A Simple Asymptotically Optimal Energy Allocation and Routing Scheme in Rechargeable Sensor Networks

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***Abstract*—In this paper, we investigate the utility maximization problem for a sensor network with energy replenishment. Each sensor node consumes energy in its battery to generate and deliver data to its destination via multi-hop communications. Although the battery can be replenished from renewable energy sources, the energy allocation should be carefully designed in order to maximize system performance, especially when the replenishment profile is unknown in advance. In this paper, we address the joint problem of energy allocation and routing to maximize the total system utility, without prior knowledge of the replenishment profile. We first characterize optimal throughput of a single node under general replenishment profile, and extend our idea to the multi-hop network case. After characterizing the optimal network utility with an upper bound, we develop a low-complexity online solution that achieves asymptotic optimality. Focusing on long-term system performance, we can greatly simplify computational complexity while maintaining high performance. We also show that our solution can be approximated by a distributed algorithm using standard optimization techniques. Through simulations with replenishment profile traces for solar and wind energy, we numerically evaluate our solution, which outperforms a state-of-the-art scheme that is developed based on the Lyapunov optimization technique.**

I. Introduction

Wireless sensor networks have been shown to be immensely useful for monitoring a wide range of environmental parameters, such as earthquake intensity, glacial movements, and water flow. Unattended operation of sensor networks for a long period is highly desirable due to typical remoteness and harshness of the environment. One of the main obstacles in developing long-lived networks is limited battery of sensor nodes. Energy harvesting from various natural sources, such as solar and vibration [1]–[3], has been shown to be effective in alleviating this problem by allowing sensor nodes to replenish their batteries. However, energy management still remains critical, in particular, when one cannot forecast the amount of energy replenishment. Keeping a high battery level may result in low network performance, while maintaining a low battery level increases risk of energy depletion.

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There are several works that address the energy allocation problem in sensor networks with energy replenishment. In [5], a solution has been developed to maximize the total utility for a satellite with energy replenishment, based on dynamic programming (DP) technique. In [6], the authors consider a network where nodes with and without replenishment coexist, and propose two heuristic routing schemes to exploit renewable energy: one scheme looks for the path with minimum number of nodes without replenishment, and the other scheme allows one relaying node to deviate from the shortest path and forward packets opportunistically to nodes with energy replenishment. A battery recharging and discharging model has been developed in [7] for energy replenishment sensor networks. A threshold-based policy has been proven to guarantee at least $\frac{3}{4}$ of the optimal performance. In [8], the authors have developed an energy-adaptive scheme that achieves order-optimal performance for a single node with energy replenishment. Lexicographically maximum rate assignment and routing for perpetual data collection has been studied in [9]. The authors have proposed a centralized solution, which can obtain the optimal lexicographic rate assignment, and a distributed solution, which reaches the optimum only in tree networks with predetermined routing paths. Task scheduling problem is considered for a single node with energy replenishment in [10]. The authors have developed two heuristic schemes that smooth the energy consumption over the running period. In [11], a power-aware routing policy has been developed. Computing a path with the least cost, the solution asymptotically achieves optimal competitive ratio as the network scales. Also, there are a few works that exploit the Lyapunov optimization technique to achieve asymptotic optimality [12], [13]. However, they require the replenishment processes to be i.i.d. or Markovian, which may not be true in practice due to fluky characteristics of renewable energy sources.

In this work, we are interested in developing low-complexity solutions that maximize the total user utility for a rechargeable sensor network, in particular, when future replenishment profile is unknown a priori. The problem can be formulated as a standard convex optimization problem with energy and routing constraints as in [4]. However, the solution requires centralized control and full knowledge of replenishment profiles in the future, which are hardly available in practice. In this paper, we characterize optimal performance and obtain insight into the asymptotical properties. Based on the time-invariant properties, we develop a low-complexity solution that is asymptotically optimal and can be approximated by a distributed algorithm. We summarize our main contributions as follows:

