireless communication and networking problems can be stated as utility maximization problems subject to resource (e.g., energy and bandwidth, rate and routes) constraints. The utility functions can be the throughput, sum rate or sum of concave shaped rate etc. These problems have been widely addressed for networks without renewable energy sources. We focus on the development of efficient resource allocation schemes in the rechargeable counterpart. In static resource allocation, decisions are usually made only once at the beginning of a long period, while in dynamic resource allocation, decisions are made in each time slot. In,[4] a convex optimization problem is formulated to fairly allocate resource while incorporating the renewable energy constraint, i.e., the long term energy consumption rate should be less than the long term replenishing rate. However, for applications with rechargeable battery, ensuring energy consumption rate being less than replenishing rate is not enough. In addition, the allocated energy in each time slot should not be larger than the available amount of energy in the battery. Dynamic formulation of utility maximization problems that make resource allocation decisions in each time slot are formulated in.[6–8] In,[5] the problem of how to efficiently route packets through rechargeable network is studied.

2. Static Resource Allocation with Renewable Energy

Similar to the non-renewable counterpart, static resource allocation problems are also studied in rechargeable networks. In static resource allocation, the allocation decision is made over a long period or at the beginning of an event, rather than updated in each time slot.

2.1. *Rate, Bandwidth and Flow Allocation with Renewable Energy*

We denote a directed wireless network by $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{L}$ is the set of directed links. We assume that our network is an acyclic sensor network with one sink node $s \in \mathcal{N}$ to be constructed for data collection as shown in Fig. 1. Similar to the non-rechargeable network counterpart,[25–27] we formulate[4] a utility maximization problem as a convex optimization problem while constructing the data collection network. For each node $n \in \mathcal{N}$, let A_n, D_n, C_n and P_n denote its ancestors, descendants, children and parent, respectively. The energy cost for sensing the environment at node n is represented by E_n^{sen}. We assume that the expected number of retransmissions over a long period is known for each link. Let $E_{n,m}^{tran}$ represent the energy cost for delivering a packet over link (n, m) and E_n^{rec} represent the energy cost for receiving a packet at node n. We consider a time slotted system and the time during a day is broken into multiple time intervals called epochs. The length of each epoch is T slots. We use $r_n(e)$ to represent the average (long term) energy replenishment rate of node n in epoch e, while $r_n(t)$ is used to represent the real instantaneous (short term) energy replenishment rate of node n in time slot t. For each epoch e, $r_n(e) = \frac{1}{T}\sum_{t=(e-1)T+1}^{eT} r_n(t)$. We assume that $r_n(e)$ can be estimated by each node with high accuracy. Moreover, we define $f_{n,m}$ to be the fractional outgoing traffic of node n that passes through a parent node m and $z_{n,m}(\vec{f})$ to be the fractional outgoing traffic of node n that passes through an ancestor m, where $\vec{f}$ is the vector of $f_{n,m}$, $\forall n, m$. Thus,

$f_{n,m} = 0$, $\forall m \neq P_n$ and $z_{n,m}(\vec{f}) = 0$, $\forall m \notin A_n$. The recursive relation between the two variables is $z_{n,m}(\vec{f}) = \sum_{k \in P_n} f_{n,k} z_{k,m}(\vec{f})$. The capacity of link (n,m) is represented by $c_{n,m}$.

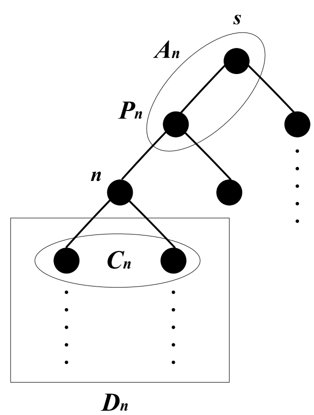

Fig. 1. Sensor Network Model.

We define the utility function $U_n(R_n)$ for node n to be $\log(R_n)$ where R_n is the sampling rate of node n. Therefore, for each epoch e, the utility optimization problem can be formulated as the following strictly concave maximization problem:

$$P(2.1) \qquad \max_{\vec{R},\vec{c},\vec{f}} \sum_n \log(R_n)$$

$$s.t. \qquad E_n^{sen} R_n + \sum_{m \in D_n} z_{m,n}(\vec{f}) E_n^{rec} R_m + \sum_{m \in P_n} E_{n,m}^{tran} c_{n,m} \leq r_n(e), \tag{1}$$

$$R_n + \sum_{m \in D_n} z_{m,n}(\vec{f}) R_m \leq \sum_{m \in P_n} c_{n,m}, \tag{2}$$

$$\vec{R} \succ \vec{0}, \quad \vec{c} \in \mathcal{C}, \quad \vec{f} \in \mathcal{F}. \tag{3}$$

Constraints 1 and 2 are the energy conservation and flow balance constraints, respectively. The flow balance constraint states that the sum of allocated capacity for each outgoing link should be greater than the total amount of traffic going through each node, including its own data. Besides these two constraints, the amount of capacity allocated on each link must be in the feasible capacity region $\mathcal{C}$. $\mathcal{F}$ is the feasible region of the routing variables $\vec{f}$. We assume the node exclusive interference model as in.[25] Thus, the feasible capacity region can be similarly defined as the convex hull of all the rate vectors of the matchings in $\mathcal{G}$.

Note that within epoch e, P(2.1) is a static problem. The standard method to solve this static problem involves the application of the dual decomposition and the subgradient methods. However, the implementation of these solutions in the network involves a large overhead due to message exchange between neighboring nodes. Consequently, the convergence time becomes an important issue. To that end, we introduce QuickFix, which, in each iteration of the subgradient method, exploits the special structure of directed acyclic network to form an efficient control message exchange scheme. This scheme is motivated by the general solution structure of a dynamic program. QuickFix is based on the hierarchical decomposition approach as the starting point. By relaxing the energy conservation and flow balance constraints in Problem P(2.1), we get the partial dual function $q(\vec{\mu}, \vec{v})$ as

$$\begin{aligned} q(\vec{\mu}, \vec{v}) = &\max_{\vec{R},\vec{c},\vec{f}} \sum_n \log(R_n) + \sum_n \mu_n \Big(r_n(e) - E_n^{sen} R_n - \sum_{m \in D_n} z_{m,n}(\vec{f}) E_n^{rec} R_m \\ &- \sum_{m \in P_n} E_{n,m}^{tran} c_{n,m} \Big) + \sum_n v_n \Big(\sum_{m \in P_n} c_{n,m} - R_n - \sum_{m \in D_n} z_{m,n}(\vec{f}) R_m \Big) \\ &s.t. \quad \vec{R} \succ \vec{0}, \quad \vec{c} \in \mathcal{C}, \quad \vec{f} \in \mathcal{F} \end{aligned}$$

The dual problem is then $\min_{\vec{\mu} \succeq \vec{0}, \vec{v} \succeq \vec{0}} q(\vec{\mu}, \vec{v})$. Since the dual function is not differentiable, the subgradient method[28] is applied to iteratively update the Lagrange Multipliers $\vec{\mu}$ and $\vec{v}$ at each node using:

$$\begin{aligned} \mu_n^{(i+1)} = &\Big[\mu_n^{(i)} - s_{\mu_n} \Big(r_n(e) - E_n^{sen} R_n - \sum_{m \in D_n} z_{m,n}(\vec{f}) E_n^{rec} R_m \\ &- \sum_{m \in P_n} E_{n,m}^{tran} c_{n,m} \Big) \Big]^+ \\ v_n^{(i+1)} = &\Big[v_n^{(i)} - s_{v_n} \Big(\sum_{m \in P_n} c_{n,m} - R_n - \sum_{m \in D_n} z_{m,n}(\vec{f}) R_m \Big) \Big]^+ \end{aligned} \tag{4}$$

where the notation $[\cdot]^+$ means projection to the positive orphan of the real line and s_{μ_n} and s_{v_n}, $\forall n$ are stepsizes.

The primal problem can be decomposed into the following subproblems:

$$\max_{R_n,\vec{f}} \quad \log(R_n) - \mu_n\Big(E_n^{sen}R_n - \sum_{m\in D_n} z_{m,n}(\vec{f})E_n^{rec}R_m\Big) - v_n\Big(R_n + \sum_{m\in D_n} z_{m,n}(\vec{f})R_m\Big)$$

$$s.t. \quad R_n > 0, \quad \vec{f} \in \mathcal{F} \tag{5}$$

$$\max_{\vec{c}} \quad \sum_{(n,m)\in\mathcal{L}} \Big(v_n - E_{n,m}^{tran}\mu_n\Big)c_{n,m}$$

$$s.t. \quad \vec{c} \in \mathcal{C} \tag{6}$$

In QuickFix, the subproblem in (6) is equivalent to the maximum weighted matching problem. Under the node exclusive interference model, it can be solved in polynomial time. However, in order to solve the maximum weighted matching problem in a distributed fashion, we utilize the heuristic algorithm in.[29] While applying the algorithm, instead of the queue difference between neighboring nodes, we use the combined energy and queue state of a node $(v_n - E_{n,m}^{tran}\mu_n)$ to modulate the MAC contention window size, when a node n attempts to transmit a packet over the link (n, m).

Since the objective function in (5) of QuickFix is strictly concave in $(\vec{R}, \vec{f})$, the unique maximizer satisfies:

$$R_n^* = \frac{1}{E_n^{sen}\mu_n + v_n + \sum_{m\in A_n} z_{n,m}(\vec{f}^*)\big(E_m^{rec}\mu_m + v_m\big)}. \tag{7}$$

We refer to $E_n^{sen}\mu_n + v_n$ in Equation (7) as node n's local price, and $W_n = \sum_{m\in A_n} z_{n,m}(\vec{f}^*)\ \big(E_m^{rec}\mu_m + v_m\big)$ as its aggregate price. Observe that if a node wants to maximize its rate, it should find the path such that its aggregate price W_n is minimized, i.e., it is a joint routing and rate control problem. Since our formulation utilizes the directed acyclic structure, this allows a node to calculate its aggregate price recursively from those of its parents as stated in Proposition 1. Furthermore, Proposition 1 implies that a node should select the parent with the minimum sum of local and aggregate prices as its relay node in the directed acyclic topology. This motivates the following distributed routing and rate control algorithm. Each node collects the local and aggregate prices from all its parents and selects the parent with the minimum sum of the local and aggregate prices as the relay node while constructing the directed acyclic network. Then, each node uses Equation (8) to calculate its own aggregate price and then applies Equation (7) to determine its rate. Having determined the local rate and the outgoing link to use, a node distributes its local and aggregate prices to its children, so that the children nodes can determine their routes and rates. This process continues until the leaf nodes are reached. Now, starting from the leaf nodes, each node reports its aggregate traffic to its parents, so that the parent nodes can have the needed information to update their local prices. Aggregate traffic F_n of node n is the total amount of traffic generated by the descendants that goes through node n. Similar to the computation of the aggregate price, a node can compute its aggregate traffic recursively using Equation (9).

