

# Network Control without CSI using Rateless Codes for Downlink Cellular Systems

Yin Sun<sup>†</sup>, C. Emre Koksal<sup>†</sup>, Sung-Ju Lee<sup>‡</sup>, and Ness B. Shroff<sup>†</sup>

<sup>†</sup>Dept. of ECE, The Ohio State University, Columbus, OH

<sup>‡</sup>Hewlett Packard Laboratories, Palo Alto, CA

**Abstract**—Wireless network scheduling and control techniques (e.g., opportunistic scheduling) rely heavily on access to Channel State Information (CSI). However, obtaining this information is costly in terms of bandwidth, time, and power, and could result in large overhead. Therefore, a critical question is *how to optimally manage network resources in the absence of such information*. To that end, we develop a cross-layer solution for downlink cellular systems with imperfect (and possibly no) CSI at the transmitter. We use rateless codes to resolve channel uncertainty. To keep the decoding complexity low, we explicitly incorporate time-average block-size constraints, and aim to maximize the system utility. The block-size of a rateless code is determined by both the network control decisions and the unknown CSI of many time slots. Therefore, unlike standard utility maximization problems, this problem can be viewed as a constrained partial observed Markov decision problem (CPOMDP), which is known to be hard due to the “curse of dimensionality.” However, by using a modified Lyapunov drift method, we develop a dynamic network control scheme, which yields a total network utility within  $O(1/L_{av})$  of utility-optimal point achieved by infinite block-size channel codes, where  $L_{av}$  is the enforced value of the time-average block-size of rateless codes. This opens the door of being able to trade complexity/delay for performance gains in the absence of accurate CSI. Our simulation results show that the proposed scheme improves the network throughput by up to 68% over schemes that use fixed-rate codes.

## I. INTRODUCTION

Over the past decade wireless scheduling and control techniques (e.g., opportunistic scheduling) have been developed to exploit opportunistic gains under the assumption of accurate channel state information (CSI) [1], [2], [3], [4]. However, obtaining this information is costly in terms of bandwidth, time, and power, and could result in incurring large overhead. Therefore, a critical question is “how to optimally manage network resources in the absence of such information?” We aim to answer this question by using *rateless codes* to jointly control power allocation, scheduling, and channel coding for downlink cellular systems with imperfect (and possibly no) CSI at the transmitter.

Rateless codes are a class of channel codes that the code-words (i.e., sequences of coded symbols or packets) with higher code-rates are prefixes of lower-rate codes. The transmitter progressively sends the coded packets to the receiver, until the receiver successfully decodes the message and sends an acknowledgment (ACK) to the transmitter. These codes are “regret-free” in the sense that the transmitter never worries

about that the selected modulation and code-rate are inappropriate such that the receiver cannot decode the message. Therefore, rateless codes work well when the CSI is not available at the transmitter due to limited-feedback and/or interference [5].

The first practical realizations of low complexity rateless codes are Luby Transform (LT) codes [6] and Raptor codes [7] for erasure channels, which have been widely used for application layer forward error correction (FEC). In the physical layer, Raptor codes for binary symmetric channel (BSC) and Gaussian channels were constructed in [8], [9], [10], where belief propagation (BP) decoding algorithms were utilized to realize a near-capacity performance over a wide range of SNR. The complexity of the BP decoding algorithms increases linearly with the number of coded packets (block-size) of rateless codes [8]. Rateless codes that simultaneously achieve the capacity of Gaussian channels at multiple SNRs were developed in [11], [5]. They use a layered encoding and successive decoding approach to achieve linear decoding complexity. Recently, a new type of rateless codes, called spinal codes, have been proposed [12], which use an approximate maximum-likelihood (ML) decoding algorithm to achieve the Shannon capacity of both BSC and Gaussian channels. The complexity of this decoding algorithm is polynomial in the size of message bits, but is still exponential in the block-size [13].<sup>1</sup>

In rateless codes, if the block-sizes are allowed to be arbitrarily large, the achievable rate will gradually approach the ergodic capacity of the channel, at the expense of unbounded decoding time. However, in practice, one cannot use rateless codes with arbitrarily large block-sizes so as to maintain manageable decoding time and complexity. Therefore, the block-size of rateless codes can be viewed as a parameter to control the throughput-complexity tradeoff.

We investigate the cross-layer design of downlink cellular systems with imperfect (possibly no) CSI at the transmitter that employ rateless codes to resolve channel uncertainty. Most of the prior work on cross-layer network control with imperfect CSI was centered on fixed-rate codes, e.g., [14], [15], which can achieve Shannon capacity for a certain channel state. However, these schemes suffer from channel outages or inefficient use of available channel rates, since the CSI information is not perfectly known at the transmitter. In

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<sup>1</sup>In [13], the rate gap  $\varepsilon$  from the capacity is inversely proportional to the block-size  $L$ , and the decoding complexity is exponential in  $1/\varepsilon$ .

contrast, rateless codes combat these issues by choosing their decoding time on the fly, at the expense of additional decoding complexity. Recently, scheduling and routing policies based on rateless codes were proposed in [16], [17] for time-invariant channel environments. Dynamic scheduling for incremental redundancy HARQ was analyzed in [18] for fading channels, which still requires feeding back ACK and realized mutual information in each slot to update the transmitter queue.

We explicitly incorporate time-average block-size constraints to keep the decoding complexity low and maximize the system utility. This utility maximization problem is challenging for two reasons: 1) the block-size of a rateless code is affected by the network control decisions of many time slots; and 2) the system is only partially observed because the accurate CSI is not available. Therefore, unlike standard utility maximization problems [1], [3], [19], [20], this problem is a constrained partial observed Markov decision problem (CPOMDP), which is generally intractable due to the ‘‘curse of dimensionality.’’ To that end, the following are the intellectual contributions of our paper:

- We formulate and solve a new utility maximization problem for downlink cellular systems, which utilizes rateless codes to resolve channel uncertainty. We develop a low-complexity dynamic network control scheme to attain a near-optimal solution to this problem. By varying the power allocation and scheduling decisions dynamically in each slot, our control scheme exploits the imperfect CSI and realizes a multi-user diversity gain. Our simulation results show that our scheme, by avoiding channel outages and utilizing the full channel rate more efficiently, improves the network throughput by up to 68%, compared with the schemes based on fixed-rate codes. To the best of our knowledge, *this is the first cross-layer network control scheme for physical layer rateless codes over time-varying noisy channels that does not require accurate CSI information.*
- One of our key technical contributions is in showing that our network control scheme meets the time-average block-size constraint of rateless codes. In doing so, we prove that the second order moment of the block-size of rateless codes is finite. This is accomplished by establishing a large-deviation principle for the reception process of rateless codes, which is difficult because the underlying Markov chain of our scheme has an uncountable state space.
- Another technical contribution is in developing a modified Lyapunov drift method to analyze the performance of our network control scheme. Conventional Lyapunov drift methods require minimizing the drift-plus-penalty of the system in each slot. However, our network control scheme generates an approximate drift-plus-penalty solution for only a portion of time slots. Nevertheless, we show that our scheme deviates from the time-average optimal utility of infinite block-size channel codes by no more than  $O(1/L_{av})$ , where  $L_{av}$  is the enforced value of the time-average block-size of rateless codes. This opens the door

of being able to trade complexity/delay for performance gains. Moreover, the feedback overhead of our scheme is at most  $1/L_{av}$  of that for fixed-rate codes when no CSI is available at the transmitter.

