Scheduling with Rate Adaptation under Incomplete Knowledge of Channel/Estimator Statistics

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***Abstract*—In time-varying wireless networks, the states of the communication channels are subject to random variations, and hence need to be estimated for efficient rate adaptation and scheduling. The estimation mechanism possesses inaccuracies that need to be tackled in a probabilistic framework. In this work, we study scheduling with rate adaptation in single-hop queueing networks under two levels of channel uncertainty: when the channel estimates are inaccurate but complete knowledge of the channel/estimator joint statistics is available at the scheduler; and when the knowledge of the joint statistics is incomplete. In the former case, we characterize the network stability region and show that a maximum-weight type scheduling policy is throughput-optimal. In the latter case, we propose a joint channel statistics learning - scheduling policy. With an associated trade-off in average packet delay and convergence time, the proposed policy has a stability region arbitrarily close to the stability region of the network under full knowledge of channel/estimator joint statistics.**

I. Introduction

Scheduling in wireless networks is a critical component of resource allocation that aims to maximize the overall network utility subject to link interference and queue stability constraints. Since the seminal paper by Tassiulas and Ephremides ([1]), maximum-weight type algorithms have been intensely studied (e.g., [2]-[8]) and found to be throughput-optimal in various network settings. The majority of existing works employing maximum-weight type schedulers are based on the assumption that full knowledge of channel state information (CSI) is available at the scheduler. In realistic scenarios, however, due to random variations in the channel, full CSI is rarely, if ever, available at the scheduler. The dynamics of the scheduling problem with imperfect CSI is, therefore, vastly different from the problem with full CSI in the following two ways (1) a non-trivial amount of network resource, that could otherwise be used for data transmission, is spent in learning the channel; (2) the acquired information on the channel is potentially inaccurate, essentially underscoring the need for intelligent rate adaptation and user scheduling. Realistic networks are thus characterized by a convolved interplay between channel estimation, rate adaptation, and multiuser scheduling mechanisms.

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These complicated dynamics are studied under various network settings in recent works ([9]-[15]). In [9], the authors study scheduling in single-hop wireless networks with Markov-modeled binary ON-OFF channels. Here scheduling decisions are made based on cost-free estimates of the channel obtained once every few slots. The authors show that a maximum-weight type scheduling policy, that takes into account the probabilistic inaccuracy in the channel estimates and the memory in the Markovian channel, is throughput-optimal. In [10], the authors study decentralized scheduling under partial CSI in multi-hop wireless networks with Markov-modeled channels. Here, each user knows its channel perfectly and has access to delayed CSI of other users' channels. The authors characterize the stability region of the network and show that a maximum-weight type threshold policy, implemented in a decentralized fashion at each user, is throughput optimal. In [12], the authors study scheduling under imperfect CSI in single-hop networks with independent and identically distributed (*i.i.d.*) channels. They consider a two-stage decision setup: in the first stage, the scheduler decides whether to estimate the channel with a corresponding energy cost; in the second stage, scheduling with rate adaptation is performed based on the outcome of the first stage. Under this setup, the authors propose a maximum-weight type scheduling policy that minimizes the energy consumption subject to queue stability.

While studying scheduling under imperfect CSI is a first step in the right direction, these works assume that complete knowledge of the channel/estimator joint statistics, which is crucial for the success of opportunistic scheduling, is readily available at the scheduler. This is another simplifying assumption that need not always hold in reality. Taking note of this, we study scheduling in single-hop networks under imperfect CSI, and when the knowledge of the channel/estimator joint statistics is incomplete at the scheduler. We propose a joint statistics learning-scheduling policy that allocates a fraction of the time slots (the exploration slots) to continuously learn the channel/estimator statistics, which in turn is used for scheduling and rate adaptation during data transmission slots. Note that our setup is similar to the setup considered in [15]. Here the author considers a two-stage decision setup. When applied to the scheduling problem, this work can be interpreted as follows. One of K

estimators is chosen to estimate the channel in the first stage, with unknown channel/estimator joint statistics. The second stage decision is made to minimize a *known* function of the estimate obtained in the first stage. Our problem is different from this setup in that the channel/estimator joint statistics is important to optimize the second stage decision in our problem - i.e., scheduling with rate adaptation. This is not the case in [15] where a *known* function of the estimate is optimized and the channel/estimator joint statistics is helpful only in the first stage that decides one of K estimators. Our contribution is two-fold:

- When complete knowledge of the channel/estimator joint statistics is available at the scheduler, we characterize the network stability region and show that a simple maximum-weight type scheduling policy is throughput-optimal. It is worth contrasting this result with those in [9]-[12]. In these works, imperfection of CSI is assumed to be caused by *specific* factors like delayed channel feedback, infrequent channel measurement, etc, whereas, in our model, since the channel/estimator joint statistics is unconstrained, the CSI inaccuracy is captured in a more general probabilistic framework.
- Using the preceding system level results as a benchmark, we study scheduling under incomplete knowledge of the channel/estimator joint statistics. We propose a scheduling policy with an in-built statistics learning mechanism and show that, with a corresponding tradeoff in the average packet delay before convergence, the stability region of the proposed policy can be pushed arbitrarily close to the network stability region under full knowledge of channel/estimator statistics.

The paper is organized as follows. Section II formalizes the system model. In Section III, we characterize the stability region of the network and propose a throughput-optimal scheduling policy. In Section IV, we study joint statistics learning-scheduling and rate adaptation when the scheduler has incomplete knowledge of channel/estimator statistics. Concluding remarks are provided in Section V.