Proposition 1. [4] *The aggregate price W_n can be recursively computed as*

$$W_n = \sum_{m \in P_n} f_{n,m} \Big(\Big(E_m^{rec} \mu_m + v_m \Big) + W_m \Big) \tag{8}$$

Proposition 2. [4] *The aggregate traffic F_n of node n can be recursively computed as*

$$F_n = \sum_{k \in C_n} f_{k,n} \Big(R_k + F_k \Big) \tag{9}$$

Fig. 2 compares the convergence time of QuickFix with the standard dual-based algorithm when a fixed $r_n(e)$ is given. Here, QuickFix is run for a directed acyclic network of 67 nodes. The improvement in convergence rate with QuickFix relative to the standard dual-based solution is apparent.

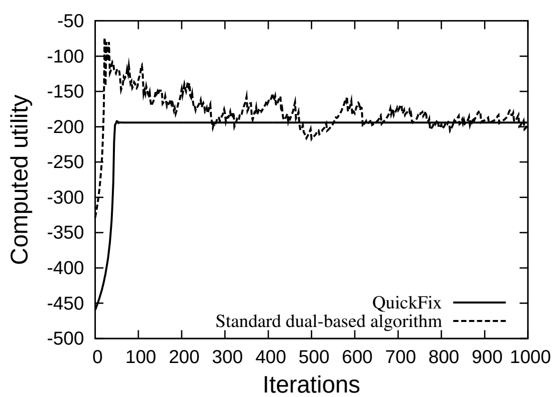

Fig. 2. The convergence time comparison between QuickFix and the standard dual-based algorithm.

2.2. *Energy-Aware Routing With Renewable Energy*

We have thus far focused on exploiting the harvested energy to improve the achievable utility of link rates in the network. An important question that[5] attempts to answer is how to efficiently route packets through the network to improve the overall performance. In this section, $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ stands for a directed wireless multihop network. Data packets arrive into the network and the algorithm developed in[5] decides whether to accept or reject them based on the energy resource on potential routes. This is an event driven problem, i.e., decisions are made every time a packet arrives into the network. The energy resource and route are allocated statically for a given packet upon its acceptance.

A 2-tuple $(E^{tran}_{n,m}, E^{rec}_{n,m})$ is associated with each edge $(n,m) \in \mathcal{L}$, where $E^{tran}_{n,m}$ is the transmission energy cost and $E^{rec}_{n,m}$ is the reception energy cost. More precisely, if a data packet of length l is sent directly from node n to node m, an amount of energy equal to $lE^{tran}_{n,m}$ will be subtracted from the residual energy of node n, and $lE^{rec}_{n,m}$ will be subtracted from the residual energy of node m. For simplicity, we assume that the size of a control packet is negligible compared to the size of a data packet. We define the unit energy cost of node n on path $\mathcal{P}$ as $E_n(\mathcal{P}) = E^{rec}_{n''(\mathcal{P}),n} + E^{tran}_{n,n'(\mathcal{P})}$, $\forall n \in \mathcal{P}$ where nodes $n''(\mathcal{P})$ and $n'(\mathcal{P})$ are the upstream and downstream neighbors of node n in path $\mathcal{P}$, respectively.

We consider a discrete time system in which each node begins with a fully charged battery that has a capacity of B_n. At the end of each time slot t, $q^b_n(t)$ is the residual energy at node n. Each node falls in one of the two categories depending on whether a renewable energy source is attached to it. We use $\mathcal{N}_r$ to denote the set of nodes with energy replenishment, and $\mathcal{N}_p$ to denote the set of nodes with no energy replenishment. At the beginning of time slot t, node $n \in \mathcal{N}_r$ receives the energy accumulated due to replenishment in the previous time slot, represented by $r_n(t-1)$. At all times, the maximum energy at node n is not allowed to exceed B^b_n. Data packet routing requests arrive to the network sequentially, the j-th of which can be described as

$$\beta_j = (s_j, d_j, l_j, t^s_j, \rho_j) \tag{10}$$

where s_j is the source node of the j-th packet routing request, d_j is the destination, l_j is the packet length, t^s_j is the arrival time of the request, and finally ρ_j is the revenue gained by routing this packet through the network. A request can be accepted only if there is at least one feasible path (that is, each node along the path must have at least $l_j E_n(\mathcal{P}_j)$ amount of residual energy) in the system when the request arrives. If the routing request is accepted and $\mathcal{P}_j$ is the route used to accommodate the request, then $l_j E_n(\mathcal{P}_j)$ will be the amount of energy expenditure at node n for $n \in \mathcal{P}_j$. We also assume that the reduction of energy is instantaneous for all the nodes along the path since the time-scale of energy replenishment is usually much larger than the time-scale of packet forwarding. In other words, we assume the delay due to packet transmission, queueing, etc., is negligible compared to the time it takes to replenish the energy consumption of transmitting/receiving one packet. For any node $n \in \mathcal{N}_r$, the energy model can be summarized by the following equation

$$\begin{aligned} q_n^b(t) = & \min[q_n^b(t-1) + r_n(t-1), B_n^b] \\ & - \mathit{1}_{[\beta_j \text{ is accepted at time } t \text{ and } n \in \mathcal{P}_j]} l_j E_n(\mathcal{P}_j), \ \forall n \in \mathcal{N}_r \end{aligned}$$

It is assumed that each node has an accurate estimate of its own short term energy replenishment process. More precisely, at time slot t, node n knows $r_n(t), r_n(t+1), \ldots, r_n(\hat{t}_n)$, where $\hat{t}_n$ is the earliest time the battery at node n would be fully recharged if no request were accepted at or after time t. It is worth noting that $\hat{t}_n$ is dependent on the residual energy of node at the arrival time of a request. In practice, this type of short term prediction can be easily implemented. We also assume that $\hat{t}_n$ is finite for $n \in \mathcal{N}_r$. More specifically, we denote $T < \infty$ as an upper bound on the time it takes to fully charge the empty battery at any given node $n \in \mathcal{N}_r$. For any node n in $\mathcal{N}_p$, the corresponding energy model can be written as

$$q_n^b(t) = q_n^b(t-1) - \mathit{1}_{[\beta_j \text{ is accepted at time } t \text{ and } n \in \mathcal{P}_j]} l_j E_n(\mathcal{P}_j), \ \forall n \in \mathcal{N}_p$$

Our goal is maximize the total revenue over some finite horizon $[0, \tau]$

$$J_\tau = \sum_{j:j \le k_\tau} \rho_j \mathit{1}_{[\beta_j \text{ is accepted}]} \tag{11}$$

where k_τ is the index of the last arrival in the time interval $[0, \tau]$.

The basic idea of our algorithm is to assign a cost to each node, which is an exponential function in its residual energy and then use shortest path routing with respect to this metric. To account for the timing relationship between the energy consumption and replenishment, we also need to measure the impact of previously accepted requests.

For any node n with renewable energy source, i.e., $n \in \mathcal{N}_r$, we begin by defining a set of parameters to describe the effect of previously accepted routing requests when considering the new request β_j. More specifically, let

$\Delta t_n(j)$ be the amount of time it takes for the incoming energy, accumulated from time slot t^s_{j-1}, to equal $B^b_n - q^b_n(t^s_{j-1})$. We define $\hat{t}_n(j) = t^s_{j-1} + \Delta t_n(j)$ as the earliest time the battery at node n would be fully recharged if no request were accepted after request β_{j-1}. It can also be written as

$$\hat{t}_n(j) = \min_{t \geq t^s_{j-1}} \left[t : \sum_{i=t^s_{j-1}}^{t-1} r_n(i) \geq \left(B^b_n - q^b_n(t^s_{j-1})\right) \right].$$

To characterize the energy consumption due to previous packets, we define the new power depletion index

$$\lambda_n(j,t) = \begin{cases} 0, & t \geq \hat{t}_n(j) \\ \lambda_n(k_t, t), & t < t^s_{j-1} \\ \frac{B^b_n - q^b_n(t^s_{j-1}) - \sum_{i=t^s_{j-1}}^{t-1} r_n(i)}{B^b_n}, & \textit{othwewise} \end{cases}$$

where $k_t = \max[j : t^s_{j-1} \leq t]$. In fact, $\lambda_n(j,t)$ is the fraction of energy consumed due to $\{\beta_1, \beta_2, \ldots, \beta_{j-1}\}$ at node n, as measured at time t. Note that new routing requests (with index greater than $j-1$) can arrive at or before time t, but their energy consumption will not be included in the calculation of $\lambda_n(j,t)$. There are three cases in the above definition.

- $t \geq \hat{t}_n(j)$: By definition of $\hat{t}_n(j)$, $\lambda_n(j,t)$ should be zero at or after time $\hat{t}_n(j)$.
- $t^s_{j-1} \leq t \leq \hat{t}_n(j)$: In this case, part of the energy consumption has been restored.
- $t < t^s_{j-1}$: In this case, the time slot t is before the arrival time of request β_{j-1}; hence, it is almost meaningless to talk about the energy consumption of $\{\beta_1, \beta_2, \ldots, \beta_{j-1}\}$ at time t. For preciseness, we define $\lambda_n(j,t)$ in this case to be $\lambda_n(k_t, t)$, where k_t is the largest request index j such that $\lambda_n(j,t)$ is "meaningful."

Fig. 3 shows the amount of energy at node n assuming that no request is accepted after request $j-1$. In reality, it is conceivable that only a fraction of the last replenishment is received by the node, due to limited battery capacity. This is taken into account in the definition of $\lambda_n(j,t)$.

For any node n with no renewable energy source, i.e., $n \in \mathcal{N}_p$, the power depletion index $\lambda_n(j)$ is defined as $\lambda_n(j) = 1 - \frac{q^b_n(t^s_j - 1)}{B^b_n}$, where $q^b_n(t^s_j - 1)$ is the residual energy at node n when considering request β_j.

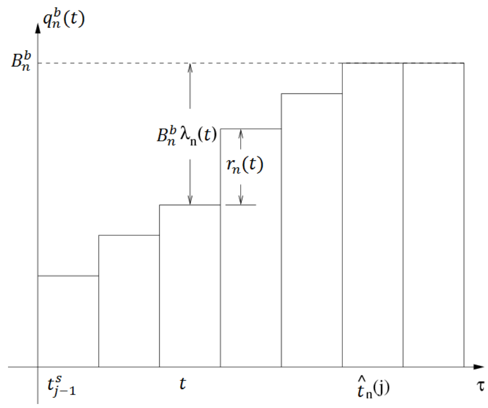

Fig. 3. Amount of energy at node n assuming that no request is accepted after request $j-1$.