## II. PROBLEM FORMULATION

We consider a time-slotted downlink cellular network with one transmitter and  $S$  receivers. The channels are assumed to be block fading with a constant channel state within each slot, and vary from one slot to another. The channel states of slot  $t$  are described as  $\mathbf{h}[t] = (h_1[t], \dots, h_S[t])$ . Each receiver has perfect knowledge of its own CSI via channel estimation. However, the transmitter only has access to an imperfect CSI  $\hat{\mathbf{h}}[t] = (\hat{h}_1[t], \dots, \hat{h}_S[t])$  due to channel fluctuation and limited feedback. We assume that  $\{h_s[t], \hat{h}_s[t]\}$  are *i.i.d.* across time and independent across receivers, and the conditional probability distribution  $f(h_s|\hat{h}_s)$  of  $h_s[t]$  based on  $\hat{h}_s[t]$  is available at the transmitter. This model has covered the special cases of no CSI feedback, i.e.,  $\hat{h}_s[t]$  is independent of  $h_s[t]$ , and perfect CSI feedback, i.e.,  $\hat{h}_s[t] = h_s[t]$ .

Let  $P[t]$  denote the transmission power in slot  $t$ . The downlink transmissions are subject to a peak power constraint

$$0 \leq P[t] \leq P_{peak}, \quad (1)$$

for all  $t$  and a time-average power constraint

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} P[t] \leq P_{av}. \quad (2)$$

The mutual information accumulated at receiver  $s$  is denoted by  $I(h_s, P)$ . We assume that  $I(h_s, P)$  is a non-decreasing and concave function of  $P$ . Moreover, there exist some finite  $I_{max}, C > 0$  such that

$$I(h_s, 0) = 0, \quad I(h_s, P_{peak}) \leq I_{max}, \quad (\text{w.p.1}) \quad (3)$$

$$\left. \frac{\partial E_{\mathbf{h}}\{I(h_s, P)|\hat{h}_s\}}{\partial P} \right|_{P=0} \leq C, \quad \forall \hat{h}_s \quad (4)$$

where w.p.1 stands for ‘‘with probability 1’’, the expectation  $E_{\mathbf{h}}$  is taken over the channel state  $\mathbf{h}$  and the upper bound  $I_{max}$  is due to the limited dynamic range of practical RF receivers.

### A. Rateless Codes and Transceiver Queues

We consider a general model for rateless codes proposed in [21], [8]. At the transmitter, the encoder generates an unlimited amount of coded packets for receiver  $s$  from a payload message with  $M_s$  bits of information. One coded packet is transmitted in each slot to a scheduled receiver. The coded packets of one receiver may be transmitted over non-sequential time slots due to user scheduling. Receiver  $s$  collects packets until its accumulated mutual information exceeds the threshold  $M_s(1 + \epsilon)$ , which  $\epsilon$  is an appropriate constant, called reception overhead [8]. The value of  $\epsilon$  is chosen such that the decoder can decode the message with high probability. For Raptor codes [8] and Strider [5] over Gaussian channels,  $\epsilon$  is nonzero for certain ranges of channel SNR. For spinal codes [12], [13] over BSC and Gaussian channels,  $\epsilon$

can be arbitrarily close to 0 by choosing the code parameter properly.

1) *Decoder Queues*: Each receiver maintains a decoder queue  $R_s[t]$ , which represents the amount of mutual information required for decoding the current message. Once  $R_s[t]$  becomes smaller than or equal to  $I(h_s[t], P[t])K$ , receiver  $s$  can decode the current message at the end of slot  $t$ . Let  $n_s[t]$  denote the index of the current rateless code of receiver  $s$ , and  $c[t]$  denote the scheduled receiver in slot  $t$ . The evolution of the decoder queue  $R_s[t]$  is determined by

$$R_s[t+1] = \begin{cases} R_s[t], & \text{if } c[t] \neq s, \\ R_s[t] - I(h_s[t], P[t])K, & \text{if } c[t] = s \text{ and } \\ & R_s[t] > I(h_s[t], P[t])K, \\ (1 + \epsilon)M_s[n_s[t] + 1], & \text{if } c[t] = s \text{ and } \\ & R_s[t] \leq I(h_s[t], P[t])K, \end{cases} \quad (5)$$

where  $M_s[n]$  is the size of the message bits for the  $n$ th rateless code of receiver  $s$ ,  $K$  is the number of symbols in each packet. For notational simplicity, we omit  $\epsilon$  in the rest of the paper. Nevertheless, one can multiply  $I(h_s, P)$  by  $1/(1 + \epsilon)$  to derive the results for non-zero  $\epsilon$ .

2) *Encoder Queues*: Since the transmitter has no access to the decoder queue  $R_s[t]$ , it updates the encoder queue  $Q_s[t]$  only based on the ACK events. Let us define an ACK variable  $a[t]$ : if  $c[t] = s$  and  $R_s[t] \leq I(h_s[t], P[t])K$ , receiver  $s$  can decode the current rateless code and send an ACK to the transmitter, hence  $a[t] = s$ ; if the transmitter receives no ACK in slot  $t$ , then  $a[t] = 0$ . Hence, the evolutions of the encoder queue  $Q_s[t]$  are given by

$$Q_s[t+1] = (Q_s[t] - M_s[n_s[t]]1_{\{a[t]=s\}})^+ + x_s[t], \quad (6)$$

where  $1_{\{A\}}$  is the indicator function of some event  $A$ ,  $(\cdot)^+ = \max\{\cdot, 0\}$ , and  $x_s[t]$  is the arrival rate of the encoder queue. We assume that the arrival rate  $x_s[t]$  is bounded by

$$0 \leq x_s[t] \leq D_s. \quad (7)$$

The code index  $n_s[t]$ , which is available to both the transmitter and receiver, evolves as

$$n_s[t+1] = n_s[t] + 1_{\{a[t]=s\}}. \quad (8)$$

### B. Decoding Complexity Control

Define  $t_{n,s} = \min\{t \geq 0 : n_s[t] = n\}$  as the time slot that the first packet of the  $n$ th rateless code for receiver  $s$  is transmitted. From (5), the block-size of the  $n$ th rateless code for receiver  $s$  turns out to be:

$$L_s[n] = \min \left\{ \sum_{t=t_{n,s}}^{t_{n,s}+l-1} 1_{\{c[t]=s\}} : M_s[n] \leq \sum_{t=t_{n,s}}^{t_{n,s}+l-1} 1_{\{c[t]=s\}} I(h_s[t], P[t])K, l \geq 1 \right\}, \quad (9)$$

which is the number of scheduled time slots for providing the amount of mutual information no smaller than  $M_s[n]$  bits.

As discussed in Section I, the block-size  $L_s[n]$  has a significant influence on the decoding time of rateless codes over time-varying noisy channels. Thus, it is important that we limit  $L_s[n]$  so as to maintain an acceptable decoding

complexity. However,  $L_s[n]$  in (9) cannot be specified before transmission, because the channel states of future slots are not available. In particular, the set of possible values for  $L_s[n]$  may have an infinite span depending on the stochastic model of the wireless channel states. Hence, in order to avoid the undesirably long block-sizes and effectively control the decoding complexity, we consider the following time-average block-size constraints

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N L_s[n] = L_{av}, \quad (10)$$

for  $L_{av} \geq 1$  and all  $s \in \{1, \dots, S\}$ .

### C. Utility Maximization Problem

Define  $\bar{x}_s = \liminf_{T \rightarrow \infty} \frac{1}{T} \sum_{n=0}^{T-1} x_s[t]$  as the time-average rate that data arrives at the encoder queue of receiver  $s$ . Each receiver is associated with a utility function  $U_s(\bar{x}_s)$ , which represents the ‘‘satisfaction’’ of receiving data at an average rate of  $\bar{x}_s$  bits/packet. We assume that  $U_s(\cdot)$  is a concave, non-decreasing, continuous differentiable function, which satisfies  $U_s(0) = 0$  and  $U'_s(0) = b_s < \infty$ .