1) We characterize an upper bound for the utility performance of a sensor network with energy replenishment, by constructing an infeasible scheme that outperforms the optimal scheme.
2) We develop a low-complexity online solution that jointly takes into account energy allocation and routing. Without advance knowledge of the future replenishment profile, our solution is provably efficient using estimation of replenishment rate and supply-demand mismatch. We show that the performance gap between our online solution and the infeasible solution for the upper bound diminishes as time tends to infinity.
3) We approximate our solution by a distributed algorithm and evaluate it through simulations based on replenishment profile traces for solar and wind energy. The results show that the solution performs close to the upper bound after a short time period, and outperforms a state-of-the-art scheme that is developed based on the Lyapunov optimization technique.

Unlike the previous works, we consider a larger class of replenishment processes, which only require the existence of a mean value rather than assumptions of i.i.d. or Markovian. To the best of our knowledge, our solution is the first one that achieves asymptotic optimality under general replenishment profiles. Also note that although the solution in [4] achieves optimal performance by making use of fluctuations of the energy replenishment process, it requires future knowledge of the replenishment profile. In contrast, our online solution here does not require such knowledge and achieves asymptotic optimality by relying on long-term characteristic of the energy replenishment process. Through successfully removing time dependency in decisions, we significantly reduce the computational complexity.

Our paper is organized as follows: In Section II, we formulate our problem as a standard utility maximization problem. In Section III, we propose a simple solution that maximizes throughput for a single node. In Section IV, we extend our results to the network case, and develop a low-complexity online solution that achieves asymptotic optimality, and approximate it by an even simpler distributed algorithm. After presenting simulation results in Section V, we conclude our paper in Section VI.

II. System Model

We consider a static sensor network, denoted by $G = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{L}$ is the set of links. We assume a time-slotted system for a period of T time slots. Each node has a battery whose size is assumed to be infinite[1]. Let $r_n(t)$ denote the amount of replenishment energy that arrives at node n in time slot t, while $e_n(t)$ denotes the allocated energy of node n in time slot t. Without loss of generality, we assume that the energy replenishment occurs at the beginning of each slot and the harvested energy is immediately stored in the battery. Let $B_n(t)$ denote the battery level of node n at the beginning of time slot t, which is assumed to be initially empty for simplicity of exposition, i.e., $B_n(0) = 0$. The energy dynamics can be depicted as follows:

$$B_n(t+1) = \max\{B_n(t) + r_n(t) - e_n(t), 0\}. \tag{1}$$

We assume that the replenishment process has a finite mean value $\bar{r}_n$, i.e.,

$$\bar{r}_n \triangleq \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} r_n(t), \tag{2}$$

which is a mild assumption including a larger class of replenishment processes than those used in the prior works [12], [13], where $r_n(t)$ is assumed to be an i.i.d. process.

There are S flows in the network, and each flow s is associated with a source node f_s and a destination node d_s. Let $\mathcal{S}$ denote the set of the source nodes. During a time slot, the data transmission of a node is characterized by a continuously nondecreasing and strictly concave rate-power function $\mu(P)$, satisfying $\mu(0)$= 0. Note that $\mu(P)$ represents the amount of data that can be transmitted using P units of energy in a time slot under a given physical layer modulation and coding strategy. (see [18] for details.)

Let $x^s(t)$ be the amount of data that is delivered from the source f_s to the destination d_s in time slot t over possibly multiple hops and multiple paths. Each user s is associated with a utility function $U_s(\bar{x}^s)$, which reflects the "satisfaction" of user s when it transmits at average data rate $\bar{x}^s \triangleq \frac{1}{T}\sum_{t=1}^{T} x^s(t)$. We assume that $U_s(\cdot)$ is a strictly concave, non-decreasing and continuously differentiable function.

A. Problem Formulation

Our objective is to develop a low-complexity online solution to the joint problem of energy allocation and data routing to maximize aggregate utility for the rechargeable sensor network. Since the rate of energy replenishment is usually much slower than the rate of energy consumption, we assume that the reduction of energy is instantaneous for all the nodes along the path as in [11]. In our work, we do not explicitly consider wireless interference. Many excellent works on scheduling in the presence of interference can be easily incorporated into our solution.