We now define our routing metric used on each node as

$$C_n(j,\mathcal{P}) = \begin{cases} \sum_{t=t_j^s}^{\hat{t}_n(j)-1} (\mu^{\lambda_n(j,t)} - 1) l_j E_n(\mathcal{P}_j), & n \in \mathcal{N}_r \\ T(\mu^{\lambda_n(j)} - 1) l_j E_n(\mathcal{P}_j) & n \in \mathcal{N}_p \end{cases} \quad (12)$$

where μ is a constant to be defined later, and $\mathcal{P}$ is a path from s_j to d_j. We recall that $T < \infty$ as an upper bound on the time it takes to fully charge the empty battery at any given node $n \in \mathcal{N}_r$. The main change in the definition of the node cost metric for $n \in \mathcal{N}_r$ is to take into account the replenishment sample in the immediate future. Again, the cost associated with $\mathcal{P}$ when considering request β_j will be calculated as $Cost_{\mathcal{P}}(j) = \sum_{n\in\mathcal{P}} C_n(j,\mathcal{P})$.

Our proposed algorithm E-WME (Energy-opportunistic Weighted Minimum Energy) simply check if the least cost route $\mathcal{P}$ from s_j to d_j satisfies $Cost_{\mathcal{P}}(j) \le \rho_j$ for an incoming routing request j. If yes, accept the request and route he packet on the least cost route. Otherwise, reject the request.

It is worth noting that the admission control of routing requests is done in an energy-opportunistic fashion. Again, we turn to the example of a sensor network powered by solar cells. Let us assume that a request arrives at the network right after sunset. Recall our assumption that each node knows its short-term energy replenishment schedule. At this moment, each node knows that the energy replenishment rate will be much smaller for the several hours to come (in practice, this type of knowledge can be gained by evaluating the energy replenishment schedule over the past few days). The $\hat{t}(\cdot)$ calculated will then be relatively large, so the cost of routing the packet will be higher than that during the daytime. As compared to its daytime policy, the network is, thus, more conservative in accepting the request, which is precisely what the network should do in this particular scenario. In a hybrid network where both kinds of nodes are present, we look at two nodes: one with energy replenishment and one without. Assuming that they both have the same residual energy and that the routing request takes the same communication costs from them, it is clear that the cost metric for the node with energy replenishment is smaller. Therefore, this node is more likely to be used than the one without energy replenishment.

We then show that E-WME is an online algorithm with asymptotically optimal competitive ratio. The competitive ratio is defined as $\sup_\tau \sup_{all\ input\ sequences\ in\ [0,\tau]} \frac{J_{\tau,off}}{J_{\tau,on}}$, where $J_{\tau,off}$ is the performance achievable by any offline algorithm and $J_{\tau,on}$ is the performance of the given online algorithm, where the performance is defined in Equation (11). A competitive ratio of r means that the performance of the online algorithm is at least $\frac{1}{r}$ that of any offline algorithm. In other words, a smaller competitive ratio means better performance.

We need the following two assumptions:

$(A1)$

$$1 \le \frac{1}{H} \cdot \frac{\rho_j}{l_j E_n(\mathcal{P}_j) T} \le F, \quad \forall n \in \mathcal{P}_j$$

$(A2)$

$$l_j E_n(\mathcal{P}_j) \le \frac{B_n^b}{\log \mu}, \quad \forall n \in \mathcal{P}_j$$

where $\mathcal{P}_j$ is the path chosen by either the online or the offline algorithm to route β_j, H is the maximum hop count allowed for any path, F is a constant chosen large enough to satisfy (A1), $T < \infty$, as defined before, is an upper bound on the time it takes to fully charge the empty battery at any given node $n \in \mathcal{N}_r$, and $\mu = 2(HFT + 1)$. Assumption (A1) requires that the revenue from a packet scales with the amount of resource it requests. This is quite reasonable and certainly agrees with the definition of revenue as throughput or weighted throughput. Assumption (A2) guarantees that the energy claimed by a packet is not larger than a certain fraction of the total energy available at any single node. Under assumptions (A1) and (A2), we have the following theorem.

Theorem 1. [5] ***Asymptotic Optimality of the E-WME Algorithm:*** *(I) The E-WME algorithm has a competitive ratio upper bounded by* $O(\log(|\mathcal{N}|))$*, where* $|\mathcal{N}|$ *is the number of nodes in the network.*

(II) The competitive ratio of any online routing scheme is lower bounded by $\Omega(\log(|\mathcal{N}|))$.
From (I) and (II), our algorithm is asymptotically optimal.

Fig. 4 shows the throughput comparison between our E-WME algorithm and other routing algorithms in the literature. It can be seen that the E-WME routing always has better throughput than the other routing algorithms*. The two main reasons are that E-WME is optimal in the sense of minimizing the competitive ratio, and that it strikes the right balance between saving communication cost and distributing the load. These characteristics are not present in other power aware routing algorithms.

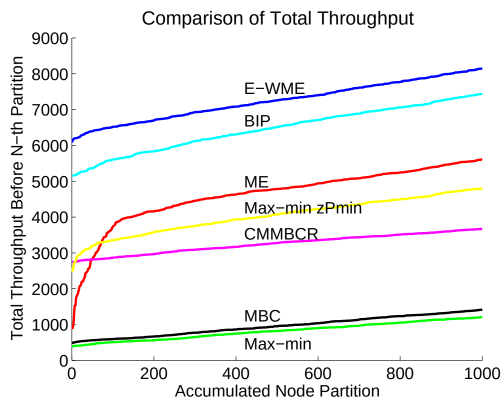

Fig. 4. Throughput comparison of E-WME to other schemes.

3. Dynamic Resource Allocation with Renewable Energy

For applications with rechargeable batteries, ensuring energy consumption rate being less than replenishing rate as formulated in Section 2.1 is not enough. In addition, as shown in Fig. 5, the allocated energy $P(t)$ in each time slot t should not be larger than the available amount of energy $q^b(t)$ in the battery with size B^b and replenishment $r(t)$, which requires a dynamic decision formulation.

*In fact the improvement in using E-WME will even be larger if replenishment rates chosen are higher.

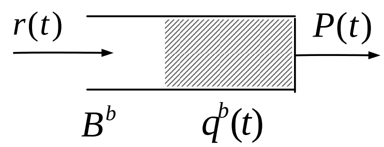

Fig. 5. Battery Queue Model.

3.1. *Basic Performance Limits and Tradeoffs with Renewable Energy*

In this section, we explain how the authors in[7] study performance limits and tradeoffs between data rate and battery outage with renewable energy under infinite time horizon. We consider a single rechargeable node with battery size B^b. We denote the available energy in the battery as $q^b(t)$ at time t. The battery replenishes at a rate $r(t)$. The process $\{r(t),\ t \geq 1\}$ is assumed to be an ergodic stochastic process with a long term mean $\lim_{T\to\infty} \sum_{t=1}^{T} r(t) \xrightarrow{a.s.} \bar{r}$. An energy management scheme $\mathcal{S}$ draws energy from this battery at a rate $P^{\mathcal{S}}(t)$ to achieve certain tasks. The success of the node in achieving these tasks is measured in terms of a utility function $U(P^{\mathcal{S}}(t))$ of the consumed energy $P^{\mathcal{S}}(t)$. We assume $U(x)$ to be a concave, non-decreasing and analytic function of x over $x \geq 0$. We define the time average utility,

$$\bar{U}^{\mathcal{S}}(T) = \frac{1}{T}\sum_{t=1}^{T} U(P^{\mathcal{S}}(t)). \tag{13}$$

We consider the optimization problem in which a node tries to maximize its long-term average utility, $\bar{U}^{\mathcal{S}} = \liminf_{T\to\infty} \bar{U}^{\mathcal{S}}(T)$, subject to battery constraints

$$\begin{aligned} &P(3.1.1) \qquad \max_{\{P^{\mathcal{S}}(t),\ t\geq 1\}} \bar{U}^{\mathcal{S}} \\ &s.t. \qquad q^b(t) = \min[B^b, q^b(t-1) + r(t) - P^{\mathcal{S}}(t-1)], \\ &\qquad\qquad P^{\mathcal{S}}(t) \leq q^b(t). \end{aligned}$$

One approach to solving this optimization problem is by using Markov decision process (MDP) techniques. Since solving MDPs is computationally intensive, these methods may not be suitable for computationally-limited sensor nodes. Consequently, we seek schemes that are easy to implement and yet achieve close to optimal performance. The next lemma gives an upper bound for the asymptotic time-average utility achieved over all ergodic energy management policies.

Lemma 1. [7] *Let $\bar{U}^{\mathcal{S}^*}$ be the optimal objective value to P(3.1.1). Then, $\bar{U}^{\mathcal{S}^*} \leq U(\mu)$.*

The proof of Lemma 1 uses Jensen's inequality and conservation of energy arguments. Lemma 1 tells us that for any ergodic energy management scheme $\mathcal{S}$, $\bar{U}^{\mathcal{S}} \leq U(\bar{r})$. With an unlimited energy reservoir (i.e., $B^b = \infty$) and average energy replenishment rate μ, if one uses $P^{\mathcal{S}}(t) = \bar{r}$ for all $t \geq 1$, this upper bound can be achieved. However, if $B^b < \infty$, achieving $\bar{U}^{\mathcal{S}} = U(\bar{r})$ using this simple scheme is not possible. Indeed, due to finite energy storage and variability in $r(t)$, the battery will occasionally get discharged completely. At such instances, $P^{\mathcal{S}}(t)$ has to be set to 0, which will reduce the time-average utility. The question we answer is, "how close can the average utility $\bar{U}^{\mathcal{S}}$ get to the upper bound asymptotically, as $B^b \to \infty$, while keeping the long-term battery discharge rate low?"

We will show that there is a trade-off between achieving maximum utility and keeping the discharge rate low. First, we make some weak assumptions on the replenishment process $r(t)$. In particular, we assume that the asymptotic semi-invariant log moment generating function,

$$\bar{\Lambda}_r(s) = \lim_{T\to\infty} \frac{1}{T} \log E\left[\exp\left(s\sum_{t=1}^{T} r(t)\right)\right], \tag{14}$$

of $r(t)$ exists for $s \in (-\infty, s_{max})$, for some $s_{max} > 0$. We also assume that the asymptotic variance $\bar{\sigma}_r^2 \triangleq \lim_{T\to\infty} \frac{1}{T} var(\sum_{t=1}^{T} r(t))$ exists[†]. Note that, in practice, the recharging process is not necessarily stationary. While this assumption does allow the possibility that the statistics of $r(t)$ has variations (e.g., due to clouds and the solar power at different times of the day), it rules out the possibility of long-range dependencies in $r(t)$.