Our goal is to solve

$$\begin{aligned} \max_{\bar{x}_s} \quad & \sum_{s=1}^S U_s(\bar{x}_s) \\ \text{s.t.} \quad & \bar{\mathbf{x}} \in \Lambda, \end{aligned} \quad (11)$$

where  $\bar{\mathbf{x}} = (\bar{x}_1, \dots, \bar{x}_S)$ , and  $\Lambda$  is the time-average rate region such that there exists a network control scheme  $\{x_s[t], c[t], P[t], M_s[n]\}$  which satisfies (1), (2), (5)-(10), and the queues  $Q_s[t]$  are rate stable, i.e., [20]

$$\lim_{t \rightarrow 0} \frac{Q_s[t]}{t} = 0. \quad (\text{w.p.1}) \quad (12)$$

The aforementioned utility maximization problem (11) is challenging for two reasons: 1) the block-size of rateless codes  $L_s[n]$  in (9) is affected by the network control decisions  $\{c[t], P[t], M_s[n]\}$  of many time slots; and 2) the system is only partially observed because the accurate CSI  $h_s[t]$  is not available. Therefore, the problem (11) belongs to the class of constrained partially observed Markov decision problems (CPOMDP), which are known to be inherently hard. However, we are able to develop a dynamic network control scheme, described next, to obtain an efficient solution to this problem.

## III. CROSS-LAYER NETWORK CONTROL

We develop a dynamic network control scheme to solve the utility maximization problem (11). We show that our scheme deviates from the optimal utility of infinite block-size channel codes by no more than  $O(1/L_{av})$ , while still ensuring that the time-average block-size of rateless codes is equal to  $L_{av}$ .

### A. Network Control Algorithm

We first define virtual queues for the time-average constraints (2) and (10), i.e.,

$$Z[t+1] = (Z[t] - P_{av})^+ + P[t], \quad (13)$$

$$W_s[n+1] = W_s[n] + L_s[n] - L_{av}. \quad (14)$$

Since the block-size  $L_s[n]$  in (9) is affected by the network control decisions  $\{c[t], P[t], M_s[n]\}$  and the unknown CSI of many time slots, conventional Lyapunov drift methods for enforcing the time-average block-size constraints (10) will result in solving a difficult partially observable Markov decision problem.

Rather, we develop a low-complexity encoding control method that increases the message size of rateless codes  $M_s[n]$ , if  $W_s[n] \geq 0$ ; and decreases  $M_s[n]$ , if  $W_s[n] < 0$ . The network control scheme  $\{x_s[t], P[t], c[t], M_s[n]\}$  is determined by the following algorithm:

**Network Control Algorithm (NCA):**

- **Encoding control:** The message size  $M_s[n]$  is given by:

$$M_s[n+1] = \begin{cases} (M_s[n] - \delta)^+, & \text{if } W_s[n] \geq 0, \\ \min\{M_s[n] + \delta, M_{\max}\}, & \text{if } W_s[n] < 0, \end{cases} \quad (15)$$

where  $\delta > 0$  and  $M_{\max} = I_{\max} L_{av} K$  are algorithm parameters.

- **Power allocation and scheduling:**

Find the receiver  $\varsigma[t]$  that satisfies

$$\varsigma[t] = \underset{s \in \{1, \dots, S\}}{\operatorname{argmax}} Q_s[t] E_{\mathbf{h}} \{ I(h_s[t], P_s[t]) | \hat{h}_s[t] \} K - Z[t] P_s[t], \quad (16)$$

where  $P_s[t]$  is determined by

$$P_s[t] = \underset{P \in [0, P_{peak}]}{\operatorname{argmax}} Q_s[t] E_{\mathbf{h}} \{ I(h_s[t], P) | \hat{h}_s[t] \} K - Z[t] P. \quad (17)$$

The power allocation and scheduling scheme is described as follows: If the transmission power  $P_{\varsigma[t]}[t]$  is within a small neighbourhood of zero, i.e.,  $P_{\varsigma[t]}[t] \in [0, \varepsilon]$ , no receiver is scheduled and  $c[t] = P[t] = 0$ . Otherwise, if  $P_{\varsigma[t]}[t] \geq \varepsilon$ , receiver  $\varsigma[t]$  is scheduled, i.e.,  $c[t] = \varsigma[t]$  and  $P[t] = P_{\varsigma[t]}[t]$ . Here,  $\varepsilon > 0$  is a very small constant parameter.

- **Rate control:** The arrival rate of the encoder queue is determined by

$$x_s[t] = \underset{x \in [0, D_s]}{\operatorname{argmax}} V U_s(x) - x^2 - 2Q_s[t]x, \quad (18)$$

where  $V > 0$  is a constant algorithm parameter.

- **Queue update:** Update the queues  $R_s[t]$ ,  $Q_s[t]$ ,  $Z[t]$ , and  $W_s[n]$  according to (5), (6), (13), and (14), respectively.

In Algorithm NCA, we introduced a transmission power lower bound  $P[t] \geq \varepsilon$ , where  $\varepsilon > 0$  is an arbitrary small constant. This additional power lower bound is useful for establishing the stability of  $W_s[n]$  in Section III-B. The impact of this power lower bound becomes negligible as  $\varepsilon$  tends to  $0^+$ .

### B. Performance Analysis

We analyze the performance of Algorithm NCA in two steps: In *Step One*, we show that the virtual queue  $W_s[n]$  is rate stable, and thereby the time-average block-size constraint (10) is satisfied with probability 1. In *Step Two*, we show that the performance of our scheme deviates from the optimal utility by no more than  $O(1/L_{av})$ .

1) *Step One:* The key idea for proving the stability of  $W_s[n]$  is to show that the second order moment of the block-size  $E\{L_s[n]^2\}$  is upper bounded uniformly for all  $n$  and  $s$ , which is stated in the following lemma:

**Lemma 1.** *Let  $\{P[t], x_s[t], c[t], M_s[n]\}$  be determined by Algorithm NCA. There then exists some  $G > 0$  such that*

$$E\{L_s[n]^2\} \leq G, \quad \forall n, s. \quad (19)$$

In order to prove Lemma 1, we need to establish a large-derivation principle for the mutual information accumulation process expressed in (9). However, there is a technical difficulty: the mutual information  $I(h_s[t], P[t])$  is non-*i.i.d.* in the scheduled slots of the rateless code. In particular, the underlying Markov chain of Algorithm NCA has an uncountable state space, which makes it difficult to check if a large derivation principle holds [22]. However, by using the transmission power lower bound  $P[t] \geq \varepsilon$  and some additional manipulations, we can obtain a lower bound on  $I(h_s[t], P[t])$  that is *i.i.d.* across the scheduled slots.