II. System Model

We consider a wireless downlink communication scenario with one base station and N mobile users. Data packets to be transmitted from the base station to the users are stored in N separate queues at the base station. Time is slotted with the slots of all the users synchronized. The channel between the base station and each user is *i.i.d.* across time slots and independent across users. We do not assign any specific distribution to the channels throughout this work. The channel state of a user in a slot denotes the number of packets that can be successfully transmitted without outage to that user, in that slot. Transmission at a rate below the channel state always succeeds, while transmission at a rate above the channel state always fails. We assume the channel state lies in a finite discrete state space $\mathcal{S}$. Let $C_i[t]$ be the random variable denoting the channel state of user i in slot t. The channel state of the network in slot t is denoted by the vector $\boldsymbol{C}[t] = \big[C_1[t], C_2[t], \cdots, C_N[t]\big] \in \mathcal{S}^N$. In each slot, the scheduler has access to estimates of the channel states, i.e., $\widehat{\boldsymbol{C}}[t] = \big[\widehat{C}_1[t], \widehat{C}_2[t], \cdots, \widehat{C}_N[t]\big] \in \mathcal{S}^N$. The estimator is fixed for each user and the estimates are independent across users. The channel/estimator joint statistics for user i is given by the $|\mathcal{S}|^2$ probabilities $P(C_i{=}c_i, \widehat{C}_i{=}\hat{c}_i)$, $\forall c_i{\in}\mathcal{S}, \hat{c}_i{\in}\mathcal{S}$.

We adopt the one-hop interference model, where, in each slot, only one user is scheduled for data transmission. The scheduler (base station), based on the channel estimate and the queue length information, decides which user to schedule and performs rate adaptation in order to maximize the overall network stability region. Let $I[t]$ and $R[t]$ denote the index of the user scheduled to transmit and the corresponding rate of transmission, respectively, at slot t. Due to potential mismatch between the channel estimates and the actual channels, it is possible that the allocated rate is larger than the actual channel rate, thus leading to outage. In this case, the packet is retained at the head of the queue and a retransmission will be attempted later. Let $Q_i[t]$ denote the state (length) of queue i at the beginning of slot t. Let $A_i[t]$ denote the number of exogenous packet arrivals at queue i at the beginning of slot t with $E[A_i[t]] = \lambda_i$. The queue state evolution can now be written as a discrete stochastic process:

$$Q_i[t{+}1] = \big[Q_i[t]{-}\mathbf{1}(I[t]{=}i)R[t]\cdot\mathbf{1}(R[t]{\le}C_i[t])\big]^+{+}A_i[t], \quad (1)$$

where $[\cdot]^+ = \max\{0,\cdot\}$. We adopt the following definition of queue stability [2]: Queue i is stable if there exists a limiting stationary distribution F_i such that $\lim_{t\to\infty} P(Q_i[t] \le q) = F_i(q)$.

III. Full Knowledge of Channel/Estimator Joint Statistics

In this section, we consider the scenario where the scheduler has full access to the channel/estimator joint statistics, i.e., $P(C_i{=}c_i, \widehat{C}_i{=}\hat{c}_i)$, $\forall c_i{\in}\mathcal{S}, \hat{c}_i{\in}\mathcal{S}$ for $i{\in}\{1,\ldots,N\}$. We characterize the network stability region next.

A. Network Stability Region

Consider the class of stationary scheduling policies G that base their decision on the current queue length information $[Q_1,\ldots,Q_N]$, the channel estimates $[\widehat{C}_i,\ldots,\widehat{C}_N]$, and full knowledge of channel/estimator joint statistics. Define the network stability region as the closure of the arrival rates that can be supported by the policies in G without leading to system instability. Let $P_{\widehat{\boldsymbol{C}}}(\hat{\boldsymbol{c}} = [\hat{c}_1,\ldots,\hat{c}_N])$ denote the probability of the channel estimate vector. Thus,

$$P_{\widehat{\boldsymbol{C}}}(\hat{\boldsymbol{c}} = [\hat{c}_1,\ldots,\hat{c}_N]) = \prod_{i=1}^{N} P(\widehat{C}_i = \hat{c}_i), \quad (2)$$

where the probabilities $P(\widehat{C}_i = \hat{c}_i)$ are evaluated from the knowledge of the channel/estimator joint statistics. Defining $\mathcal{CH}[\mathcal{A}]$ as the convex hull ([16]) of set $\mathcal{A}$ and $\vec{\mathbf{1}}_i$ as the i^{th} coordinate vector, we record our result on the network stability region below.

Proposition 1. *The stability region of the network is given by*

$$\begin{aligned}\boldsymbol{\Lambda} \;=\; &\sum_{\hat{\boldsymbol{c}}\in\mathcal{S}^N} P_{\widehat{\boldsymbol{C}}}(\hat{\boldsymbol{c}})\cdot \\ &\mathcal{CH}\Big[\,\mathbf{0}, P(C_i{\ge}r_i^*(\hat{c}_i)\big|\widehat{C}_i{=}\hat{c}_i\,)r_i^*(\hat{c}_i)\cdot\vec{\mathbf{1}}_i; i=1,\cdots,N\Big],\end{aligned}$$

where $r_i^*(\hat{c}_i) = \arg\max_{r\in\mathcal{S}} \left\{P(C_i \geq r \middle| \widehat{C}_i = \hat{c}_i\ ) \cdot r\right\}$ *and the conditional probabilities* $P(C_i \geq r_i^*(\hat{c}_i)|\widehat{C}_i = \hat{c}_i)$ *are evaluated from the knowledge of the channel/estimator joint statistics.*

Proof Outline: The proof contains two parts. We first show that any rate vector $\boldsymbol{\lambda}$ strictly within $\boldsymbol{\Lambda}$ is stably supportable by some randomized stationary policy. In the second part, we establish that any arrival rate $\boldsymbol{\lambda}$ outside $\boldsymbol{\Lambda}$ is not supportable by any policy. We show this by first identifying a hyperplane that separates $\boldsymbol{\lambda}$ and $\boldsymbol{\Lambda}$ using the strict separation theorem ([17]). We then define an appropriate Lyapunov function and show that, for any scheduling policy, there exists a positive drift, thus rendering the queues unstable ([18]). Details of the proof are available in our online technical report ([25]).

B. Optimal Scheduling and Rate Allocation

In this section, we propose a maximum-weight type scheduling policy with rate adaptation and show that it is throughput-optimal, i.e., it can support any arrival rate that can be supported by any other policy in G. The policy is introduced next.

Scheduling Policy Ψ

At time slot t, the base station makes the scheduling and rate adaptation decisions based on the channel/estimator joint statistics and the channel estimate vector $\widehat{\boldsymbol{C}} = \hat{\boldsymbol{c}}$ (the time index is dropped for notational simplicity).