From previous discussion, we can infer that by choosing a battery drift, defined as $r(t) - P(t-1)$, that goes to zero with increasing battery size, one might achieve a long-term average utility that is close to $U(\bar{r})$ as B^b increases. However, smaller drift away from the empty battery state implies a more frequent occurrence of the complete battery discharge event. In the following theorem, we quantify this tradeoff between the achievable utility and the battery discharge rate, asymptotically in the large battery regime. In this regime, the battery size B^b is large enough for the variations in $r(t)$

[†]Examples of valid processes include the following. 1) Any i.i.d. process with a sample distribution that has finite moments of all orders; 2) All Gaussian processes with an auto-covariance function that has a finite integral; 3) The process obtained by adding a deterministic periodic function of time (to mimic the daily cycles of solar radiation) to the aforementioned processes in 1), 2).

to average out nicely over the time scale that $q^b(t)$ changes significantly. Consequently, we now define the long-term battery discharge rate as the probability of discharge, i.e., $p^o(B^b) \triangleq \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} 1^o(t)$, where the indicator $1^o(t) = 1$ if $q^b(t) = 0$ and is identical to 0 otherwise.

Next, we show that one can achieve a battery discharge probability that exhibits a polynomial decay of arbitrary order with the battery size, and at the same time achieves a utility that approaches the maximum achievable utility as $(\log B^b)^2/(B^b)^2$.

Theorem 2. [7] *Consider any continuous, concave, nondecreasing, and analytic utility function $U(P(t))$ over the nonnegative real line such that $|\frac{\partial^2 U(x)}{\partial x^2}| < \infty$ for all $x > 0$. Given any $\beta \geq 2$, there exists an energy management scheme $\mathcal{B}$ such that the associated battery discharge probability $p^{o,\mathcal{B}}(B^b) = \Theta((B^b)^{-\beta})$ and $U(\bar{r}) - \bar{U}^{\mathcal{B}} = \Theta((\frac{\log B^b}{B^b})^2)$*

Consider the allocation scheme $\mathcal{B}$ in which

$$P^{\mathcal{B}}(t) = \begin{cases} \min[\bar{r} - \delta^{\mathcal{B}}, q^b(t)], & q^b(t) < \frac{B^b}{2} \\ \bar{r} + \delta^{\mathcal{B}}, & q^b(t) \geq \frac{B^b}{2} \end{cases} \tag{15}$$

for some $\delta^{\mathcal{B}} > 0$. As shown in Fig. 6 (a), the instantaneous utility associated with Scheme $\mathcal{B}$ alternates between U^- and U^+, depending on the battery state. By choosing $\delta^{\mathcal{B}} = \beta\bar{\sigma}_r^2 \frac{\log B^b}{B^b}$ for some $\beta \geq 2$, we show that long-term maximum utility $U(\bar{r})$ can be achieved asymptotically while achieving decay, as a polynomial of arbitrarily high order, for the battery discharge probability. We note that while the order of the polynomial decay β can be made arbitrarily large, it comes at the expense of slower convergence (by some constant factor) to the maximum utility.

Here, we illustrated that with a simple scheme, it is possible to achieve desirable scaling laws for the performance of a given task, under the assumption that the asymptotic moment generating function of the replenishment process exists. To illustrate the theorem we consider a specific example.

Example 1. Achievable Rate in a Gaussian Channel: We study the basic limits of point to point communication with finite but replenishing energy stores. For simplicity, we consider the static Gaussian channel. At time t, the transmitter transmits a complex valued block (vector of symbols) $X(t)$ of unit power and the receiver receives $Y(t)$. We have,

$$Y(t) = hX(t) + W(t), \tag{16}$$

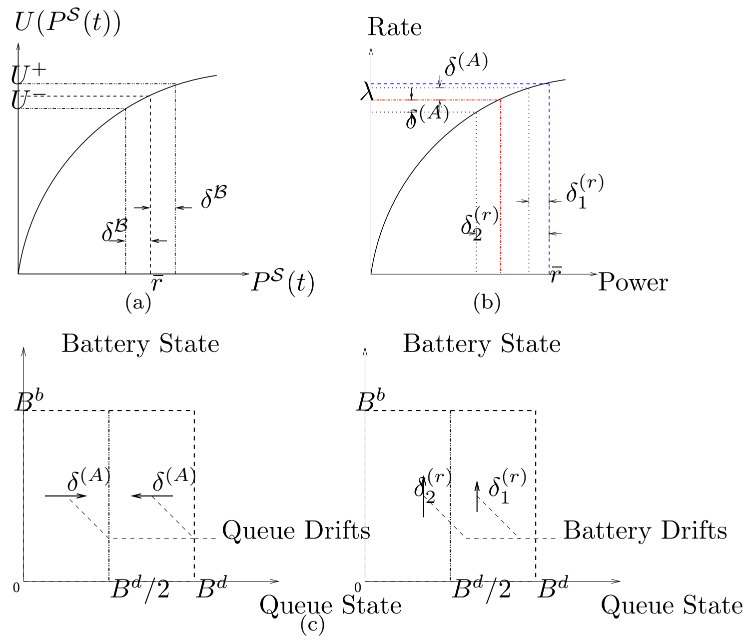

Fig. 6. (a) With scheme $\mathcal{B}$, utility alternates between U^+ and U^-, (b) Relationship between $\delta^{(A)}, \delta_1^{(r)}$ and $\delta_2^{(r)}$, and (c) Drifts of the data queue and battery state for scheme $\mathcal{Q}$

where the channel gain h is a complex constant and $W(t)$ is additive Gaussian noise with sample variance N_0. We define the channel SNR as $\gamma \triangleq \frac{|h|^2}{N_0}$. The maximum amount of data that could be reliably communicated over this channel with an amount of energy $P(t)$ at time t is

$$\mu(P(t)) = \log_2(1 + P(t)\gamma) \text{ bits/channel use,} \tag{17}$$

assuming the block size is long enough so that sufficient averaging of additive noise is possible. Thus, the rate at which reliable communication can be achieved at a given block is a concave non-decreasing function of the transmit power and it can be viewed as our utility function. Consequently, using a constant power $\bar{r}$, the maximum utility of $\bar{\mu} = \mu(\bar{r})$ can be achieved, which is the famous Gaussian channel capacity result. Clearly, the capacity is possibly achievable, only if the energy store is infinite.

With an energy store that is not capable of providing power at a constant rate (e.g., an energy replenishing battery), one may observe outages due to occurrences of complete discharge at times. Thus, for such stores, it is not possible to achieve the aforementioned Gaussian channel capacity. However, we can show that, using our simple energy management scheme, one can achieve an average rate that converges to the capacity at an outage probability that converges to zero asymptotically as $B^b \to \infty$. We assume that each time slot is large enough for sufficiently long code blocks to be formed.

We simply substitute $U(\cdot)$ with $\mu(\cdot)$ in Equation (13) to get the relevant optimization problem. With an unlimited energy store ($B^b = \infty$) of limited average power $\bar{r}$, the maximum achievable long term average rate is identical to the channel capacity, i.e., $\mu(\bar{r}) = \log_2(1+\bar{r}\gamma)$ bits/channel use. By using the energy management scheme $\mathcal{B}$ given in Equation (15), an average rate $\bar{\mu}^{\mathcal{B}}$ can be achieved such that $\mu(\bar{r}) - \bar{\mu}^{\mathcal{B}} = \Theta(\frac{(\log B^b)^2}{(B^b)^2})$ while the battery discharge (i.e., the outage) probability follows $p^{o,\mathcal{B}}(B^b) = \Theta((B^b)^{-\beta})$ for any given $\beta \geq 2$.

To understand the strength of Theorem 2, we note that it is not trivial to achieve decaying discharge probability and maximum utility with increasing battery size. In fact, an ergodic[‡] energy management scheme cannot achieve exponential decay in discharge probability and convergence (even asymptotically) to the maximum average utility function simultaneously. We formalize this statement in the following theorem.

Theorem 3. [7] *Consider any continuous, concave and nondecreasing utility function $U(\cdot)$. If an ergodic energy management scheme $\mathcal{S}$ has a discharge probability $p^{o,\mathcal{S}}(B^b) = \Theta(\exp(-\alpha_c B^b))$ for some constant $\alpha_c > 0$, then the time average utility, $\bar{U}^{\mathcal{S}}$, for Scheme $\mathcal{S}$ satisfies $U(\bar{r}) - \bar{U}^{\mathcal{S}} = \Omega(1)$.*

We then extend the problem to the case when data packets arrive at a node and are kept in a finite buffer before transmission. Hence, the task is to transmit packets arriving at the data buffer without dropping them due to exceeding the buffer capacity. We define $q^d(t)$ as the data queue state at time t, and the data buffer size $B^d < \infty$. The data arrival process $A(t)$, represents the amount of data (in bits) arriving at the data buffer in the time slot t. The process $\{A(t),\ t \geq 1\}$ is an ergodic process independent of the energy replenishment process $\{r(t),\ t \geq 1\}$ and $E[A(t)] = \lambda$. We assume that the process $A(t)$ has a finite asymptotic variance $\bar{\sigma}_A^2 = \lim_{T\to\infty} \frac{1}{T} var(\sum_{t=1}^{T} A(t))$. The energy replenishment model is the same as used previously. We use $\mu(\cdot)$ as given in Equation (17) as the

[‡]An ergodic energy management scheme $P^{\mathcal{S}}(t)$ is the one that satisfies $\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} P^{\mathcal{S}}(t) = E[P^{\mathcal{S}}(t)]$

rate-power function (continuous, concave, non-decreasing, and analytic) for the wireless channel and assume that data is served at that rate as a function of the consumed energy $P(t)$ at time t. We also assume that $\lambda < \mu(\bar{r})$. Without this condition, there exists no joint energy and data buffer control policy that can simultaneously keep the long-term battery discharge and data loss rates arbitrarily low asymptotically, as $B^d, B^b \to \infty$.

The objective of an efficient energy management scheme in this case is to maximize the average utility function of the data transmitted subject to battery and data buffer constraints:

$$
\begin{aligned}
P(3.1.2) \qquad & \max_{P(t),\ \forall t \geq 1} \liminf_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} U_D(\mu(P(t))) \\
s.t. \quad & q^b(t) = \min[B^b, q^b(t-1) + r(t) - P(t-1)], \\
& q^d(t) = \min[B^d, q^d(t-1) + A(t) - \mu(P(t-1))], \\
& 0 \leq P(t) \leq q^b(t), \quad \mu(P(t)) \leq q^d(t).
\end{aligned}
$$

Here, $U_D(\mu(P))$ is a non-decreasing, concave, and analytic utility gained by transmitting $\mu(P)$ bits. Since $\lambda < \mu(\bar{r})$, we know that $U_D(\lambda)$ is an upper bound on the achievable long-term utility with any energy management scheme. Solution of P(3.1.2) jointly controls the data queue state and the battery state to avoid energy outage and data overflow while maximizing the utility. The main complexity in such an approach stems from the fact that the drifts of $q^d(t)$ and $q^b(t)$ are dependent. With this dependence, a critical factor one needs to take into consideration is the relative "size" of the data buffer with respect to the battery. In the sequel, we assume a large battery regime, which implies that, within the duration that some change occurs in $q^b(t)$, $q^d(t)$ may fluctuate significantly. Technically, for an Gaussian channel with an SNR γ, this assumption implies $B^b \gg \frac{1}{\gamma}(2^{\lambda} - 1)B^d$, i.e., the total amount of energy in the battery is much larger than that required to serve a full data buffer worth of packets. In the subsequent asymptotic results, in which both $B^d, B^b \to \infty$, the large battery regime implies the following. For all sequences of values, B_n^d, B_n^b, where both sequence goes to ∞ as $n \to \infty$, we assume $\frac{B_n^d}{B_n^b}$ as $n \to \infty$.