We start with the definition of *rate region* for a node under energy replenishment profile $\vec{r}_n = (r_n(1), r_n(2), \cdots, r_n(T))$.

Definition 1 *(Rate region):* The rate region Λ_n of node n is defined as the set of all vectors $\vec{v}_n = (v_n(1), v_n(2), \cdots, v_n(T))$, such that for any $\vec{v}_n \in \Lambda_n$, there

[1]We refer to our technical report [18] about relaxing the infinite-battery assumption.

exists some energy allocation $\vec{e}_n$ that achieves $\vec{v}_n$, i.e., $v_n(t) = \mu(e_n(t))$, for all $t \in (1, \cdots T)$.

It has been shown that the rate region Λ_n of node n is convex (see Lemma 4 in [4]). Let $w_{ij}^d(t)$ denote the amount of data on the outgoing link $(i,j) \in \mathcal{L}$ for destination node d in time slot t, and we denote its vector as $\vec{w}_{ij} = (\sum_d w_{ij}^d(1), \sum_d w_{ij}^d(2), \cdots, \sum_d w_{ij}^d(T))$. We formulate the utility maximization problem as follows:

$$\text{Problem } \mathbf{A}: \quad \max_{\vec{w}_{ij}, \vec{x}^s, \vec{e}_n} \quad \sum_s U_s \left(\frac{1}{T} \sum_{t=1}^{T} x^s(t) \right)$$

$$\text{subject to} \quad w_{ij}^d(t) \geq 0, \ \ \forall t, \ \forall d, \ \forall (i,j) \in \mathcal{L},$$

$$\sum_{t=1}^{T} \sum_j w_{ij}^d(t) - \sum_{t=1}^{T} \sum_j w_{ji}^d(t) - \sum_{t=1}^{T} \sum_{s: f_s = i, d_s = d} x^s(t) \geq 0,$$

$$\forall d, \text{ and for all } i \neq d,$$

$$\sum_{j:(i,j)\in\mathcal{L}} \vec{w}_{ij} \in \Lambda_i, \ \ \text{for all node } i \in \mathcal{N}, \tag{3}$$

where the second constraint means that total amount of data for destination d into node i is less than or equal to total amount of data out of the node. If any node does not have enough data for a flow to send over all outgoing links, null bits are delivered.

The solution to Problem **A** will determine i) the amount of energy $e_n(t)$ that should be spent for each node $n \in \mathcal{N}$ in time slot t, ii) the amount of data $x^s(t)$ that should be transmitted by each flow $s \in \mathcal{S}$ in time slot t, and iii) routing decisions for each node i, i.e., choosing $w_{ij}^d(t)$ for each link (i,j) and each destination node d.

It has been shown in [4] that Problem **A** is a convex optimization problem and can be solved using the standard convex duality approach if full knowledge of the replenishment profile including for the future is provided. However, such knowledge is difficult to obtain in practice. Furthermore, even if such knowledge is assumed, this problem is computationally highly complex. The culprit is the "time coupling property", which is reflected in the last constraint $\sum_{j:(i,j)\in\mathcal{L}} \vec{w}_{ij} \in \Lambda_i$. In this paper, we show an upper bound on optimal performance that can be obtained by solving Problem **A**. We also provide a low-complexity online solution, the performance of which forms a lower bound. Moreover, we show that the lower bound can get arbitrarily close to the upper bound, when T tends to infinity, which implies that our solution is asymptotically optimal.

III. Throughput Maximization: A Single Node Case

We first investigate throughput performance of optimal energy allocation scheme for a single node. In this section, we omit the subscript n from all the notations defined in the previous section, since all results are for a single node n.