(1) Rate Adaptation:

For each user i, assign rate R_i such that,

$$R_i = \arg\max_{r\in\mathcal{S}} \left\{P(C_i \geq r \middle| \widehat{C}_i = \hat{c}_i) \cdot r\right\}$$

(2) Scheduling Decision:

Schedule the user I that maximizes the queue-weighted rate R_i, as follows:

$$I = \arg\max_{i} \left\{Q_i \cdot P(C_i \geq R_i \middle| \widehat{C}_i = \hat{c}_i) \cdot R_i\right\}$$

Note that when the channel state estimation is accurate, the conditional probability $P(C_i \geq r \middle| \widehat{C}_i = \hat{c}_i)$ will be a step function, with Ψ essentially becoming the classic maximum-weight policy in [2]. The next proposition establishes the throughput optimality of policy Ψ. Details are provided in [25].

Proposition 2. *The scheduling policy* Ψ *stably supports all arrival rates that lie in the interior of the stability region* $\boldsymbol{\Lambda}$.

Proof Outline: The proof proceeds as follows. Consider a Lyapunov function $L(\boldsymbol{Q}[t]) = \sum_{i=1}^{N} Q_i^2[t]$. For any arrival rate $\boldsymbol{\lambda}$ that lies strictly within the stability region $\boldsymbol{\Lambda}$, we know it is stably supportable by some policy G_0. Under G_0, we show that the corresponding Lyapunov drift is negative. We then show that policy Ψ minimizes the Lyapunov drift and hence it will have a negative drift, thus establishing the throughput optimality of Ψ.

The results obtained thus far when the channel/estimator joint statistics is available at the scheduler are along expected lines. Nonetheless, they serve as a benchmark to the rest of the work under incomplete knowledge of the channel/estimator joint statistics, which is *the main focus of the paper*.

IV. Incomplete Knowledge of Channel/Estimator Joint Statistics

In this section, we study scheduling with rate adaptation when the scheduler only has knowledge of the marginal statistics of the estimator, i.e., $P(\widehat{C}_i = \hat{c}_i)$, $\forall c_i \in \mathcal{S}$, $i \in \{1, \ldots, N\}$, and hence, the knowledge of the channel/estimator joint statistics is incomplete at the scheduler. We first illustrate, with a simple example, that significant system level losses are incurred when no effort is made to learn these statistics, and hence no rate adaptation is performed.

A. Illustration of the Gains from Rate Adaptation

With incomplete information on the channel-estimator joint statistics, the scheduler naively trusts the channel estimates to be actual channel states and transmits at the rate allowed in this state. Under this scheduling structure, for the single-hop network we consider, the stability region is given in [25] by

$$\begin{aligned}\tilde{\boldsymbol{\Lambda}} \quad = \quad & \sum_{\hat{\boldsymbol{c}}\in\mathcal{S}^N} P_{\widehat{\boldsymbol{C}}}(\hat{\boldsymbol{c}}) \cdot \\ & \mathcal{CH}\Big[\ \mathbf{0}, P(C_i{\geq}\hat{c}_i \big| \widehat{C}_i{=}\hat{c}_i\ )\hat{c}_i \cdot \vec{\mathbf{1}}_i; i = 1, \cdots, N\Big]. \quad (3)\end{aligned}$$

For a two-user single-hop network, this region is plotted in Fig. 1 along-side the network stability region when full knowledge of the channel/estimator joint statistics is available at the scheduler and hence rate adaptation is performed. The channel between the base station and each user is independent and binary ($\mathcal{S} = \{0.2, 1\}$) with $P(C_i = 1) = 0.8,\ for\ i = 1, 2$. For different mismatch between the channel and the estimate, Fig. 1 plots the stability region of the system when rate adaptation is performed and when it is not. Note the significant reduction in the stability region when rate adaptation is not performed. This loss increases with increase in the degree of channel-estimator mismatch. The preceding example underscores the importance of rate adaptation and hence the need to learn the channel/estimator joint statistics. We now proceed to introduce our joint statistics learning-scheduling policy.

B. Joint Statistics Learning - Scheduling Policy

We design the policy with the following main components: (1) The fraction of time slots the policy spends in learning the channel/estimator joint statistics is fixed at $\gamma \in (0, 1)$,

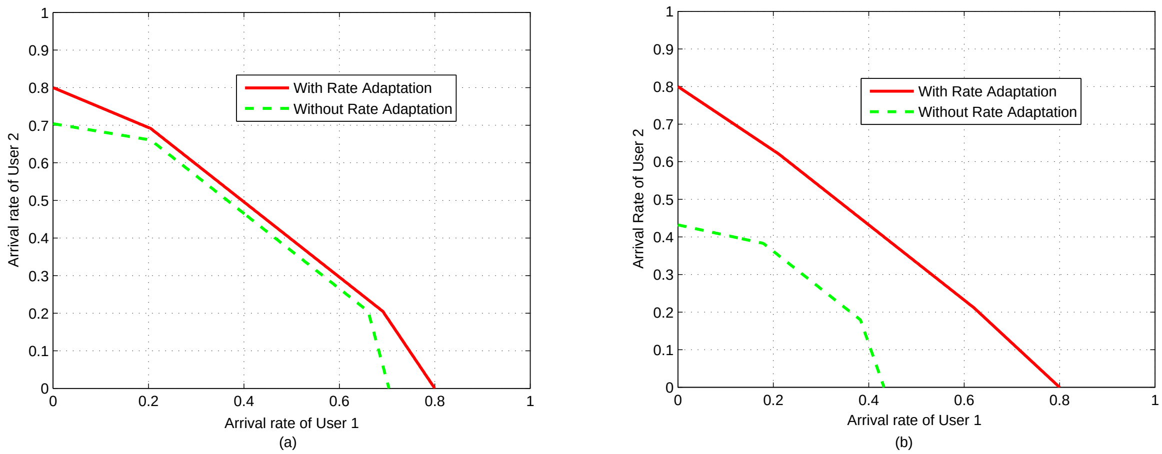

Fig. 1. Illustration of the system level gains associated with joint scheduling and rate control. $P_i(\widehat{C}_i = k \big| C_i = k)$ denotes the probability that the channel estimate of user i is k given the actual channel of user i is k. (a) $P_i(\widehat{C}_i = k \big| C_i = k) = 0.8,\ \ k \in \{0.2, 1\},\ i \in \{1, 2\}$; (b) $P_i(\widehat{C}_i = k \big| C_i = k) = 0.4$, $k \in \{0.2, 1\},\ i \in \{1, 2\}$.