Intuitively, in large battery regime, an energy control algorithm should give "priority" to adjusting the queue state to achieve a high performance. Consequently, it should choose $P(t)$ such that the drift of $q^d(t)$ is always toward a desired queue state even though this may cause battery drift to be negative. Since battery size is large, such temporary negative drifts are expected to affect the battery discharge rate only minimally. With these observations, we state the following theorem, which indeed verifies our intuition. This theorem shows an asymptotic tradeoff between the achieved utility and the long-term rates of discharge and data loss as $B^d \to \infty$. In this regime, the data buffer size is large enough for the variations in $A(t)$ to average out over the time scale that $q^d(t)$ changes significantly. Consequently, we now define the long-term data loss rate as the data loss probability, i.e., $p^d(B^d) \triangleq \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} 1^d(t)$, where the indicator variable $1^d(t) = 1$ if $q^d(t) = B^d$ and is identical to 0 otherwise.

Theorem 4. [7] *Consider any non-decreasing concave utility function $U_D(\cdot)$ such that $|\frac{\partial^2 U_D(\mu(x))}{\partial x^2}| < \infty$ for all $x > 0$ and a rate-power function $\mu(\cdot)$, both of which are analytic in the non-negative real line. For any $\lambda < \mu(\bar{r})$, there exists some $\beta > 0$ for which an energy management scheme $\mathcal{Q}$ achieves a data loss probability $p^{d,\mathcal{Q}}(B^b) = O((B^d)^{-\beta})$, a battery discharge probability $p^{o,\mathcal{Q}} = O(\exp(-\alpha_{\mathcal{Q}} B^b))$ for some $\alpha_{\mathcal{Q}} > 0$ and a utility that satisfies $U_D(\lambda) - \bar{U}^{\mathcal{Q}} = \Theta(\frac{(\log B^d)^2}{(B^d)^2})$ under the large battery regime.*

Theorem 4 states that it is possible to have an exponential decay (with B^b) for the battery discharge probability and a polynomial decay (with B^d) for the data loss probability and at the same time achieve a time average utility that approaches the upper bound on the achievable long-term utility, $U_D(\lambda)$, as $\frac{(\log B^d)^2}{(B^d)^2}$. Note that $U_D(\lambda)$ can only be achieved with an infinite battery and data buffer sizes.

Consider the energy management scheme $\mathcal{Q}$, where

$$P^{\mathcal{Q}}(t) = \begin{cases} \min[\bar{r} - \delta_1^{(r)}, q^b(t)], & q^d(t) \geq \frac{B^d}{2} \\ \min[\bar{r} - \delta_2^{(r)}, q^b(t)], & q^d(t) < \frac{B^d}{2} \end{cases}, \tag{18}$$

and the drifts $\delta_1^{(r)}$ and $\delta_2^{(r)}$ are chosen to satisfy the relationship

$$\mu(\bar{r} - \delta_1^{(r)}) - \lambda = \lambda - \mu(\bar{r} - \delta_2^{(r)}) = \beta_{\mathcal{Q}} \bar{\sigma}_A^2 \frac{\log B^d}{B^d} \tag{19}$$

where $\beta_{\mathcal{Q}}$ is constant greater than 2. From Fig. 6 (b), we note that this choice of energy drifts correspond to a queue drift of $|\delta^{(A)}| = \beta_{\mathcal{Q}} \bar{\sigma}_A^2 \frac{\log B^d}{B^d}$, toward the state $\frac{B^d}{2}$, regardless of the queue state $q^d(t)$. The queue and battery drifts with scheme $\mathcal{Q}$ are illustrated in Fig. 6 (c). We observe that even though scheme $\mathcal{Q}$ regulates the data queue to a desired state (i.e., $\frac{B^d}{2}$), the battery is always regulated towards full state (i.e., B^b). State equation for $q^d(t)$ is given by,

$$q^d(t+1) = \begin{cases} \min[B^d, q^d(t) + A(t) - \lambda - \delta^{(A)}], & q^d(t) \geq \frac{B^d}{2} \\ \max[0, q^d(t) + A(t) - \lambda + \delta^{(A)}], & q^d(t) < \frac{B^d}{2} \end{cases}, \qquad (20)$$

and the state equation for $q^b(t)$ is given by,

$$q^b(t+1) = \begin{cases} (\min[B^b, q^b(t) + r(t) - \bar{r} + \delta_1^{(r)}])^+, & q^d(t) \geq \frac{B^d}{2} \\ (\min[B^b, q^b(t) + r(t) - \bar{r} + \delta_2^{(r)}])^+, & q^d(t) < \frac{B^d}{2} \end{cases}, \qquad (21)$$

where $(x)^+ = \max[0, x]$.

The following theorem quantifies tradeoff between the probabilities of battery discharge and data loss.

Theorem 5. [7] *For a channel with a rate-power function $\mu(\cdot)$ that is continuous at $\bar{r}$, there exists an energy management scheme $\mathcal{E}$ that simultaneously achieves*

$$\lim_{B^b \to \infty} \lim_{\lambda \uparrow \mu(\bar{r})} \frac{1}{B^b \delta^{(r)}} \log p^{o,\mathcal{E}}(B^b) = -\frac{2}{\bar{\sigma}_r^2},$$
$$\lim_{B^d \to \infty} \lim_{\lambda \uparrow \mu(\bar{r})} \frac{1}{B^d \delta^{(A)}} \log p^{d,\mathcal{E}}(B^d) = -\frac{2}{\bar{\sigma}_A^2},$$

where $\delta^{(r)} = v(\bar{r} - \mu^{-1}(\lambda)), \delta^{(A)} = \mu(\bar{r} - \delta^{(r)}) - \lambda$ for any $v \in (0, 1)$.

Theorem 5 shows that, in the heavy traffic limit, we observe an exponential decay for both the battery underflow and the buffer overflow probabilities, with the battery size and the data buffer size respectively. However, one can also see that, there is a tradeoff in the decay exponents of these two probabilities. More specifically, by varying $\delta^{(r)}$, it is possible to increase (or decrease) the decay exponent for the data loss probability. However this will result a proportional decrease (or increase) in the decay exponent for the battery discharge probability.

3.2. *Finite Horizon Power Allocation with Renewable Energy*

In,[6] the authors consider a time-slotted system with a finite-time operation of T time slots. Let B^b denote the battery size. Let $r(t)$ denote the amount of replenishment in slot t, while $P(t)$ denotes the allocated energy in slot t. For simplicity of exposition, we assume that the harvested energy arrives at the beginning of each slot and is immediately stored in the battery. We also assume that the initial battery is empty. At all times, the stored energy is never allowed to exceed B^b. Let $R(t)$ represent the cumulative harvested energy from slot 1 to slot t, i.e.,

$$R(t) = \sum_{\tau=1}^{t} r(\tau),\ \forall t \in \{1, 2, \dots, T\}. \tag{22}$$

For each t, $R(t)$ can be viewed as representing a point on a graph with time as the x-axis and the cumulated harvested energy as the y-axis. We connect all the neighboring points $R(t)$ and $R(t+1)$, for all $t \in \{1, 2, \dots, T-1\}$ with line segments. It immediately follows that $R(t)$ is a continuous, nondecreasing function of t that passes through points $(0, 0)$ and (T, K), where $K = R(T)$.

Similarly, we define $E(t)$ as the cumulative energy consumption from time slot 1 to slot t, i.e.,

$$E(t) = \sum_{\tau=1}^{t} P(\tau),\ \forall t \in \{1, 2, \dots, T\}. \tag{23}$$

We assume that $R(0) = E(0) = 0$. Define $\vec{P} = (P(1), P(2), \dots, P(T))$ and $\vec{E} = (E(1), E(2), \dots, E(T))$. Note that $\vec{E}$ and $\vec{P}$ are related by a 1-1 mapping because $P(t) = E(t) - E(t-1)$, for all $t \in \{1, 2, \dots, T\}$. Henceforth, we will interchangeably call both $\vec{E}$ and $\vec{P}$, the energy allocation scheme.

We investigate the finite-horizon throughput maximization problem for a single node assuming that the replenishment rate profile for the entire finite-horizon period is known in advance. During a time slot, the throughput of the node is characterized by a nondecreasing and strictly concave rate-power function $\mu(P)$, satisfying $\mu(0) = 0$. Recall that $\mu(P)$ represents the amount of data that can be transmitted using P units of energy in a slot under a given physical layer modulation and coding strategy. We are interested in finding an energy allocation policy $\vec{P} = (P(1), P(2), \dots, P(T))$ that maximizes the throughput during T time slots. Since the cumulative used energy cannot be greater than the cumulative harvested energy for any slot t, a natural constraint is given as follows:

$$E(t) \le R(t),\ \forall t = 1, 2, \dots, T. \tag{24}$$

Since computing $\vec{E}$ is equivalent to computing $\vec{P}$, the problem is formulated as follows:

$$P(3.2.1) \qquad \max_{\vec{P}} \sum_{t=1}^{T} \mu(P(t))$$
$$s.t. \qquad E(t) \leq R(t), \ \forall t \in \{1, 2, \ldots, T\} \tag{25}$$

Note that an optimal solution $\vec{E}^*$ must satisfy that at all time slots, the residual energy, $R(t) - E^*(t)$, is no greater than the battery capacity B^b. Otherwise, some energy will be lost due to the finite battery size, and we can easily find another energy allocation that achieves a greater throughput than $\vec{E}^*$, contradicting the optimality of $\vec{E}^*$. Hence, together with Equation (24), we obtain

$$R(t) - B^b \leq E(t) \leq R(t), \ \forall t \in \{1, 2, \ldots, T\}. \tag{26}$$

Therefore, P(3.2.1) can be formulated as

$$P(3.2.2) \qquad \max_{\vec{P}} \sum_{t=1}^{T} \mu(P(t))$$
$$s.t. \qquad R(t) - B^b \leq E(t) \leq R(t), \ \forall t \in \{1, 2, \ldots, T\} \tag{27}$$

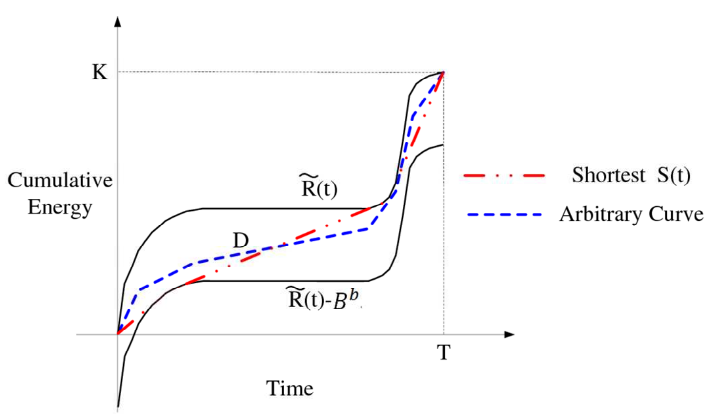

Fig. 7. The feasible domain D and the shortest curve $S(t)$

Let domain D denote all possible values that $\vec{E}$ can take such that Equation (26) is satisfied, in other words, the area that is surrounded by the curves $R(t)$, $R(t) - B^b$, the two vertical lines crossing node $(0, 0)$ and (T, K) as shown in Fig. 7. Note that D is a simply-connected space.