*Proof:* Let us consider the  $n$ th rateless code of receiver  $s$ . Suppose that the  $L_s[n]$  packets of this rateless code are transmitted in the time slots  $t \in \{t_{n,s,1} = t_{n,s}, t_{n,s,2}, \dots, t_{n,s,L_s[n]}\}$ . The tail probability of  $\Pr\{L_s[n] > l\}$  satisfies

$$\begin{aligned} & \Pr\{L_s[n] > l\} \\ &= \Pr \left\{ \sum_{\tau=1}^l I(h_s[t_{n,s,\tau}], P[t_{n,s,\tau}]) K < M_s[n] \right\} \\ &\stackrel{(a)}{\leq} \Pr \left\{ \sum_{\tau=1}^l I(h_s[t_{n,s,\tau}], P[t_{n,s,\tau}]) K < M_{\max} \right\} \\ &\stackrel{(b)}{\leq} \Pr \left\{ \sum_{\tau=1}^l I(h_s[t_{n,s,\tau}], \varepsilon) K < M_{\max} \right\}, \end{aligned} \quad (20)$$

where step (a) is due to  $M_s[n] \leq M_{\max}$  in (15) and step (b) is due to the transmission power lower bound  $P[t] \geq \varepsilon$ . The mutual information lower bound  $I(h_s[t], \varepsilon)$  is still non-*i.i.d.* across the scheduled slots, due to receiver scheduling. Therefore, for large enough  $l$ , we make further modifications:

$$\begin{aligned} & \Pr\{L_s[n] > l\} \\ &\stackrel{(c)}{\leq} \Pr \left\{ \sum_{\tau=1}^l \min_u [I(h_u[t_{n,s,\tau}], \varepsilon)] K < M_{\max} \right\} \\ &\stackrel{(d)}{\leq} \Pr \left\{ \sum_{\tau=1}^l \min_u [I(h_u[t_{n,s,\tau}], \varepsilon)] - la < 0 \right\}, \end{aligned} \quad (21)$$

where step (c) is due to  $\min_u \{I(h_u[t], \varepsilon)\} \leq I(h_s[t], \varepsilon)$  and step (d) due to the choice of  $l \geq \frac{2M_{\max}}{E\{\min_u [I(h_u, \varepsilon)]\} K}$  and  $a = 1/2E\{\min_u [I(h_u, \varepsilon)]\}$ . Here, by choosing the smallest mutual information over all receivers,  $\min_u [I(h_u[t], \varepsilon)]$  is *i.i.d.* across the scheduled slots. According to (3), there exists some  $\theta > 0$  such that

$$E\{e^{\theta \min_u [I(h_u, \varepsilon)]}\} < \infty. \quad (22)$$

Therefore, we can use the large derivation theory [23] to show that there exist some  $\gamma(a) > 0$  and  $N$ , such that the inequality

$$\Pr \left\{ \sum_{\tau=1}^l \min_u [I(h_u[t_{n,s,\tau}], \varepsilon)] - la < 0 \right\} < e^{-l\gamma(a)} \quad (23)$$

holds for all  $l > N$ . Therefore, we have

$$\begin{aligned}
& E\{L_s[n]^2\} \\
&= \sum_{l=1}^{\infty} [(l+1)^2 - l^2] \Pr\{L[n] > l\} \\
&\leq \sum_{l=1}^N [(l+1)^2 - l^2] + \sum_{l=N+1}^{\infty} [(l+1)^2 - l^2] \Pr\{L_s[n] > l\} \\
&\leq (N+1)^2 + \sum_{l=N+1}^{\infty} (l+1)^2 e^{-l\gamma(a)} \\
&\leq (N+1)^2 + \int_0^{\infty} (\tau+1)^2 e^{-(\tau-1)\gamma(a)} d\tau. \quad (24)
\end{aligned}$$

Since both terms of (24) are upper bounded, there must exist some  $G > 0$  such that

$$E\{L_s[n]^2\} \leq G. \quad (25)$$

Since the distribution of  $\min_u [I(h_u[t_{n,s,\tau}], \varepsilon)]$  does not rely on any particular choice of  $n$  and  $s$ , (23)-(25) hold uniformly for all  $n$  and  $s$ , and the asserted statement is proved. ■

We now analyze the evolution of the virtual queue  $W_s[n]$ : Suppose  $W_s[n] < 0$  and  $W_s[n+1] \geq 0$ . By (15), the message size  $M_s[n]$  starts to decrease. As long as  $W_s[n] \geq 0$ ,  $M_s[n]$  keeps decreasing. Once  $M_s[n]$  decreases to 0, we have  $L_s[n] = 1 \leq L_{av}$  and  $W_s[n]$  stops increasing. Since the step size of (15) is  $\delta$ ,  $W_s[n]$  either stops increasing or drops back to  $W_s[n] < 0$ , within  $\kappa = \lceil M_{\max}/\delta \rceil$  rateless codes. Therefore, the virtual queue  $W_s[n]$  is upper bounded by

$$W_s[n + \kappa] \leq \sum_{m=0}^{\kappa} L_s[n + m], \quad \forall n. \quad (26)$$

On the other hand, if  $W_s[n] \geq 0$  and  $W_s[n+1] < 0$ , by (15), the message size  $M_s[n]$  starts to increase. As long as  $W_s[n] < 0$ ,  $M_s[n]$  keeps increasing. Once  $M_s[n]$  reaches  $M_{\max}$ , we have  $L[n] \geq L_{av}$ , since  $I(h_s, P) \leq I_{\max}$ . Therefore, within  $\kappa$  rateless codes,  $W_s[n]$  either stops decreasing or grows up to  $W_s[n] \geq 0$ . Therefore,  $W_s[n]$  is lower bounded by

$$W_s[n + \kappa] \geq -(\kappa + 1)L_{av}, \quad \forall n. \quad (27)$$

Using these observations, we show the following theorem:

**Theorem 1.** *Let  $\{P[t], x_s[t], c[t], M_s[n]\}$  be determined by Algorithm NCA, then the virtual queues  $W_s[n]$  are rate stable, i.e.,*

$$\lim_{n \rightarrow \infty} \frac{W_s[n]}{n} = 0. \quad (\text{w.p.1}) \quad (28)$$

Hence, the time-average constraint (10) is satisfied with probability 1.

*Proof:* If  $W_s[n + \kappa] \geq 0$ , according to (26) and Lemma 1, the second moment of  $W_s[n]$  is upper bounded by

$$E\{W_s[n + \kappa]^2\} \leq E\left\{\left[\sum_{m=0}^{\kappa} L_s[n + m]\right]^2\right\} \leq (\kappa + 1)^2 G. \quad (29)$$

By (27) and (29), there exists some  $D > 0$  such that

$$E\{W_s[n]^2\} \leq D, \quad \forall n.$$

By Markov's inequality, for any  $\varepsilon > 0$ , we have

$$\Pr\left\{\frac{W_s[n]}{n} > \varepsilon\right\} \leq \frac{E\{W_s[n]^2\}}{n^2 \varepsilon^2} \leq \frac{D}{n^2 \varepsilon^2},$$

and thus

$$\sum_{n=1}^{\infty} \Pr\left\{\frac{W_s[n]}{n} > \varepsilon\right\} < \infty.$$

Then, (28) follows from the Borel-Cantelli lemma [23]. ■

2) *Step Two:* We now utilize a modified Lyapunov drift method to analyze the performance of Algorithm NCA. One difficulty is that the rate region  $\Lambda$  is not directly accessible. For this, we construct a larger rate region  $\Lambda_{out}$  satisfying  $\Lambda \subseteq \Lambda_{out}$ , and show that the performance of Algorithm NCA is within  $O(1/L_{av})$  from the optimum of the following problem:

$$\begin{aligned}
& \max_{x_s} \quad \sum_{s=1}^S U_s(x_s) \\
& \text{s.t.} \quad \mathbf{x} \in \Lambda_{out}.
\end{aligned} \quad (30)$$

To construct the outer rate region  $\Lambda_{out}$ , we consider the following genie-assisted policy: The transmitter has access to the perfect CSI  $\mathbf{h}[t]$  for coding control, while the power allocation and scheduling scheme is determined by only the imperfect CSI  $\hat{\mathbf{h}}[t]$ . This policy achieves the rate region  $\Lambda_{out}$  such that for each point  $\mathbf{x} = (x_1, \dots, x_S) \in \Lambda_{out}$  there exists a network control scheme  $\{c[t], P[t]\}$  satisfying

$$x_s \leq \liminf_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} [I(h_s[t], P[t]) K 1_{\{c[t]=s\}}], \quad (31)$$

$$0 \leq x_s \leq D_s, \quad (32)$$

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} P[t] \leq P_{av}, \quad (33)$$

$$0 \leq P[t] \leq P_{peak}, \quad (34)$$

where  $\{c[t], P[t]\}$  is determined by  $\hat{\mathbf{h}}[t]$ , but not  $\mathbf{h}[t]$ . We note that one can choose  $M_s[n] = I(h_s[t], P[t])K$  in the genie-assisted policy such that the mutual information in each slot is fully utilized. An alternative to this genie-assisted policy is to use infinite block-size channel codes to fully exploit the mutual information, which achieves the same rate region  $\Lambda_{out}$ , but results in unbounded decoding time. In [24], we prove that  $\Lambda \subseteq \Lambda_{out}$ . Hence, the performance of problem (11) is upper bounded by (30). Note that the key issue of fixed-rate codes with imperfect CSI are that the mutual information is underutilized if the transmitter has imperfect CSI and the code-rate is different from the mutual information  $I(h_s[t], P[t])K$ .