(2) The worst-case rate of convergence of the statistics learning process is maximized. We formally introduce the policy next, followed by a discussion on the policy design.

Joint statistics learning-scheduling policy (parameterized by γ)

(1) In each slot, the scheduler first decides whether to explore the channel of one of the users or transmit data to one of the users. Specifically, it randomly decides to explore the channel of user i with probability $x^i_{\hat{c}_i}/N$ where $\sum_{i=1}^{N} x^i_{\hat{c}_i}/N < 1$. The quantity $x^i_{\hat{c}_i} \in (0, 1]$ is a function of γ and the channel estimate, $\hat{c}_i$, of user i. It is optimized to maximize the worst-case rate of convergence of the statistics learning mechanism subject to the γ constraint. We postpone the discussion on this optimization to Proposition 4. Note that, we have dropped the time index from the estimates for ease of notation.

(2) If a user is chosen for exploration, this time slot becomes an observing slot. Call the chosen user as e. The scheduler now sends data at a rate r that is chosen uniformly at random from the set $\mathcal{S}$. Let the quantity $\xi(t)$ indicate whether the transmission was successful or not:

$$\xi(t) = \mathbf{1}\big(c_e \geq r\big),$$

where, recall, c_e denotes the current channel state of user e. Let $\Theta_{i,\hat{c},r}$ denote the set of exploration time slots when the channel estimate of user i was $\hat{c}$ and user i was explored with rate r. Thus, the current slot is added to the set $\Theta_{e,\hat{c}_e,r}$. Now, an estimate of the quantity $P(C_e \geq r \Big| \widehat{C}_e = \hat{c}_e)$ is obtained using the following update:

$$\widehat{P}_t(C_e \geq r \Big| \widehat{C}_e = \hat{c}_e) = \frac{\sum_{k \in \Theta_{e,\hat{c}_e,r}} \xi(k)}{|\Theta_{e,\hat{c}_e,r}|}$$

where $|\mathcal{V}|$ denotes the cardinality of set $\mathcal{V}$. We assume $\widehat{P}_t(C_e | \widehat{C}_e)$ to be uniform when $\Theta_{e,\hat{c}_e,r} = \emptyset$, i.e., $\widehat{P}_t(C_e \geq r \Big| \widehat{C}_e = \hat{c}_e) = 1 - r/|\mathcal{S}|$.

(3) With probability $1 - \sum_{i=1}^{N} \frac{x^i_{\hat{c}_i}}{N}$, no user is chosen for exploration and the slot is used for data transmission. The scheduler follows policy Ψ introduced in the previous section with $P(C_i \geq r \Big| \widehat{C}_i = \hat{c}_i\ )$ replaced by the estimate $\widehat{P}_t(C_i \geq r \Big| \widehat{C}_i = \hat{c}_i\ )$.

An illustration of the proposed policy is provided in Fig. 2. We now discuss the design of the quantities $x^i_{\hat{c}_i}$, $\hat{c}_i \in \mathcal{S}$, $i \in \{1, \ldots, N\}$. Let $\eta_{i,\hat{c}_i} = P(\widehat{C}_i = \hat{c}_i)\frac{x^i_{\hat{c}_i}}{N}$ be a measure of how often the channel of user i is explored when the estimate is $\hat{c}_i$. For fairness considerations, we impose the following constraint in addition to the γ-constraint discussed earlier:

$$\sum_{\hat{c}_i \in \mathcal{S}} \eta_{i,\hat{c}_i} = \gamma/N.$$

The preceding constraint ensures that each user's channel is explored for an equal fraction, γ/N, of the total time slots. From strong law of large numbers, with probability one, $\widehat{P}_t(C_i \geq r \Big| \widehat{C}_i = \hat{c}_i)$ will converge to $P(C_i \geq r \Big| \widehat{C}_i = \hat{c}_i)$ as t tends to infinity. The rate of convergence of the channel/estimate joint statistics, parameterized by the user and the channel estimate, is given by the following lemma. Henceforth, we drop the suffix i from $\hat{c}_i$ for notational convenience.

Lemma 3.

$$\limsup_{t \to \infty} \frac{\widehat{P}_t(C_i \geq r \big| \widehat{C}_i = \hat{c}) - P(C_i \geq r \big| \widehat{C}_i = \hat{c})}{\sqrt{\frac{\log\log(\frac{\eta_{i,\hat{c}}t}{|\mathcal{S}|})}{(\frac{\eta_{i,\hat{c}}t}{|\mathcal{S}|})}}} = \sqrt{2}\sigma$$