Definition 1. Feasible Curve: Any nondecreasing curve, defined on integer-valued t and located in the domain D, is said to be a feasible curve. From Equation (26), it can be seen that there is a 1-1 mapping between any feasible curve in D and an energy allocation scheme $\vec{E}$. For example, in Fig. 7, the dashed curve and the dot-and-dash curve represent two different energy allocation schemes. Furthermore, we consider two feasible curves f and g to be identical, if they have the same value at every integer point, i.e., $f(t) = g(t)$ for all $t = 1, 2, \ldots, T$. Also the length of a curve $f(t)$ in an interval $t \in [a, b]$ is defined as the sum of Euclidean lengths of $\{(x, f(x)), (x+1, f(x+1))\}$ in the interval, i.e., $\sum_{x=a}^{b-1} \sqrt{1 + (f(x+1) - f(x))^2}$.

Definition 2. Shortest Path: A curve that connects two points $(0, 0)$ and (T, K) in the domain D is said to be the shortest path $\vec{S}$, if its Euclidean length is the smallest among all feasible curves.

In Fig. 7, the shortest path is depicted by the dot-and-dash curve. In the following Lemma, we show the existence and feasibility of the shortest path.

Lemma 2. [6] *The shortest path $\vec{S}$ exists in domain D, and is feasible.*

Let $s(t) = S(t) - S(t-1)$. We know that $\vec{s} = (s(1), s(2), \ldots, s(T))$ is a feasible energy allocation scheme by Lemma 2. We first prove a property of the shortest path $S(t)$.

Lemma 3. [6] *The shortest path $S(t)$ is concave at any point t in the set $\{t : S(t) < R(t)\}$, and is convex at any point t in the set $\{t : S(t) > R(t) - B^b\}$, except for the boundary points $t = 0$ and $t = T$.*

Now, we claim the optimality of the energy allocation scheme $\vec{s}$ via the following theorem:

Theorem 6. [6] *The energy allocation scheme $\vec{s}$, each element of which satisfies $s(t) = S(t) - S(t-1)$, maximizes the throughput of a single node with rechargeable energy, resulting in an optimal solution to P(3.2.2).*

There are some observations related to the shortest-path solution.
1) We can see how the battery size B^b influences the optimal energy allocation solution. If B^b is large enough, we can see that $R(t) - B^b$ will always be less than 0. As a result, the domain D only has an upper bound $R(t)$. On the other hand, if B^b is very small, in particular, when $B^b = 0$, then

$R(t)$ and $R(t) - B^b$ will coincide to become one curve, which is also the only feasible curve. The corresponding energy allocation scheme is then to spend all the energy harvested in the current time slot. This corresponds to the correct intuition that if the energy buffer size is zero, the best scheme is to spend all the harvested energy, since no energy can be stored.
2) Note that Theorem 6 holds for any nondecreasing and concave function $\mu(P)$. We can also incorporate the energy cost for sensing data. Let $\varphi(P)$ represent the amount of data generated using P units of energy for sensing. Typically, $\varphi(P)$ is assumed to be linear, i.e., $\varphi(P) = \gamma P$, where γ is a constant scaler. We can prove that the amount of data generated by sensing and then transmitting is also a concave function of P. Therefore, the shortest-path scheme is still the solution to the problem of maximizing the amount of data sensed and then transmitted in the period $[0, T]$.

3.3. *A Cross-Layer Framework for Multihop Networks with Renewable Energy*

For finite horizon dynamic decision problem investigated in Section 3.2, the future information is assumed to known by the controller which is not practical. Although Section 3.1 does not require future information as in Section 3.2, some statistical assumptions such as Equation (14) are required. Furthermore, schemes in Sections 3.2 and 3.1 proposed for single node are not easy to extend to a network setting. In this section, we describe how the authors in[8] propose a general cross-layer resource allocation framework for multihop rechargeable networks as shown in Fig.8.

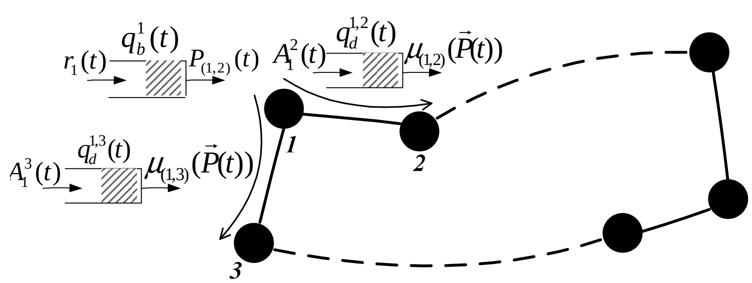

Fig. 8. Multihop Network Model.

We consider a multihop wireless network $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ with N nodes and L links. We assume a slotted system and each node $n \in \mathcal{N} = \{1, 2, \ldots, N\}$ is attached to power sources for replenishment. Let $A_n^e(t)$ (which is upper bounded by $A_{\max}$, where $0 < A_{\max} < \infty$) and $R_n^e(t)$ denote the amount of available data for sensing and the actual amount of sensed data, to node n that are destined to node e (possibly through multiple hops) in slot t. We assume that each node n maintains a separate data buffer with size $B_{n,e}^d$ (either $B_{n,e}^d < \infty$ or $B_{n,e}^d = \infty$) and state $q_{n,e}^d(t)$ for flows destined to e, and also maintains a battery buffer with size B_n^b (again, either $B_n^b < \infty$ or $B_n^b = \infty$) and state $q_n^b(t)$. Let $r_n(t)$ denote the replenishment at node n in time slot t. The transmit power is chosen to be $P_l(t)$ (which is upper bounded by P_{peak}, where $0 < P_{\text{peak}} < \infty$) over link l. In the formulation, we assume that the power the receiving node consumes to receive and decode the packet is identical to $P_l(t)$ as well. The sole reason for this is simplicity and the generalization to the asymmetric case is straightforward (By defining a receiving transmitting power ratio, we can extend it to the general case with no technique difference). We use the node-exclusive interference model. Under this model, a node can only receive from or transmit to at most one node at any time slot. In each time slot t, with the assigned power $P_l(t)$, the achieved data rate at link l is $\mu_l(P_l(t))$ in that time slot, where the rate function $\mu_l(\cdot)$ is a non-decreasing, concave and differentiable function on the half real line $\Re^+ \bigcup \{0\}$ satisfying $\mu_l(0) = 0$. Let Ω_n and Θ_n denote the set of directed links originated from node n and terminate at node n, respectively. We say $\vec{P} = [P_1(t), P_2(t), \cdots, P_L(t)]$ satisfies the node-exclusive model if $P_l(t) > 0$ for some $l \in \Omega_n \cup \Theta_n$, then $P_{l'} = 0$ for all $l' \in (\Omega_n \cup \Theta_n) \setminus \{l\}$. Our general objective is to maximize the long-term average sensing rate subject to the QoS constraints on both data and battery queues:

$$P(3.3) \qquad \max_{\vec{P},\vec{R}} \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{n,e\in\mathcal{N}} R_n^e(t)$$

$$s.t. \quad \vec{P}(t) \text{ satisfies the node-exclusive model,} \tag{28}$$

$$q_{n,e}^d(t+1) \le \min\left[\left(q_{n,e}^d(t) - \sum_{l\in\Omega_n} \mu_l^e\big(P_l(t)\big)\right)^+ + R_n^e(t) + \sum_{l\in\Theta_n} \mu_l^e(P_l(t)), B_{n,e}^d\right],\ n \ne e, \tag{29}$$

$$q_n^b(t+1) = \min\left[q_n^b(t) - \sum_{l\in\Omega_n\cup\Theta_n} P_l(t) + r_n(t), B_n^b\right], \tag{30}$$

$$0 \leq \sum_{l \in \Omega_n \cup \Theta_n} P_l(t) \leq \min\left[q_n^b(t), P_{\text{peak}}\right], \tag{31}$$

$$\sum_{e=1}^{N} \mu_l^e\big(P_l(t)\big) = \mu_l\big(P_l(t)\big),\ R_n^e(t) \leq A_n^e(t), \tag{32}$$

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q_{n,e}^d(t) < \infty,\ n \neq e,\ (B_{n,e}^d = \infty),$$

$$or\ p_{n,e}^d \leq \eta_{n,e}^d,\ (B_{n,e}^d < \infty) \tag{33}$$

$$p_n^o \leq \eta_n^o, \tag{34}$$

where $(\cdot)^+ = \max[\cdot, 0]$, $\vec{P}(t)$ is the power assignment vector for all links in slot t, $\vec{P}$ is the power assignment for all links over all time slots, and $\vec{R}$ is the actual sensing data vector for all node-destination pairs over all time slots, and

$$p_{n,e}^d = \begin{cases} 0, & \text{if } \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\big(R_n^e(t) + \sum_{l\in\Theta_n} f_l^e(t)\big) = 0 \\ \limsup_{T\to\infty} \frac{\frac{1}{T}\sum_{t=0}^{T-1} D_n^e(t)}{\frac{1}{T}\sum_{t=0}^{T-1}\big(R_n^e(t) + \sum_{l\in\Theta_n} f_l^e(t)\big)}, & \text{otherwise} \end{cases}, n \neq e \tag{35}$$

$$p_n^o = \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} 1_n^o(t) \tag{36}$$

are the long-term data loss ratio with an upper bound $\eta_{n,e}^d$, and the frequency of visits to zero battery state with given threshold η_n^o, respectively, where $f_l^e(t)$ is the actual amount of data transmitted through link l destined to node e in slot t, and

$$D_n^e(t) = \left(\left(q_{n,e}^d(t) - \sum_{l\in\Omega_n} \mu_l^e\big(P_l(t)\big)\right)^+ + R_n^e(t) + \sum_{l\in\Theta_n} f_l^e(t) - B_{n,e}^d\right)^+, \tag{37}$$

$$\begin{aligned} 1_n^o(t) &= \textit{indicator that battery hits zero state in slot } t \textit{ for node } n \\ &= \begin{cases} 0 \text{ if } \sum_{l\in\Omega_n \bigcup \Theta_n} P_l(t) < q_n^b(t) \\ 1 \text{ otherwise} \end{cases} \end{aligned} \tag{38}$$

are the amount of data loss and the indicator that the battery discharges completely in time slot t, respectively. Some applications may require the battery state to be always above certain positive level, then Equation (38) can be easily modified to represent how often the battery is below the desired level, and our solution structure works as well. Note that, we do not assume ergodicity of the system parameters, but if they are ergodic, then p_n^o represents the actual probability of a complete discharge event as $t \to \infty$.