Another difficulty is that  $I(h_s[t], P[t])$  is not directly associated to the service process of the encoder queue  $Q_s[t]$ . For this, we define an auxiliary queue

$$Y_s[t] = Q_s[t] + R_s[t] - M_s[n_s[t]]. \quad (35)$$

From (5) and (6), the evolution of  $Y_s[t]$  is given by

$$\begin{aligned}
Y_s[t+1] &= (Y_s[t] - 1_{\{c[t]=s, a[t]=0\}} I(h_s[t], P[t])K \\
&\quad - 1_{\{a[t]=s\}} R_s[t])^+ + x_s[t].
\end{aligned} \quad (36)$$

Therefore, the service process of  $Y_s[t]$  is given by the mutual information  $I(h_s[t], P[t])K$ , if  $c[t] = s$  and  $a[t] = 0$  (i.e., the scheduled slot is not the last reception slot of a rateless code). This motivates us to utilize the auxiliary queue  $Y_s[t]$  to construct the Lyapunov drift.

Now, we still need to solve the following two remaining difficulties: 1) the transmitter only has access to  $Q_s[t]$  but not the auxiliary queue  $Y_s[t]$ ; and 2) the obtained power allocation and scheduling scheme is optimal only when  $c[t] = s$  and  $a[t] = 0$ . The first problem is solved by a delayed queue analysis. Since  $Q_s[t] - M_{\max} \leq Y_s[t] \leq Q_s[t]$ , we can show that replacing  $Y_s[t]$  with  $Q_s[t]$  does not affect the attained performance significantly. Second, although the power allocation and scheduling scheme is not optimal when either  $c[t] = 0$  (i.e., no user is scheduled due to the transmission power lower bound) or  $a[t] \geq 1$  (i.e., the scheduled slot is the last reception slot of a rateless code), we show that the performance loss in these two cases are not significant, if  $\varepsilon$  tends to  $0^+$  and  $L_{av}$  is not too small. In particular, the following statement holds:

**Lemma 2.** Define the Lyapunov function  $\Psi(Y_s, Z) = \sum_{s=1}^S Y_s^2 + Z^2$ . If  $Q_s[0] = Z[0] = 0$  and problem (30) has a feasible solution based on  $\hat{\mathbf{h}}[t]$  and  $\{P[t], x_s[t], c[t], M_s[n]\}$  are determined by Algorithm NCA, then

$$\begin{aligned} & E \left\{ \Psi(Y_s[t+1], Z[t+1]) - \Psi(Y_s[t], Z[t]) \right. \\ & \quad \left. - V \sum_{s=1}^S U_s(x_s[t]) \left| Q_s[t], Z[t] \right. \right\} \\ & \leq V B_1 \varepsilon + V B_2 E \{ 1_{\{a[t] \geq 1\}} | Q_s[t], Z[t] \} + B_3 - V \sum_{s=1}^S U_s(x_s^*), \end{aligned} \quad (37)$$

where  $\sum_{s=1}^S U_s(x_s^*)$  is the optimal value of problem (30) and

$$\begin{aligned} B_1 &= \max_{\{s=1, \dots, S\}} \{b_s\} CK, \\ B_2 &= \max_{\{s=1, \dots, S\}} \{b_s\} I_{\max} K, \\ B_3 &= I_{\max}^2 K^2 + P_{peak}^2 + P_{av}^2 + \sum_{s=1}^S D_s^2 + 2M_{\max} I_{\max} K. \end{aligned}$$

*Proof:* See Appendix A. ■

In the proof of Lemma 2, we have used the following result:

**Lemma 3.** If  $Q_s[0] = Z[0] = 0$  and  $\{P[t], x_s[t], c[t], M_s[n]\}$  are determined by Algorithm NCA, then the queue backlogs  $Q_s[t]$  and  $Z[t]$  satisfy

$$Q_s[t] \leq \frac{b_s V}{2}, \quad \forall t \geq 0, \quad (38)$$

$$Z[t] \leq \max_s \{b_s\} \frac{CVK}{2} + P_{peak}, \quad \forall t \geq 0. \quad (39)$$

Therefore, the encoder queues  $Q_s[t]$  are rate stable, and the time-average power constraint (2) holds with probability 1.

The proof of Lemma 3 is provided in our technical report [24] and is omitted here due to space limitations.

Lemma 2 suggests that Algorithm NCA has a performance close to that of problem (30), if  $\varepsilon$  is very small,  $V$  is very large, and the ACK event  $a[t] \geq 1$  does not occur too often. On the other hand, according to Theorem 1, we can obtain

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} 1_{\{a[t] \geq 1\}} \leq \frac{1}{L_{av}}, \quad (\text{w.p.1}) \quad (40)$$

and thereby the ACK event  $a[t] \geq 1$  only happens in no more than  $1/L_{av}$  time slots. In [24], we substitute (40) into Lemma 2 to establish the following theorem:

**Theorem 2.** If  $Q_s[0] = Z[0] = 0$  and problem (30) has a feasible solution based on  $\hat{\mathbf{h}}[t]$  and  $\{P[t], x_s[t], c[t], M_s[n]\}$  are determined by Algorithm NCA, then the achieved time-average rate  $\bar{x}_s$  satisfies

$$\sum_{s=1}^S U_s(\bar{x}_s) \geq \sum_{s=1}^S U_s(x_s^*) - B_1 \varepsilon - \frac{B_2}{L_{av}} - \frac{B_3}{V}, \quad (\text{w.p.1}) \quad (41)$$

where  $x_s^*$ ,  $B_1$ ,  $B_2$ , and  $B_3$  are defined in (37).

Thus, by setting  $\varepsilon \rightarrow 0^+$  and increasing the values of  $V$  and  $L_{av}$ , we can get arbitrarily close to the optimal system utility of problem (30).

Theorem 2 allows trading complexity/delay for performance gains in the absence of accurate CSI: For a large  $V$  parameter, the optimal network utility of infinite block-size codes is reached as  $O(1/L_{av})$ , where  $L_{av}$  is the determinant of the decoding complexity for our rateless code scheme. On the other hand, conventional schemes for fixed-rate codes can only get close to the performance upper bound when the difference between  $\mathbf{h}[t]$  and  $\hat{\mathbf{h}}[t]$  is very small.

We finally note that our scheme significantly reduces the feedback overhead (in terms of bandwidth, time, and power) when no CSI is available to the transmitter: According to (40), the amount of ACK feedback in our scheme is at most  $1/L_{av}$  of those for fixed-rate codes, where an ACK feedback is required in each slot.