Let $\vec{e}^* = (e^*(1), e^*(2), \ldots, e^*(T))$ denote the optimal energy allocation that maximizes throughput of a single node under energy replenishment $\vec{r} = (r(1), r(2), \cdots, r(T))$. Let $J_{one}^*(T)$ denote the optimal throughput achieved by $\vec{e}^*$, that is,

$$J_{one}^*(T) \triangleq \frac{1}{T} \sum_{t=1}^{T} \mu(e^*(t)). \tag{4}$$

In the following, we provide an upper and a lower bound for $J_{one}^*(T)$, whose difference can be arbitrarily small as T tends to infinity.

A. Upper Bound

Let $\bar{r}$ denote the average replenishment rate, defined as $\bar{r} \triangleq \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} r(t)$.

Proposition 1*:* When T tends to infinity, $J_{one}^*(T)$ is upper bounded by $\mu(\bar{r})$.

Proof: From Eqn. (4) and Jensen's inequality with the concavity of $\mu(\cdot)$, we have that

$$J_{one}^*(T) = \frac{1}{T} \sum_{t=1}^{T} \mu(e^*(t)) \leq \mu(\frac{\sum_{t=1}^{T} e^*(t)}{T}) \leq \mu(\frac{\sum_{t=1}^{T} r(t)}{T}), \tag{5}$$

where the second inequality holds because the total allocated energy can be no greater than the total harvested energy. By taking the limsup on both sides, we can obtain that

$$\limsup_{T\to\infty} J_{one}^*(T) \leq \limsup_{T\to\infty} \mu \left(\frac{\sum_{t=1}^{T} r(t)}{T} \right) = \mu(\bar{r}). \tag{6}$$

■

Proposition 1 also implies that for any $\vec{v} \in \Lambda$, we have $\limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} v(t) \leq \mu(\bar{r})$. Hence, for any $\epsilon > 0$, there exists T_0, such that for all $T > T_0$, we have

$$\frac{1}{T} \sum_{t=1}^{T} v(t) \leq \mu(\bar{r}(1+\epsilon)). \tag{7}$$

This equation will be used later in the proof of the network case.

B. Lower Bound

We consider the following energy allocation scheme, denoted by Scheme-LBONE:

- In each time slot t, average harvested energy is estimated as follows:

$$\hat{r}(t) \triangleq \frac{1}{t} \sum_{\tau=1}^{t} r(\tau). \tag{8}$$

- Using the estimation, energy is allocated as:

$$e(t) = \begin{cases} (1-\epsilon)\hat{r}(t), & \text{if } B(t) + r(t) \geq (1-\epsilon)\hat{r}(t), \\ B(t) + r(t), & \text{otherwise}, \end{cases} \tag{9}$$

  where $\epsilon > 0$ is a system parameter that can be chosen to be arbitrarily small.

We denote the throughput of Scheme-LBONE by $J_{one}^{lb}(T) \triangleq \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[\mu(e(t))]$, where the expectation is taken with respect to the sample space of the replenishment process. We will obtain a lower bound for $J_{one}^*(T)$ by the following proposition.

Proposition 2: When T tends to infinity, $J^*_{one}(T)$ is lower bounded by $\mu((1-\epsilon)^2\bar{r})$.

Proof: From Eqn. (2), we have $\lim_{t\to\infty}\hat{r}(t)=\bar{r}$, which follows that for any $\epsilon>0$, there exists T_1, such that $|\hat{r}(t)-\bar{r}|<\epsilon\bar{r}$ holds for all $t>T_1$. Thus, we have that $(1-\epsilon)\bar{r}<\hat{r}(t)<(1+\epsilon)\bar{r}, \forall t>T_1$. It follows that

$$(1-\epsilon)\hat{r}(t)<(1+\epsilon)(1-\epsilon)\bar{r}<\bar{r}, \quad \forall t>T_1. \tag{10}$$