In P(3.3), constraint (28) is the interference constraint. Constraints (29) and (30) describe how the data and battery queues evolve, respectively. Note that the destination node of each flow does not need to maintain a data buffer for that flow, as indicated in (29). Constraints (31) are the energy conservation equations stating that we cannot oversubscribe the energy that is unavailable in the battery nor can we exceed the peak power level. Constraints (32) are the rate conservation equations that bound the actual amount of sensed data $R_n^e(t)$ by the available amount of data $A_n^e(t)$, and share the transmission rate of a link among all the destinations in slot t. Constraint (33) is the QoS constraint for data queue: if $B_{n,e}^d = \infty$, we need to keep the data queue stable, and if $B_{n,e}^d < \infty$, the data loss ratio is required under a given threshold $\eta_{n,e}^d$. Constraint (34) is the battery QoS constraint of the desired battery discharge rate η_n^o.

We define $\tilde{q}_{n,e}^d$ and $\tilde{q}_n^b$ as the virtual data and battery queues. The virtual queues evolve according to the following Lindley's queue evolution equations:

$$\tilde{q}_{n,e}^d(t+1) = \Bigg(\Big(\tilde{q}_{n,e}^d(t) - \eta_{n,e}^d \big(R_n^e(t) + \sum_{l \in \Theta_n} f_l^e(t) \big) \Big)^+ - \sum_{l \in \Omega_n} \mu_l^e\big(P_l(t)\big) + R_n^e(t) + \sum_{l \in \Theta_n} f_l^e(t) + I_n^e(t) \Bigg)^+, \quad (39)$$

$$\tilde{q}_n^b(t+1) = \Big(\big(\tilde{q}_n^b(t) - \eta_n^o \big)^+ + \sum_{l \in \Omega_n \cup \Theta_n} P_l(t) - r_n(t) + M_n(t) + \mathit{1}_n^o(t) \Big)^+, \quad (40)$$

where $I_n^e(t) = \big(\sum_{l \in \Omega_n} \mu_l^e\big(P_l(t)\big) - \tilde{q}_{n,e}^d(t) \big)^+$ is the amount of transmitted idle packets when there is no enough data to transmit using the allocated energy, $M_n(t) = \big(q_n^b(t) - \sum_{l \in \Omega_n \cup \Theta_n} P_l(t) + r_n(t) - B_n^b \big)^+$ is the amount of missed replenishing energy due to full battery when $B_n^b < \infty$, and $\mathit{1}_n^o(t)$ is defined in Equation (38). Note that if $B_n^b = \infty$, then $M_n(t) = 0$ and Equation (40) reduces to $\tilde{q}_n^b(t+1) = \Big(\big(\tilde{q}_n^b(t) - \eta_n^o \big)^+ + \sum_{l \in \Omega_n \cup \Theta_n} P_l(t) - r_n(t) + \mathit{1}_n^o(t) \Big)^+$.

Fig. 9 shows the relationship between the actual and virtual battery queues when the battery size is finite. The amount of change in $q_n^b(t)$ from time t to $t+1$ is $\sum_{l \in \Omega_n \cup \Theta_n} P_l(t) - r_n(t) + M_n(t)$. Since the battery has a

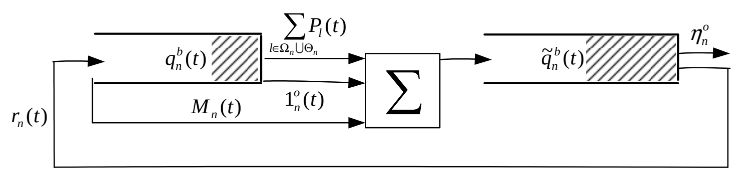

Fig. 9. Battery Queue and Virtual Battery Queue

finite size, this term vanishes when averaged over an infinitely long period of time. Then, $p_n^o = \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} 1_n^o(t)$ and η_n^o can be viewed as the long-term input and output rate of $\tilde{q}_n^b(t)$, respectively. Thus, it is reasonable to expect that $\tilde{q}_n^b(t)$ being stable implies $p_n^o \le \eta_n^o$.

Without loss of generality, the initial state $\tilde{q}_n^b(0)$ and $\tilde{q}_{n,e}^d(0)$ can be set to be zero. The following proposition shows that if the virtual queues $\tilde{q}_{n,e}^d(t)$, $\tilde{q}_n^b(t)$ and the actual battery queue $q_n^b(t)$ are all strongly stable, $p_{n,e}^d$ and p_n^o are guaranteed to meet their constraints.

Proposition 3. [8] *If all the virtual queues $\tilde{q}_{n,e}^d(t)$, $\tilde{q}_n^b(t)$ and the actual battery queue $q_n^b(t)$, $\forall n, e \in \mathcal{N}$ are all strongly stable, then $p_{n,e}^d \le \eta_{n,e}^d$ and $p_n^o \le \eta_n^o$, $\forall n, e \in \mathcal{N}$.*

The joint rate control, power allocation and routing algorithm for multihop networks can either be implemented in a centralized or distributed manner. For the centralized solution, we use the classical Maximal Weighted Matching (MWM) based algorithm and for the distributed algorithm, we can use the Maximal Matching (MM) based algorithms as in.[30]

Multihop Rate Control (MRC):
We define $0 < V < \infty$ to be the control parameter of our algorithm. Let $Q_{n,e}^d(t) = q_{n,e}^d(t)$ when $B_{n,e}^d = \infty$, and let $Q_{n,e}^d(t) = (1-\eta_{n,e}^d)\tilde{q}_{n,e}^d(t)$ when $B_{n,e}^d < \infty$. Depending on whether Maximum Weight Matching or Maximal Matching is employed by the scheduler, there is a slight difference in the implementation of MRC:
Maximum Weighted Matching (MWM): If $Q_{n,e}^d(t) \le \frac{V}{2}$, node n chooses to sense all the available data packets, i.e., $R_n^e(t) = A_n^e(t)$; otherwise, reject all the arrivals, i.e., $R_n^e(t) = 0$.
Maximal Matching (MM): If $Q_{n,e}^d(t) \le V$, node n chooses to sense all the available data packets, i.e., $R_n^e(t) = A_n^e(t)$; otherwise, reject all the arrivals, i.e., $R_n^e(t) = 0$.

Multihop Power Allocation (MPA):
Here the goal is to ensure that no node transfers data of a flow to a relay

node that is not the destination of that flow, unless the differential backlog for that flow is greater than a fixed value $\gamma > 0$. We will choose the value of γ such that the resulting backlog of the receiving node is not larger than that of the transmitting node after the transmission. This pushes the data flow from the source to the destination with a positive back pressure. Let tran(l) and rec(l) denote the transmitting and receiving node of link l, respectively. We first define

$$\gamma_l^e = \begin{cases} \gamma \text{ if } \mathrm{rec}(l) \neq e \\ 0 \text{ otherwise} \end{cases}$$

where $\gamma > 0$ is some constant. Let $e_l(t) = \arg\max_e \Big\{ Q^d_{\mathrm{tran}(l),e}(t) - Q^d_{\mathrm{rec}(l),e}(t) - \gamma_l^e \Big\}$ be the flow on link l that has the maximal modified differential backlog, and $w_l(t) = \max\Big[Q^d_{\mathrm{tran}(l),e_l(t)}(t) - Q^d_{\mathrm{rec}(l),e_l(t)}(t) - \gamma_l^{e_l(t)}, 0 \Big]$ is the nonnegative differential backlog of l at time t.

For each link l, solve

$$\max_{P_l(t)\in\Pi_l(t)} w_l(t)\mu_l\big(P_l(t)\big) - \Big(\tilde{q}^b_{\mathrm{tran}(l)}(t) + \tilde{q}^b_{\mathrm{rec}(l)}(t)\Big) P_l(t) \tag{41}$$

where $\Pi_l(t) = \Big\{ P_l(t) : 0 \leq P_l(t) \leq \min\Big[q^b_{\mathrm{tran}(l)}(t), q^b_{\mathrm{rec}(l)}(t), P_{\mathrm{peak}} \Big] \Big\}$.

Let $P_l^s(t)$ be the solution for link l. With the calculated power $P_l^s(t)$, let $W_l(t) = w_l(t)\mu_l\big(P_l^s(t)\big) - \Big(\tilde{q}^b_{\mathrm{tran}(l)}(t) + \tilde{q}^b_{\mathrm{rec}(l)}(t)\Big) P_l^s(t)$ be the weight on link l.

For the whole network, we either use the MWM or MM as described below.

Maximum Weighted Matching Algorithm: link l has weight $W_l(t)$, then the weight of a matching $\mathcal{M}$ is $W_{\mathcal{M}}(t) = \sum_{l\in\mathcal{M}} W_l(t)$. The network chooses a maximum weighted matching in a centralized manner, the links in the chosen matching become active with the calculated transmitting power, and other links are not activated.

Maximal Matching Algorithm: the network calculates a maximal matching that achieves at least half of the total weight of MWM in a fully distributed manner as in.[30] The links in the chosen matching become active with the calculated transmitting power, and other links are not activated.