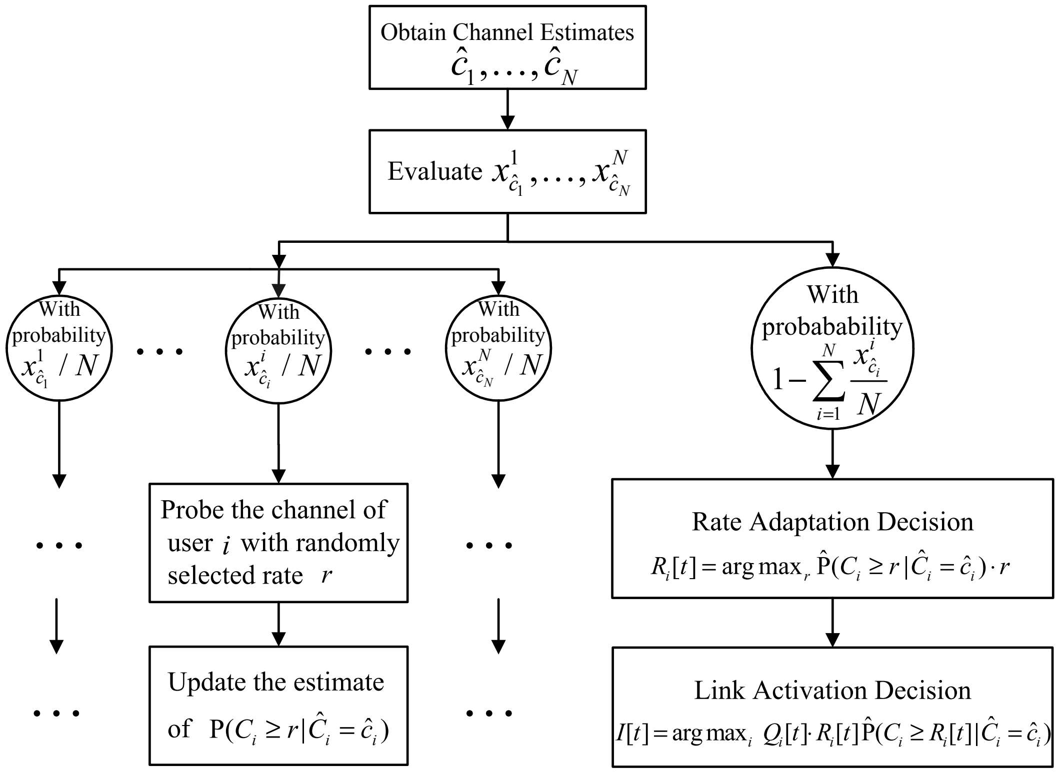

Fig. 2. Illustration of the joint statistics learning - scheduling policy.

almost surely (a.s.), where

$$\sigma=\sqrt{P(C_i\geq r\Big|\widehat{C}_i=\hat{c})(1-P(C_i\geq r\Big|\widehat{C}_i=\hat{c})}).$$

Proof: We use $N_r[t]$ to denote the number of exploration slot corresponding to estimated channel $\widehat{C}_i=\hat{c}$ and rate r. We express the left hand side of the equation in the lemma as follows.

$$\begin{aligned}&\limsup_{t\to\infty}\frac{\widehat{P}_t(C_i\geq r\Big|\widehat{C}_i=\hat{c})-P(C_i\geq r\Big|\widehat{C}_i=\hat{c})}{\sqrt{(2\log\log(\eta_{i,\hat{c}}t/|\mathcal{S}|))/(\eta_{i,\hat{c}}t/|\mathcal{S}|)}}\\ =&\limsup_{t\to\infty}\frac{\widehat{P}_t(C_i\geq r\Big|\widehat{C}_i=\hat{c})-P(C_i\geq r\Big|\widehat{C}_i=\hat{c})}{\sqrt{(2\log\log N_r[t])/N_r[t]}}\cdot\\ &\sqrt{\frac{\log\log N_r[t]}{\log\log(\eta_{i,\hat{c}}t/|\mathcal{S}|)}\cdot\frac{\eta_{i,\hat{c}}t/|\mathcal{S}|}{N_r[t]}}\end{aligned}\tag{4}$$

From Law of Iterated Logarithm ([19]), we get

$$\limsup_{t\to\infty}\frac{\widehat{P}_t(C_i\geq r\Big|\widehat{C}_i=\hat{c})-P(C_i\geq r\Big|\widehat{C}_i=\hat{c})}{\sqrt{(2\log\log N_r[t])/N_r[t]}}=\sigma\tag{5}$$

almost surely. We also have

$$\begin{aligned}\frac{\log\log N_r[t]}{\log\log(\eta_{i,\hat{c}}t/|\mathcal{S}|)}=&1+\frac{\log\log N_r[t]-\log\log(\eta_{i,\hat{c}}t/|\mathcal{S}|)}{\log\log(\eta_{i,\hat{c}}t/|\mathcal{S}|)}\\ =&1+\frac{\log(1+\frac{\log[N_r[t]/(\eta_{i,\hat{c}}t/|\mathcal{S}|)]}{\log(\eta_{i,\hat{c}}t/|\mathcal{S}|)})}{\log\log(\eta_{i,\hat{c}}t/|\mathcal{S}|)}.\end{aligned}\tag{6}$$

Because $\{N_r[t]\}$ is a renewal process ([20]) with inter-renewal time $(\eta_{i,\hat{c}}/|\mathcal{S}|)^{-1}$, we will have

$$\lim_{t\to\infty}N_r[t]/(\eta_{i,\hat{c}}t/|\mathcal{S}|)=1\quad almost\ surely.$$

Hence (6) tends to 1 almost surely. Substituting equation (5) and (6) into (4) we get

$$\limsup_{t\to\infty}\frac{\widehat{P}_t(C_i\geq r\Big|\widehat{C}_i=\hat{c})-P(C_i\geq r\Big|\widehat{C}_i=\hat{c})}{\sqrt{(2\log\log(\eta_{i,\hat{c}}t/|\mathcal{S}|))/(\eta_{i,\hat{c}}t/|\mathcal{S}|)}}=\sigma,$$

almost surely. ■

Note from the preceding lemma that, for each $\{i,\hat{c}\}$, the higher the quantity $\eta_{i,\hat{c}}$, the faster the convergence of $\widehat{P}_t(C_i\geq r\Big|\widehat{C}_i=\hat{c})$. Also note that, for each user i, the channel estimate $\hat{c}$ with the slowest convergence affects the overall convergence performance for that user i. Taking note of this, we proceed to design $x^i_{\hat{c}}$ that maximizes the lowest convergence rate – the bottleneck.

The optimization problem (U) for user i is given by

$$\begin{aligned}\max_{x^i_{\hat{c}}}\quad&\min_{\hat{c}}\quad\eta_{i,\hat{c}}=\frac{1}{N}P(\widehat{C}_i=\hat{c})x^i_{\hat{c}}\\ s.t.\quad&\sum_{\hat{c}\in\mathcal{S}}\eta_{i,\hat{c}}=\frac{\gamma}{N}\\ &0<x^i_{\hat{c}}\leq 1,\quad\text{for all}\quad\hat{c}\in\mathcal{S}\end{aligned}$$

For ease of exposition, we assume, without loss of generality, that $P(\widehat{C}_i=s_1)\leq P(\widehat{C}_i=s_2)\leq\cdots\leq P(\widehat{C}_i=s_{|\mathcal{S}|})$. Let $[x^{i*}_{s_1},x^{i*}_{s_2},\cdots,x^{i*}_{s_{|\mathcal{S}|}}]$ be the optimal solution to the above