From Eqn. (9), we consider the battery level $B(t)$ as a queue, and Scheme-LBONE as a work-conserving server with service rate $(1-\epsilon)\hat{r}(t)$, which is strictly less than the average arrival rate $\bar{r}$, for $t>T_1$. Hence, when T tends to infinity, the battery level will increase to infinity almost surely. This implies that the probability that the available energy is greater than $\bar{r}$ tends to one as t tends to infinity, i.e., $\lim_{t\to\infty}P(B(t)+r(t)\geq\bar{r})=1$. Combining with Eqn. (10), we can obtain

$$\lim_{t\to\infty}P\big(B(t)+r(t)>(1-\epsilon)\hat{r}(t)\big)=1. \tag{11}$$

From Eqn. (9), since $e(t)=\min\{(1-\epsilon)\hat{r}(t),B(t)+r(t)\}$, together with Eqn. (11), together with $(1-\epsilon)\hat{r}(t)>(1-\epsilon)^2\bar{r},\ \forall t>T_1$, we have that

$$\lim_{t\to\infty}P(e(t)>(1-\epsilon)^2\bar{r})=1. \tag{12}$$

Eqn. (12) implies that the probability that the allocated energy is great than $(1-\epsilon)^2\bar{r}$ is one.

Next, we will use epsilon-delta arguments to show that $\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}P(e(t)>(1-\epsilon)^2\bar{r})=1$. According to Eqn. (12), it follows that, for any $\delta>0$, there exists T_2, such that for all $t>T_2$, $\big|P(e(t)>(1-\epsilon)^2\bar{r})-1\big|<\frac{\delta}{2}$.

Let $T_3=\frac{4T_2}{\delta}$, now $\forall T>T_3$, we have

$$\begin{aligned}&\left|\frac{1}{T}\sum_{t=1}^{T}P(e(t)>(1-\epsilon)^2\bar{r})-1\right|\\ \leq&\frac{1}{T}\sum_{t=1}^{T_2}\big\{\,\big|P(e(t)>(1-\epsilon)^2\bar{r})\big|+1\big\}\\ +&\frac{1}{T}\sum_{t=T_2+1}^{T}\big|P(e(t)>(1-\epsilon)^2\bar{r})-1\big|\\ \leq&\frac{2T_2}{T_3}+\frac{(T-T_2)}{T}\frac{\delta}{2}\\ \leq&\delta.\end{aligned} \tag{13}$$

Therefore, according to epsilon-delta arguments, it follows that

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}P(e(t)>(1-\epsilon)^2\bar{r})=1. \tag{14}$$

Now we can obtain the performance bound of Scheme-LBONE as follows:

$$\begin{aligned}&J^{lb}_{one}(T)=\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}[\mu(e(t))]\\ &=\frac{1}{T}\sum_{t=1}^{T}\Big\{\mathbb{E}[\mu(e(t))|e(t)>(1-\epsilon)^2\bar{r}]\cdot P(e(t)>(1-\epsilon)^2\bar{r})\\ &+\mathbb{E}[\mu(e(t))|e(t)\leq\ (1-\epsilon)^2\bar{r}]\cdot P(e(t)\leq(1-\epsilon)^2\bar{r})\Big\}\end{aligned} \tag{15}$$

$$\begin{aligned}&\geq\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}[\mu(e(t))|e(t)>(1-\epsilon)^2\bar{r}]\cdot P(e(t)>(1-\epsilon)^2\bar{r})\\ &>\mu((1-\epsilon)^2\bar{r})\cdot\frac{1}{T}\sum_{t=1}^{T}P(e(t)>(1-\epsilon)^2\bar{r}).\end{aligned} \tag{16}$$

where Eqn. (15) holds because of $\mathbb{E}[X]=\mathbb{E}[X|A]P(A)+\mathbb{E}[X|A^c]P(A^c)$. By taking liminf on both sides of Eqn. (16), we can obtain from Eqn. (14) that

$$\liminf_{T\to\infty}J^{lb}_{one}(T)\geq\mu((1-\epsilon)^2\bar{r}). \tag{17}$$

Since Scheme-LBONE is a feasible energy allocation scheme, we have that $\liminf_{T\to\infty}J^*_{one}(T)\geq\mu((1-\epsilon)^2\bar{r})$. ■

Comment: Note that Scheme-LBONE is an online scheme and does not require knowledge of the future replenishment profile. Hence, for a single node case, Propositions 1 and 2 imply that Scheme-LBONE can achieve the performance arbitrarily close to the optimum by choosing ϵ sufficiently small.