#### IV. SIMULATION RESULTS

We present simulation results of Algorithm NCA. In our theoretical analysis, we assume that  $\{h_s[t], \hat{h}_s[t]\}$  are *i.i.d.* across time. Here, we check if our proposed Algorithm NCA is robust for time-correlated wireless channels. To illustrate this, we consider a first order autoregressive (AR) Rayleigh fading process in our simulations. In particular, the channel states  $\{h_s[t], \hat{h}_s[t]\}$  are modeled by

$$h_s[t+1] = \sqrt{0.1} h_s[t] + \sqrt{0.9} n_s[t], \quad (42)$$

$$\hat{h}_s[t] = \sqrt{\rho} h_s[t] + \sqrt{1-\rho} \hat{n}_s[t], \quad (43)$$

where  $n_s[t]$  and  $\hat{n}_s[t]$  are *i.i.d.* circular-symmetric zero-mean complex Gaussian processes, and  $\rho$  represents the accuracy of the imperfect CSI  $\hat{h}_s[t]$ . The mutual information is expressed by  $I(h_s, P) = \max\{\log_2(1 + |h_s|^2 P), I_{\max}\}$ , where the additional upper bound  $I_{\max}$  is due to the limited dynamic range of practical RF receivers. The utility function is determined by  $U_s(x_s) = \ln(1 + x_s/K)$ . The average SNR is given by  $E\{|h_s[t]|^2\} P_{av} = 12$  dB. The results for the case of *i.i.d.* channel is similar, and is omitted here due to space limitation.

Two reference strategies are considered for the purpose of performance comparison: The first one uses infinite block-size channel codes (or equivalently the genie-assisted policy in Section III-B2), which achieves the performance upper bound in problem (30), but is infeasible to implement in

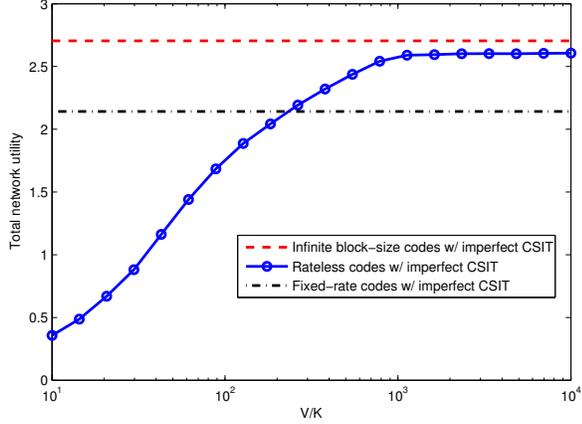


Fig. 1. Simulation results of total network utility versus the algorithm parameter  $V$  for  $L_{av} = 10$ ,  $S = 3$  and  $\rho = 0.8$ .

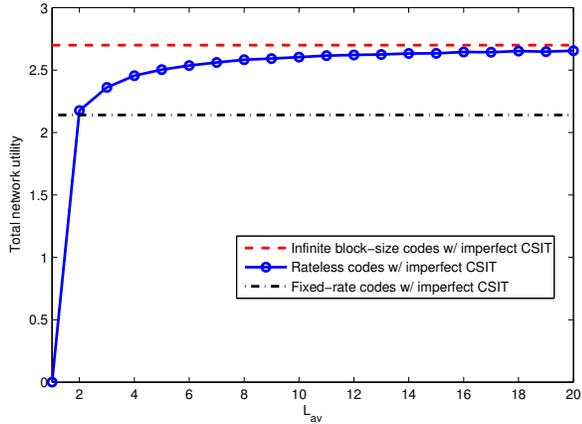


Fig. 2. Simulation results of total network utility versus the time-average block-size  $L_{av}$  for  $V = 10000K$ ,  $S = 3$  and  $\rho = 0.8$ .

a practical system. The second one uses fixed-rate channel codes, where the code-rate  $R$  is selected to maximize the goodput  $R \Pr\{I(h_s, P)K \geq R|h_s\}$ . Network control schemes are designed for these two reference strategies to maximize their corresponding total network utility.

Figure 1 compares the results of total network utility versus the algorithm parameter  $V$  for  $L_{av} = 10$ ,  $S = 3$  and  $\rho = 0.8$ , where  $K$  is the number of symbols in each packet, and ‘‘CSIT’’ stands for CSI at the transmitter. The performance of rateless codes first improves as  $V$  increases, and then tends to a constant value. For sufficiently large  $V$ , the total network utility of rateless codes is much larger than that of fixed-rate codes and is quite close to that of infinite block-size codes. Figure 2 illustrates the complexity/delay vs. utility tradeoff, as it plots the total network utility versus the time-average block-size  $L_{av}$  for  $V = 10000K$ ,  $S = 3$  and  $\rho = 0.8$ . The performance of rateless codes improves as  $L_{av}$  increases. When  $L_{av} \geq 2$ , rateless codes can realize a larger network utility than fixed-rate codes that are also optimized for this system. Figure 3 provides the results of total network utility versus the CSI accuracy  $\rho$  for  $V = 10000K$ ,  $L_{av} = 10$  and  $S = 3$ . The performance of all three strategies improves as  $\rho$  increases. When  $\rho = 0$ , the cumulative spectral efficiency

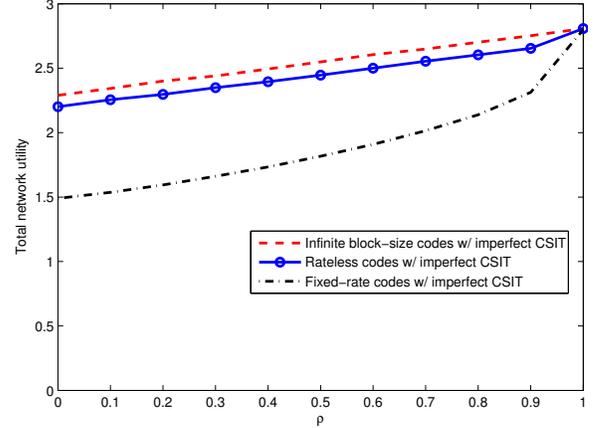


Fig. 3. Simulation results of total network utility versus the CSI accuracy  $\rho$  for  $V = 10000K$ ,  $L_{av} = 10$  and  $S = 3$ .

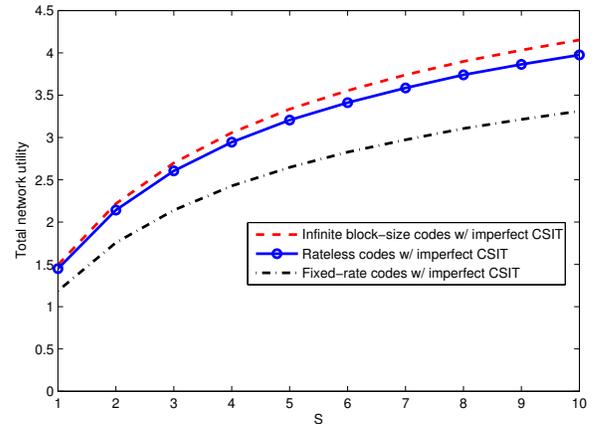


Fig. 4. Simulation results of total network utility versus the number of receivers  $S$  for  $V = 10000K$ ,  $L_{av} = 10$  and  $\rho = 0.8$ .

of rateless codes and fixed-rate codes are given by 3.246 bits/s/Hz and 1.93 bits/s/Hz, respectively, which corresponds to a throughput improvement of 68%. When  $\rho = 1$ , the CSI  $\mathbf{h}[t]$  is perfectly known to the transmitter, and we modify the encoding control scheme (15) by choosing the message size as  $M_s[n] = I(h_s[t_{n,s}], P[t_{n,s}])K$  to eliminate the rate loss as in problem (30). By this, all three strategies achieve the same performance. Finally, Fig. 4 shows the network utility results versus the receiver number  $S$  for  $V = 10000K$ ,  $L_{av} = 10$  and  $\rho = 0.8$ . The performance of all three strategies improves as  $S$  increases, which exhibits a multi-user diversity gain.