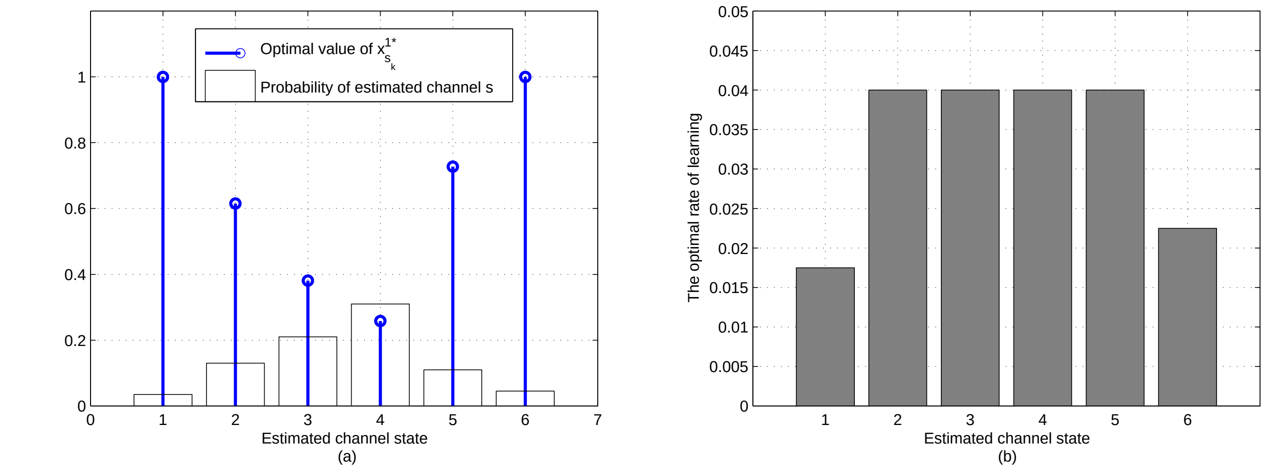

Fig. 3. Illustration of the design of optimal $x_{s_k}^{1*}$ when $N = 2$, $\gamma = 0.2$ and $\mathcal{S} = \{1, \ldots, 6\}$.

problem. We now record the structural properties of the optimal solution.

Proposition 4. *The solution $x_{s_k}^{i*}$, $\forall k \in \{1, \ldots, |\mathcal{S}|\}$, to the optimization problem (U) can be obtained with the following algorithm:*

(1) *Initialization: Let $k = 1$; $\Gamma = \emptyset$, $\omega = 0$;*

(2) *If $P(\widehat{C} = s_k) \geq \frac{\gamma - \sum_{s_j \in \Gamma} P(\widehat{C}_i = s_j)}{|\mathcal{S}| - w}$, then,*

$$\forall l \geq k, \;\; x_{s_l}^{i*} = \frac{\gamma - \sum_{s_j \in \Gamma} P(\widehat{C}_i = s_j)}{(|\mathcal{S}| - \omega) \cdot P(\widehat{C} = s_l)}.$$

Algorithm terminates.

(3) *Otherwise $x_{s_k}^{i*} = 1$, $\Gamma = \Gamma \cup s_k$, $\omega = \omega + 1$, $k = k + 1$. If $\Gamma = \mathcal{S}$, algorithm terminates, otherwise repeat Step (2).*

Proof Outline: The proof proceeds by establishing two crucial properties of the optimal solution. First, define Ω_i as the set of all channel estimates s_k such that the optimal $x_{s_k}^{i*} = 1$. Thus $\Omega_i = \cup_k \{s_k : \; x_{s_k}^{i*} = 1\}$. If no such estimate exists, $\Omega_i = \emptyset$. The optimal solution has the following properties:

(1) If $\Omega_i = \emptyset$ then $P(\widehat{C}_i = s_k) x_{s_k}^{i*} = \gamma / |\mathcal{S}|, \forall k$.
(2) If $\Omega_i \neq \emptyset$, then $x_{s_1}^{i*} = 1$.

Recall that the channel states are ordered such that $P(\widehat{C}_i = s_1) \leq P(\widehat{C}_i = s_2) \leq \cdots \leq P(\widehat{C}_i = s_{|\mathcal{S}|})$. The first property essentially says that if there does not exist a channel estimate s, for which $x_s^{i*} = 1$, then the optimal solution is such that the learning rate ($\frac{P(\widehat{C} = s_k) x_{s_k}^{i*}}{N}$) is uniform ($\frac{\gamma / |\mathcal{S}|}{N}$) for all s_k, $k \in \{1, \ldots, |\mathcal{S}|\}$. Because, otherwise, there is always room to improve the bottleneck convergence rate by redesigning the quantities $x_{s_k}^{i*}$. The second property says that whenever there exists an estimate $s_{k, k \neq 1}$ for which $x_{s_k}^{i*} = 1$, the estimate s_1 acts as a bottleneck, and the optimal value of $x_{s_1}^{i*}$ must be 1. The proposed algorithm now checks whether a solution yielding uniform convergence rate is feasible. If so, the solution is trivially given by $x_{s_k}^{i*} = \frac{1}{P(\widehat{C}_i = s_k)} \frac{\gamma}{|\mathcal{S}|}$, for all $k \in \{1, \ldots, |\mathcal{S}|\}$. Otherwise, using the preceding properties, the algorithm assigns $x_{s_1}^{i*} = 1$ and goes on to solve the reduced optimization problem over $x_{s_2}^{i*} \ldots, x_{s_{|\mathcal{S}|}}^{i*}$, iteratively. Details of the proof can be found in [25].

The proposed algorithm is illustrated in Fig. 3 when $\gamma = 0.2$, $N = 2$ and $\mathcal{S} = \{1, \ldots, 6\}$. Focusing on User 1, Fig. 3(a) plots the probability of the estimated channels and the optimal values of x_s^{1*}, $s \in \mathcal{S}$. Note that, the lower the value of $P(\widehat{C}_1 = s)$, the higher the assigned x_s^{1*}, since the algorithm maximizes the bottleneck convergence rate $\frac{P(\widehat{C} = s) x_s^{1*}}{N}$. This is further illustrated in Fig. 3(b) where the optimized convergence rate is shown to be 'near uniform', underlining the minmax nature of the optimization. Note that the structure of the minmax algorithm bears some similarity with the water-filling algorithm used in power allocation across parallel channels ([21]). There the algorithm tries to 'equalize' the sum of two components (signal and noise powers) across channels, while the minmax algorithm we propose tries to 'equalize' the *product* of two components ($P(\widehat{C}_i = s)$ and x_s^{i*}).