Multihop Routing:

When $w_l(t) > 0$, transmit for flow that is destined to $e_l(t)$ with rate $\mu_l\big(P_l(t)\big)$, i.e., $\mu_l^{e_l(t)}\big(P_l(t)\big) = \mu_l\big(P_l(t)\big)$ and $\mu_l^e\big(P_l(t)\big) = 0$, $\forall e \neq e_l(t)$. Note that *MRC* and routing can be done by each node independently. We then give our main theorem for the multihop scenario:

Theorem 7. [8] *If*
(1) $\mu_l(\cdot)$ *is concave on* $\Re^+\bigcup\{0\}$, *and its slope at* 0 *satisfies* $0 \le \beta = \mu_l'(0) < \infty$, $\forall l \in \mathcal{L}$,
(2) $\forall n \in \mathcal{N}$: $0 < r_n(t) \le r_{max}$, $\forall t \ge 0$,

then the maximum weighted matching based joint rate control MRC, *power allocation* MPA, *and routing algorithm achieves:*

$$Q^d_{n,e}(t) \le \frac{V}{2} + A_{max}, \tag{42}$$

$$\tilde{q}^b_n(t) \le \beta\Big(\frac{V}{2} + A_{max}\Big), \tag{43}$$

$$\sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_e R^e_n(t) \right\} \ge \sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_e R^{e*}_n(t) - \eta^o_n(\mu_{max} + \beta) - \sum_e g^e_n(V, B^d_{n,e}, B^b_n) \right\} - O(\frac{1}{V}), \tag{44}$$

and the maximal matching based joint rate control MRC, *power allocation* MPA, *and routing algorithm achieves:*

$$Q^d_{n,e}(t) \le V + A_{max}, \tag{45}$$

$$\tilde{q}^b_n(t) \le \beta(V + A_{max}), \tag{46}$$

$$\sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_e R^e_n(t) \right\} \ge \sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_e \frac{R^{e*}_n(t)}{2} - \eta^o_n(\mu_{max} + \beta) - \sum_e g^e_n(V, B^d_{n,e}, B^b_n) \right\} - O(\frac{1}{V}). \tag{47}$$

where $\mu_{max} = \mu(P_{peak})$ *is the upper bound for the transmission rate, and*

$$g^e_n(V, B^d_{n,e}, B^b_n) = \begin{cases} 0, & \text{if } B^d_{n,e} = \infty,\ B^b_n = \infty \\ O(\frac{(\beta V - B^b_n)^+}{V}), & \text{if } B^d_{n,e} = \infty,\ B^b_n < \infty \\ O(\frac{(V - B^d_{n,e})^+}{V}), & \text{if } B^d_{n,e} < \infty,\ B^b_n = \infty \\ O(\frac{(\beta V - B^b_n)^+}{V}) + O(\frac{(V - B^d_{n,e})^+}{V}), & \text{if } B^d_{n,e} < \infty,\ B^b_n < \infty \end{cases}.$$

In Theorem 7, V is a finite tunable approximation parameter that controls the efficiency of the algorithm. Observe Equation (44) and (47), which compares the performance of our algorithm with that of the optimal solution of P(3.3), the term $\eta^o_n(\mu_{\max} + \beta)$ captures the influence of battery outage, and it is small since the battery outage threshold η_n^o is usually set to be very small to avoid network disconnection. Function $g_n^e(V, B_{n,e}^d, B_n^b)$ represents the asymptotical property of the gap with respect to buffer sizes.

We consider a network topology, shown in Fig. 10 (a). There are 6 nodes, 7 links, and 2 flows with source-destination pair $(3, 1)$ and $(5, 2)$, respectively. In all simulations, the simulation time is $T = 10^6$ time slots. We use the rate power function $\mu_l(P_l) = 10 \log_2(1 + \frac{g_l P_l}{N_l})$ packets/slot $\forall l \in \mathcal{L}$. Let the power of the background noise $N_l = 1.6 \times 10^{-14} W$, $\forall l \in \mathcal{L}$, and the channel gains $g_l = 1.6 \times 10^{-13}$, $\forall l \in \mathcal{L}$. Each node is equipped with an infinite data buffer for each flow through it. The number of arrivals $A_3^1(t),\ t \geq 0$ and $A_5^2(t),\ t \geq 0$, are modeled as independent Poisson random variables with mean $\lambda = 20$ packets/slot and $A_{\max} = 30$ packets/slot. We set η_n^o, the threshold of battery outage probability to 0.03 for all $n \in \mathcal{N}$ and the peak power $P_{\text{peak}} = 1.5W$. The backlog threshold $\gamma = 80 \geq \max_{n,e} \left(\sum_{\text{rec}(l)=n} \mu_l + A_n^e \right) = 2 \times 10 \log_2 (1 + 10 P_{\text{peak}})$, so that the resulting backlog of the receiving node is not longer than that of the transmitting node.

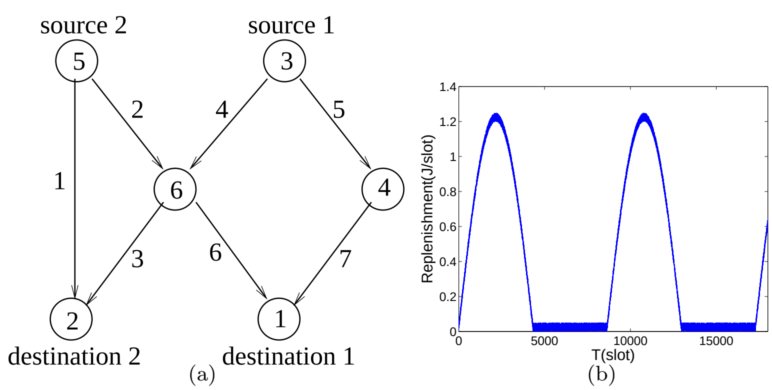

Fig. 10. (a) Network Topology and (b) A Sample Replenishing Process

Scenario 1: We first use a replenishment process which is formed by a periodic deterministic sine waveform ($r_{\max} = 1.2$ and period 8000) plus independent Gaussian noise with zero mean and variance 0.01, as shown in Fig. 10 (b) (The cycles imitate the daily solar cycles for a solar battery and the average replenishing can be simply calculated $\bar{r} = 0.2$). All the battery buffer sizes are set to be $B_b = 100J$. We simulate both MWM based and MM based algorithms. We choose different values of the control coefficient

V for the proposed algorithm and compare the results with the optimal value[§]. From Fig. 11 (a), we see that as V increases, the average total sensing rates of the MWM and MM based algorithm keep increasing and get closer to the optimum and a value that is much larger than half optimum, respectively. This is consistent with Equation (44) and Equation (47). From Fig. 11 (b), we see that as V increases, the average data queue length (we here only plot the data queue length of node 3 for flow 1 due to space limitation) keeps increasing but is upper bounded by the bound we get in Equation (42) and Equation (45). This means the queueing delay increases as we improve the sensing rate, which can be viewed as a tradeoff. From Fig. 11 (c) we observe that the battery discharge probability (we only plot for node 5 here) increases to the threshold as V increases.

Scenario 2: We use different replenishment processes: $r_2(t)$ and $r_5(t)$ are i.i.d Bernoulli random variables $Bernoulli(0.5)$ (i.e., $r_2(t) = 1$ w.p. 0.5 and $r_2(t) = 0$ w.p. 0.5); replenishing at all other nodes are independent Bernoulli random variables $0.2 \times Bernoulli(0.5)$ in even number slots, and $0.6 \times Bernoulli(0.5)$ in odd number slots ($\bar{r}_2 = \bar{r}_5 = 0.5$ and $\bar{r} = 0.2$ for other nodes), all plus Gaussian noise with zero mean and variance 0.01. This replenishing process is faster time-varying than the one in *Scenario 1*. We simulate three different battery sizes $B_b = 100J$, $B_b = 10J$ and $B_b = 1J$ (all nodes has the same battery sizes) for the MWM based algorithm. From Fig.. 11 (d), we can see that the sensing rate increases as battery size increases. However, as long as the battery is large compared to the average replenishing rate, the improvement diminishes with increasing battery sizes.

3.4. *Summary*

The three problems discussed in Section 3 have the same dynamic battery model and all of them aim to maximize a system utility. In,[6] the time horizon is finite while[7] and[8] consider a infinite horizon problem. In,[7] the authors characterize the basic limitation and tradeoff between achieving the maximum objective and the battery discharge or data overflow probability in a single link communication model. The authors in[6] provide an optimal algorithm for a single link system to achieve maximum utility subject to

[§]The exact optimal objective value for Problem (B) is hard to obtain, so we here use an upper bound for the optimum. For this example, an upper bound for the optimum can be obtained by equal time sharing of schedules $\{1, 4, 7\}$ and $\{1, 5, 6\}$, and utilizing the link rate $\mu(\bar{r})$ under infinite battery size and no discharge constraint. In *Scenario 1*, the optimum is $2\mu(\bar{r}) = 30$. In *Scenario 2*, since $\mu(\bar{r}_2) = \mu(\bar{r}_5) > \lambda$, the optimum is $\lambda + \mu(\bar{r}_3) = 35$.

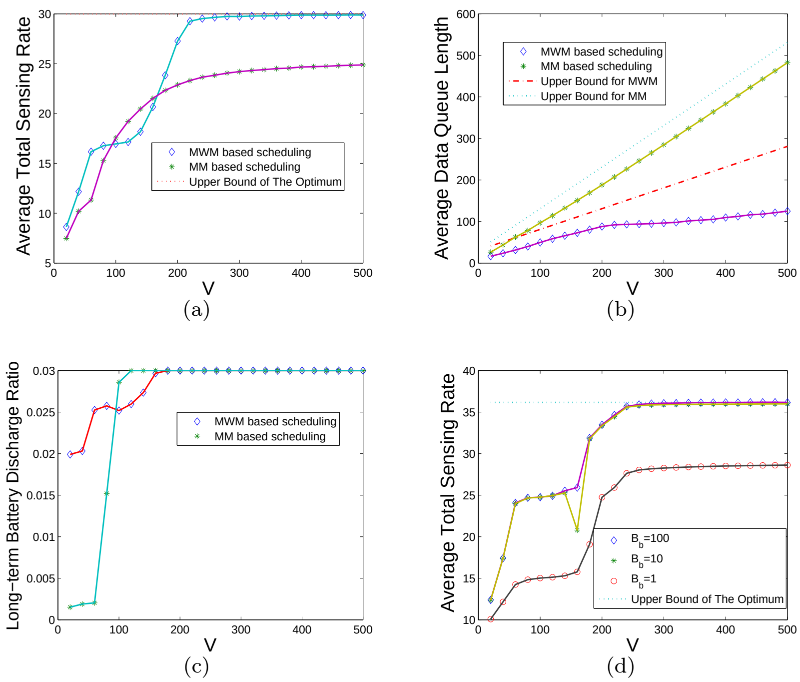

Fig. 11. Performance of the MWM and MM based algorithms. Impact of the control parameter V on (a) the average total sensing rate, (b) average data queue length, and (c) the battery discharge probability for *Scenario 1*. Impact of battery size on (d) the average total sensing rate for *Scenario 2*.

the battery constraint when knowing the future information. This gives an upper bound of the performance for system with a rechargeable battery. The authors in[8] study the utility maximization problem with predetermined QoS requirements on battery discharge and data overflow and then provide a simple online algorithm which is also extendable to a multihop network with distributed implementation.

4. Conclusion

In this chapter, we discussed thoroughly how resource allocation is different in networks with renewable energy and their non-rechargeable counterpart. For rechargeable networks, conservative energy expenditure may lead to missed recharging opportunities due to battery overflow, and aggressive usage of energy may lead to battery outage. Consequently, the efficient dynamic algorithms need to switch between aggressive and conservative resource allocation, depending on the instantaneous battery state. We investigate the joint resource allocation of energy, data rates, bandwidth and routes etc. across different layers of the network under different scenarios. Basic performance limits for various buffer sizes are illustrated and optimal or near optimal algorithms are proposed. The common features of all of the algorithms we studied in this chapter are their low complexity and their possibility of distributed implementation, which are essential for practical networks.

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