## V. CONCLUSION

We have attempted to answer an important question of how to appropriately manage network resources in the absence of (or with imperfect) CSI. To that end, we developed a cross-layer solution for downlink cellular systems with imperfect CSI at the transmitter, which utilize rateless codes to resolve channel uncertainty. To keep the decoding complexity low, we explicitly incorporated time-average block-size constraints in our formulation, subject to which we maximized the system utility. Our network control scheme jointly controls transmission power, scheduling, and channel coding, and exhibits

an elegant utility-complexity tradeoff. Our simulation results suggest that rateless codes can improve the network throughput by up to 68% in certain scenarios, compared with solutions that maximize the utility using fixed-rate codes.

APPENDIX A  
PROOF OF Lemma 2

We need to use the following lemma:

**Lemma 4.** *If the problem (30) has a feasible solution and  $\hat{\mathbf{h}}[t]$  is i.i.d. across time, then for any  $\delta > 0$  there is an  $\hat{\mathbf{h}}$ -only stationary and randomized control scheme  $\{x_s^*, P^*[t], c^*[t]\}$  that satisfies  $0 \leq P^*[t] \leq P_{peak}$ ,  $0 \leq x_s^* \leq D_s$ , and*

$$opt^* \leq \sum_{s=1}^S U_s(x_s^*) + \delta, \quad (44)$$

$$x_s^* \leq E \{ I(h_s[t], P^*[t]) K 1_{\{c^*[t]=s\}} \} + \delta, \quad (45)$$

$$E\{P^*[t]\} \leq P_{av} + \delta, \quad (46)$$

where  $opt^*$  is the maximum network utility of problem (30).

*Proof:* The proof of Lemma 4 is provided in our technical report [24].  $\blacksquare$

*Proof of Lemma 2:* By (13) and (36), we can show that

$$Z[t+1]^2 - Z[t]^2 \leq P_{peak}^2 + P_{av}^2 + 2Z[t]P[t] - 2Z[t]P_{av},$$

and

$$\begin{aligned} & Y_s[t+1]^2 - Y_s[t]^2 \\ & \leq I_{\max}^2 K^2 1_{\{c[t]=s\}} + x_s[t]^2 + 2Y_s[t]x_s[t] - 2Y_s[t]R_s[t]1_{\{a[t]=s\}} \\ & \quad - 2Y_s[t]I(h_s[t], P[t])K1_{\{c[t]=s, a[t]=0\}}. \end{aligned}$$

In Algorithm NCA, if  $c[t] = 0$ , we have  $P[t] = 0$ . Otherwise, if  $c[t] = s \geq 1$ , we can obtain  $P[t] = P_s[t]$ . This further suggests  $P[t] = \sum_{s=1}^S P_s[t]1_{\{c[t]=s\}} = \sum_{s=1}^S P_s[t](1_{\{c[t]=s, a[t]=0\}} + 1_{\{a[t]=s\}})$ . Thus, the drift-plus-penalty can be expressed as

$$\begin{aligned} & E \left\{ \Psi(Y_s[t+1], Z[t+1]) - \Psi(Y_s[t], Z[t]) \right. \\ & \quad \left. - V \sum_{s=1}^S U_s(x_s[t]) \middle| Q_s[t], Z[t] \right\} \\ & \leq I_{\max}^2 K^2 + P_{peak}^2 + P_{av}^2 - 2Z[t]P_{av} \\ & \quad + \sum_{s=1}^S E \left\{ -VU_s(x_s[t]) + x_s[t]^2 + 2Y_s[t]x_s[t] \middle| Q_s[t], Z[t] \right\} \\ & \quad + E \left\{ \sum_{s=1}^S 1_{\{c[t]=s, a[t]=0\}} [-2Y_s[t]I(h_s[t], P_s[t])K \right. \\ & \quad \left. + 2Z[t]P_s[t] \middle| Q_s[t], Z[t] \right\} \\ & \quad + E \left\{ \sum_{s=1}^S 1_{\{a[t]=s\}} [-2Y_s[t]R_s[t] + 2Z[t]P_s[t] \middle| Q_s[t], Z[t] \right\}. \quad (47) \end{aligned}$$

Since  $Q_s[t] - M_{\max} \leq Y_s[t] \leq Q_s[t]$ , from (18), we can obtain

$$\begin{aligned} & E \{ -VU_s(x_s[t]) + x_s[t]^2 + 2Y_s[t]x_s[t] \middle| Q_s[t], Z[t] \} \\ & \leq E \{ -VU_s(x_s[t]) + x_s[t]^2 + 2Q_s[t]x_s[t] \middle| Q_s[t], Z[t] \} \\ & \leq E \{ -VU_s(x_s^*) + x_s^{*2} + 2Q_s[t]x_s^* \middle| Q_s[t], Z[t] \} \\ & \leq D_s^2 - VU_s(x_s^*) + 2Q_s[t]x_s^*. \quad (48) \end{aligned}$$

If Algorithm NCA yields  $c[t] = 0$  and  $P[t] = 0$  in slot  $t$ , we have

$$\begin{aligned} 0 & \leq E \{ 2Q_{\varsigma[t]}[t] E_{\mathbf{h}} \{ I(h_{\varsigma[t]}[t], P_{\varsigma[t]}[t]) K | \hat{h}_{\varsigma[t]}[t] \} \middle| Q_s[t], Z[t] \} \\ & \quad + E \left\{ 1_{\{c[t]=0\}} \sum_{s=1}^S 1_{\{\varsigma[t]=s\}} [-2Q_s[t]K \right. \\ & \quad \left. \times E_{\mathbf{h}} \{ I(h_s[t], P_s[t]) | \hat{h}_s[t] \} + 2Z[t]P_s[t] \middle| Q_s[t], Z[t] \right\} \\ & \stackrel{(a)}{\leq} 2Q_{\varsigma[t]}[t] K E \{ E_{\mathbf{h}} \{ I(h_{\varsigma[t]}[t], \varepsilon) | \hat{h}_{\varsigma[t]}[t] \} \middle| Q_s[t], Z[t] \} \\ & \quad + E \left\{ 1_{\{c[t]=0\}} \sum_{s=1}^S 1_{\{c^*[t]=s\}} [-2Q_s[t]K \right. \\ & \quad \left. \times E_{\mathbf{h}} \{ I(h_s[t], P^*[t]) | \hat{h}_s[t] \} + 2Z[t]P^*[t] \middle| Q_s[t], Z[t] \right\} \\ & \stackrel{(b)}{\leq} V \max_s \{ b_s \} K C \varepsilon + E \left\{ 1_{\{c[t]=0\}} \sum_{s=1}^S 1_{\{c^*[t]=s\}} [-2Q_s[t]K \right. \\ & \quad \left. \times E_{\mathbf{h}} \{ I(h_s[t], P^*[t]) | \hat{h}_s[t] \} + 2Z[t]P^*[t] \middle| Q_s[t], Z[t] \right\}, \quad (49) \end{aligned}$$

where step (a) is due to  $P_{\varsigma[t]}[t] < \varepsilon$  and (16)-(17), step (b) is due to  $Q_s[t] \leq b_s V/2$  in (38) and  $E_{\mathbf{h}} \{ I(h_s[t], \varepsilon) | \hat{h}_s[t] \} \leq C\varepsilon$  by (3)-(4) and the concavity of  $I(h_s, P)$ .