We now perform a stability region analysis of the proposed policy. Define the stability region of a policy as the exhaustive set of arrival rates such that the network queues are rendered stable under the policy. The stability region of the proposed policy, parameterized by $\gamma \in (0, 1)$, is recorded below.

Proposition 5. *The stability region Λ'_γ of the proposed policy is given by*

$$\mathbf{\Lambda}'_\gamma = \{\boldsymbol{\lambda} \;\; s.t. \;\; \frac{\boldsymbol{\lambda}}{1 - \gamma} \in \mathbf{\Lambda}\} \triangleq (1 - \gamma)\mathbf{\Lambda}.$$

where $\mathbf{\Lambda}$ is the stability region of the network when complete channel/estimator joint statistics is available at the scheduler.

Proof Outline: The proof proceeds by showing that, under the proposed joint statistics learning - scheduling policy, the

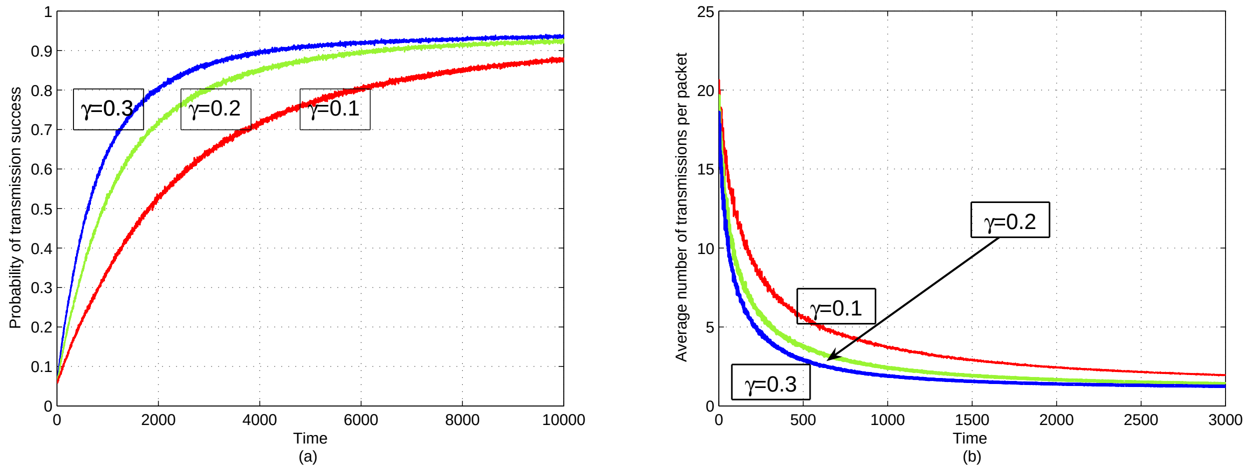

Fig. 4. Illustration of the time evolution of the probability of successful packet transmission and average number of transmissions needed per packet, for various values of γ.

instantaneous maximal sum of the queue weighted achievable rates, with sufficient time, can be arbitrarily close to the case when perfect knowledge of the statistics is available. Details are provided in [25].

C. Throughput - Delay Tradeoff

As $\gamma \to 0$, the proposed policy has a stability region that can be arbitrarily close to the system stability region $\mathbf{\Lambda}$. The trade-off involved here is the speed of convergence and hence queueing delays before convergence. Since an analytical study of this trade-off appears complicated, we proceed to perform a numerical study. The simulation setup is described next.

We use *i.i.d.* Rayleigh fading channels with minimum mean square error (MMSE) channel estimator as seen in [22] and [23]. The channel model is given by

$$Y = \sqrt{\rho} h X + \nu,$$

where X, Y correspond to transmitted and received signals, ρ is the average SNR at the receiver, and ν is the additive noise. Both h and ν are zero-mean complex Gaussian random variables, i.e., with probability density $\mathcal{CN}(0, 1)$. Let $\hat{h}$ denote the estimate of the channel and $\tilde{h}$ denote the estimation error. Under the channel statistics assumed, $\tilde{h}$ is zero-mean complex Gaussian with variance β, where the value of β depends on the resources allocated for estimation ([24]). Given the value of h, the channel rate is $R = \log(1+\rho|h|^2)$. We quantize the transmission rate to make the channel state space to be discrete and finite. We assume a two-user network and fix $\beta = 0.1$ and $\rho = 50$ for both users' channels. We study the average behavior of the proposed policy by implementing it over 10000 parallel queuing systems.

We first study the time evolution of the probability of transmission success for different values of γ. Fig. 4(a) shows that, for any γ, the probability of successful transmission increases as the accuracy of the estimate of the channel/estimator joint statistics improves with time. Also,

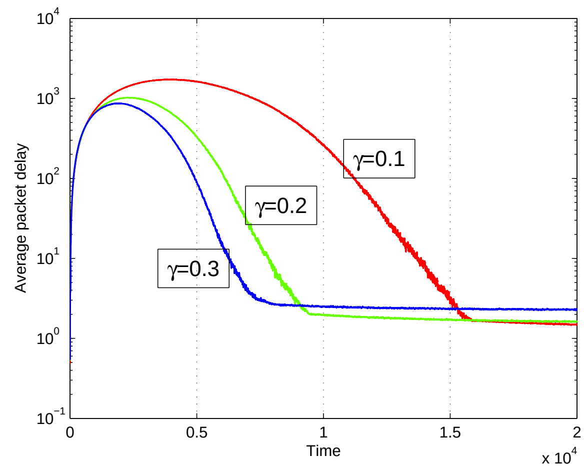

Fig. 5. Illustration of the average packet delay over time for various values of γ.

as expected, the larger the value of γ is, the faster is the improvement in the probability of successful transmission. Note that higher transmission success probability essentially means lesser number of retransmissions. This is illustrated in Fig. 4(b).