Similarly, if  $c[t] \geq 1$  and  $a[t] = 0$ , we attain

$$\begin{aligned} & E \left\{ \sum_{s=1}^S 1_{\{c[t]=s, a[t]=0\}} [-2Y_s[t]I(h_s[t], P_s[t])K \right. \\ & \quad \left. + 2Z[t]P_s[t] \middle| Q_s[t], Z[t] \right\} \\ & \stackrel{(a)}{\leq} E \left\{ 1_{\{c[t] \geq 1, a[t]=0\}} \sum_{s=1}^S 1_{\{\varsigma[t]=s\}} [2M_{\max} I_{\max} K - 2Q_s[t]K \right. \\ & \quad \left. \times E_{\mathbf{h}} \{ I(h_s[t], P_s[t]) | \hat{h}_s[t] \} + 2Z[t]P_s[t] \middle| Q_s[t], Z[t] \right\} \\ & \stackrel{(b)}{\leq} 2M_{\max} I_{\max} K + E \left\{ 1_{\{c[t] \geq 1, a[t]=0\}} \sum_{s=1}^S 1_{\{c^*[t]=s\}} [-2Q_s[t]K \right. \\ & \quad \left. \times E_{\mathbf{h}} \{ I(h_s[t], P^*[t]) | \hat{h}_s[t] \} + 2Z[t]P^*[t] \middle| Q_s[t], Z[t] \right\}, \quad (50) \end{aligned}$$

where step (a) is due to  $Q_s[t] - M_{\max} \leq Y_s[t]$ ,  $I(h_s, P_{peak}) \leq I_{\max}$ , and  $c[t] = \varsigma[t]$ , and step (b) is due to (16) and (17).

If  $c[t] = a[t] \geq 1$ , the last term of (47) satisfies

$$\begin{aligned} & E \left\{ \sum_{s=1}^S 1_{\{a[t]=s\}} [-2Y_s[t]R_s[t] + 2Z[t]P_s[t] \middle| Q_s[t], Z[t] \right\} \\ & \leq E \left\{ \sum_{s=1}^S 1_{\{a[t]=s\}} 2Z[t]P_s[t] \middle| Q_s[t], Z[t] \right\} \end{aligned}$$

$$\begin{aligned}
&= E \left\{ \sum_{s=1}^S \mathbf{1}_{\{a[t]=s\}} [2Q_s[t] E_{\mathbf{h}} \{ I(h_s[t], P_s[t]) | \hat{h}_s[t] \} K \right. \\
&\quad \left. - 2Q_s[t] E_{\mathbf{h}} \{ I(h_s[t], P_s[t]) | \hat{h}_s[t] \} K + 2Z[t] P_s[t] \right] \Big| Q_s[t], Z[t] \Big\} \\
&\stackrel{(a)}{=} E \left\{ \mathbf{1}_{\{a[t] \geq 1\}} \sum_{s=1}^S \mathbf{1}_{\{c[t]=s\}} [2Q_s[t] K E_{\mathbf{h}} \{ I(h_s[t], P_s[t]) | \hat{h}_s[t] \} \right. \\
&\quad \left. - 2Q_s[t] K E_{\mathbf{h}} \{ I(h_s[t], P_s[t]) | \hat{h}_s[t] \} + 2Z[t] P_s[t] \right] \Big| Q_s[t], Z[t] \Big\} \\
&\stackrel{(b)}{\leq} 2KE \left\{ \mathbf{1}_{\{a[t] \geq 1\}} \max_s [Q_s[t] E_{\mathbf{h}} \{ I(h_s[t], P_s[t]) | \hat{h}_s[t] \}] \right. \\
&\quad \left. Q_s[t], Z[t] \right\} + E \left\{ \mathbf{1}_{\{a[t] \geq 1\}} \sum_{s=1}^S \mathbf{1}_{\{c^*[t]=s\}} [-2Q_s[t] K \right. \\
&\quad \left. \times E_{\mathbf{h}} \{ I(h_s[t], P^*[t]) | \hat{h}_s[t] \} + 2Z[t] P^*[t] \right] \Big| Q_s[t], Z[t] \Big\} \\
&\stackrel{(c)}{\leq} V \max_s \{b_s\} K I_{\max} E \{ \mathbf{1}_{\{a[t] \geq 1\}} | Q_s[t], Z[t] \} \\
&\quad + E \left\{ \mathbf{1}_{\{a[t] \geq 1\}} \sum_{s=1}^S \mathbf{1}_{\{c^*[t]=s\}} [-2Q_s[t] K \right. \\
&\quad \left. \times E_{\mathbf{h}} \{ I(h_s[t], P^*[t]) | \hat{h}_s[t] \} + 2Z[t] P^*[t] \right] \Big| Q_s[t], Z[t] \Big\}, \quad (51)
\end{aligned}$$

where step (a) is due to  $c[t] = a[t] = \varsigma[t]$ , step (b) is due to (16)-(17), and step (c) is due to  $I(h_s, P_{peak}) \leq I_{\max}$  and  $Q_s[t] \leq b_s V/2$  in (38).

Taking the summation of the last terms in (49)-(51), we obtain

$$\begin{aligned}
&E \left\{ (\mathbf{1}_{\{c[t]=0\}} + \mathbf{1}_{\{c[t] \geq 1, a[t]=0\}} + \mathbf{1}_{\{a[t] \geq 1\}}) \sum_{s=1}^S \mathbf{1}_{\{c^*[t]=s\}} [-2Q_s[t] K \right. \\
&\quad \left. \times E_{\mathbf{h}} \{ I(h_s[t], P^*[t]) | \hat{h}_s[t] \} + 2Z[t] P^*[t] \right] \Big| Q_s[t], Z[t] \Big\} \\
&\stackrel{(a)}{=} E \left\{ \sum_{s=1}^S \mathbf{1}_{\{c^*[t]=s\}} [-2Q_s[t] K E_{\mathbf{h}} \{ I(h_s[t], P^*[t]) | \hat{h}_s[t] \} \right. \\
&\quad \left. + 2Z[t] P^*[t] \right] \Big| Q_s[t], Z[t] \Big\} \\
&\stackrel{(b)}{=} -2Q_s[t] \sum_{s=1}^S E \{ I(h_s[t], P^*[t]) K \mathbf{1}_{\{c^*[t]=s\}} \} + 2Z[t] E \{ P^*[t] \}, \quad (52)
\end{aligned}$$

where step (a) is due to  $\mathbf{1}_{\{c[t]=0\}} + \mathbf{1}_{\{c[t] \geq 1, a[t]=0\}} + \mathbf{1}_{\{a[t] \geq 1\}} = 1$ , step (b) is due to the fact that  $\{h_s[t], \hat{h}_s[t]\}$  and the stationary and randomized control scheme  $\{P^*[t], c^*[t]\}$  are independent of  $Q_s[t], Z[t]$ .

By substituting (48)-(52) back to (47), and invoking Lemma 4 with  $\delta \rightarrow 0$ , we can derive

$$\begin{aligned}
&E \left\{ \Psi(Y_s[t+1], Z[t+1]) - \Psi(Y_s[t], Z[t]) \right. \\
&\quad \left. - V \sum_{s=1}^S U_s(x_s[t]) \right] \Big| Q_s[t], Z[t] \Big\} \\
&\leq I_{\max}^2 K^2 + P_{peak}^2 + P_{av}^2 + \sum_{s=1}^S D_s^2 - V opt^* \\
&\quad + 2M_{\max} I_{\max} K + V \max_s \{b_s\} C K \varepsilon \\
&\quad + V \max_s \{b_s\} K I_{\max} E \{ \mathbf{1}_{\{a[t] \geq 1\}} | Q_s[t], Z[t] \}
\end{aligned}$$

$$= V B_1 \varepsilon + V B_2 E \{ \mathbf{1}_{\{a[t] \geq 1\}} | Q_s[t], Z[t] \} + B_3 - V opt^*,$$

and the asserted statement is proved.

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