In Fig. 5, we study the time evolution of the average packet delay - the delay between the time a packet enters the queue and the time it leaves the head of the queue - for various values of γ. Note that γ influences the average delay through (1) the average number of retransmissions and (2) the fraction of time slots available for transmissions. It is expected that the nature of the influence of γ on the average delay depends on whether the estimate of the channel/estimator joint statistics has reached convergence or not. After convergence, the average delay is influenced by γ solely through the fraction of time slots available for transmissions. Thus, after convergence, the higher the value of γ, the higher the average delay. This is illustrated in Fig. 5. *Before* convergence, however, the effect of γ on the average delay is not straightforward. Fig. 5, along with the fact that higher γ results in faster convergence, suggests the following: before convergence, γ influences the average delay predominantly through the average number of retransmissions, resulting in decreasing average delay for increasing γ. In fact, Fig. 5 suggests the existence of a larger phenomenon: the trade-off between throughput (the stability region) and the delay before convergence.

V. Conclusion

We studied scheduling with rate adaptation in single-hop queueing networks, under imperfect channel state information. Under complete knowledge of the channel/estimator joint statistics at the scheduler, we characterized the network stability region and proposed a maximum-weight type scheduling policy that is throughput optimal. Under incomplete knowledge of the channel/estimator joint statistics, we designed a joint statistics learning - scheduling policy that maximizes the worst case rate of convergence of the statistics learning mechanism. We showed that the proposed policy can be tuned to achieve a stability region arbitrarily close to the network stability region with a corresponding trade-off in the average packet delay before convergence and the time for convergence.

References

[1] L. Tassiulas, A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Transactions on Automatic Control,* vol. 37, no. 12, pp. 1936-1948, Dec. 1992.

[2] L. Tassiulas, A. Ephremides, "Dynamic server allocation to parallel queues with randomly varying connectivity," *IEEE Transactions on Information Theory,* vol. 39, no. 2, pp. 466-478, Mar. 1993.

[3] X. Lin, N. B. Shroff. "Joint rate control and scheduling in multihop wireless networks," *Proceedings of IEEE Conference on Decision and Control,* Paradise Island, Bahamas, Dec. 2004.

[4] X. Lin, N. B. Shroff, "The impact of imperfect scheduling on cross-Layer congestion control in wireless networks," *IEEE/ACM Transaction on Networking,* vol. 14, no. 2, pp. 302-315, Apr. 2006.

[5] A. Eryilmaz, R. Srikant, "Fair resource allocation in wireless networks using queue-length based scheduling and congestion control," *IEEE/ACM Transaction on Networking,* vol. 15, no. 6, pp. 1333-1344, Dec. 2007.

[6] A. Eryilmaz, R. Srikant, "Joint congestion control, routing and MAC for stability and fairness in wireless networks," *IEEE Journal on Selected Areas in Communications,* vol. 24, no. 8, pp. 1514-1524, Aug. 2006.

[7] A. Stolyar, "Maximizing queueing network utility subject to stability: greedy primal-dual algorithm," *Queueing Systems,* vol. 50, no.4, pp.401-457, Aug. 2005.

[8] M. J. Neely, E. Modiano, C. Li, "Fairness and optimal stochastic control for heterogeneous networks," *IEEE Transactions on Information Theory,* vol. 52, no. 7, pp. 2915-2934, Jul. 2006.

[9] K. Kar, X. Luo, S. Sarkar, "Throughput-optimal scheduling in multichannel access point networks under infrequent channel measurements," *IEEE Transactions on Wireless Communications,* vol. 7, no. 7, pp. 2619-2629, Jul. 2008,

[10] L. Ying, S. Shakkottai, "Scheduling in mobile Ad Hoc networks with topology and channel-State Uncertainty," *Proceedings of IEEE INFOCOM,* Rio de Janeiro, Brazil, Apr. 2009.

[11] A. Gopalan, C. Caramanis, S. Shakkottai,"On wireless scheduling with partial channel-state information," *Proceedings of Allerton Conference on Communication, Control, and Computing,* Monticello, IL, Sept. 2007.

[12] C. Li, M. J. Neely, "Energy-optimal scheduling with dynamic channel acquisition in wireless downlinks," *Proceedings of IEEE Conference on Decision and Control,* New Orleans, LA, Dec. 2007.

[13] A. Pantelidou, A. Ephremides, A.L. Tits, "Joint scheduling and routing for Ad-hoc networks under channel state uncertainty," *Proceedings of IEEE Intl. Symp. on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt),* Limassol, Cyprus, Apr. 2007.

[14] R. Aggarwal, P. Schniter, C. E. Koksal, "Rate adaptation via link-layer feedback for goodput maximization over a time-varying channel," *IEEE Transactions on Wireless Communications,* vol. 8, no. 8, pp. 4276-4285, Aug. 2009.

[15] M. J. Neely, "Max weight learning algorithms with application to scheduling in unknown environments," *arXiv:0902.0630v1,* Feb. 2009.

[16] S. Boyd, L. Vandenberghe, *"Convex optimization,"* Cambridge University Press, 2004.

[17] D. Bertsekas, *"Nonlinear Programming,"* Belmont, MA: Athena Scientific, 1995.

[18] S. Meyn, *"Control techniques for complex networks,"* Cambridge University Press, 2007.

[19] P. Billingsley, *"Probability and measure, 3rd edition,"* Wiley, New York, 1995.

[20] R. G. Gallager, *"Discrete stochastic processes,"* Boston, MA: Kluwer, 1996.

[21] D. Tse, P. Viswanath, *"Fundamentals of wireless communication,"* Cambridge University Press, 2005.

[22] D. Zheng, S. Pun, W. Ge, J. Zhang, H.V. Poor, "Distributed opportunistic scheduling for Ad-Hoc communications under noisy channel estimation," *Proceedings of IEEE ICC,* Beijing, China May 2008.

[23] A. Vakili, M. Sharif, B. Hassibi, "The effect of channel estimation error on the throughput of broadcast channels," *Proceedings of IEEE Int. Conf. Acoust. Speech Signal Processing,* Toulouse, France, May 2006.

[24] T. Kailath, A.H. Sayed, B. Hassibi, *"Linear estimation,"* Prentice-Hall, 2000.

[25] W. Ouyang, S. Murugesan, A. Eryilmaz, N. B. Shroff, "Scheduling with rate adaptation under incomplete knowledge of channel/estimator statistics," *Online Technical Report,* (www.ece.osu.edu/∼ouyangw/rate_adaptation.pdf).