IV. Utility Maximization: A Network Case

In this section, we investigate the problem of maximizing utility over the network with energy replenishment. In our formulation Problem **A**, we denote the achievable maximum utility by $J^*(T)\triangleq\max\sum_s U_s\left(\frac{1}{T}\sum_{t=1}^{T}x^s(t)\right)$. We first provide an upper bound on $J^*(T)$ using an infeasible scheme, and then propose a low-complexity online scheme that does not require future knowledge of replenishment profile. We show that the performance of our proposed scheme approaches the upper bound as time T tends to infinity.

A. Upper Bound

We consider a fictitious infeasible scheme, denoted by Scheme-UB, which not only knows in advance the average energy harvesting rate $\bar{r}_i$ for all $i\in\mathcal{N}$, but also can allocate more energy than the harvested energy. Scheme-UB works as follows:

- *Energy allocation*: each node i spends a fixed amount of energy $\bar{r}_i(1+\epsilon)$ in all time slots, i.e.,

$$e_i(t)=\bar{r}_i(1+\epsilon),\ \text{for all } i \text{ and } t. \tag{18}$$

Clearly, this is more than the average replenishment rate and thus infeasible.

- *Routing*: the routing in each time slot t is determined by solving the following strictly convex optimization problem:

$$\max_{\vec{w}_{ij},\vec{x}^s} \quad \sum_s U_s\big(x^s(t)\big)$$
$$\text{subject to} \quad w_{ij}^d(t) \geq 0,\ \forall d,\ \forall (i,j)\in\mathcal{L},$$
$$\sum_{j:(i,j)\in\mathcal{L}} w_{ij}^d(t) - \sum_{j:(i,j)\in\mathcal{L}} w_{ji}^d(t) - \sum_{f_s=i,d_s=d} x^s(t) \geq 0,$$
$$\forall d,\ \text{and}\ \forall i\neq d$$
$$\sum_{j:(i,j)\in\mathcal{L}} \sum_d w_{ij}^d(t) \leq \mu(\bar{r}_i(1+\epsilon)), \quad \forall i\in\mathcal{N}. \tag{19}$$

In contrast to Problem **A**, the third constraint in the above problem is not coupled across time, which implies that routing decision in each time slot t can be solved independently. We denote the unique solution to Eqn. (19) by $\vec{x}_{ub}(t) = [x_{ub}^s(t)]$. Though Scheme-UB is an infeasible scheme, we will show that its performance, defined as $J^{ub}(T) \triangleq \sum_s U_s\left(\frac{1}{T}\sum_{t=1}^T x_{ub}^s(t)\right)$, dominates the optimal performance $J^*(T)$. Also since the energy allocation and routing in Scheme-UB do not change over time, it follows that $x_{ub}^s(t)$ is the same in all time slots, which we denote as x_{ubc}^s. By denoting $J_c^{ub} \triangleq \sum_s U_s\left(x_{ubc}^s\right)$, we have $J^{ub}(T) = J_c^{ub}$.

Proposition 3*:* When T tends to infinity, $J^*(T)$ is upper bounded by $J^{ub}(T)$, and we have that $\limsup_{T\to\infty} J^*(T) \leq \limsup_{T\to\infty} J^{ub}(T) = J_c^{ub}$.

We refer to Appendix A for the proof.

B. Lower Bound

In this subsection, we propose a low-complexity online scheme, denoted by Scheme-LB, and show that its performance approaches the upper bound obtained in the previous section, when T tends to infinity. We begin with the algorithm description of Scheme-LB:

- *Energy allocation*: as in Scheme-LBONE, in each time slot t, each node estimates its average harvested energy as:

$$\hat{r}_i(t) \triangleq \frac{1}{t}\sum_{\tau=1}^{t} r_i(\tau). \tag{20}$$

Then energy is allocated as

$$e_i(t) = \begin{cases} (1-\epsilon)\hat{r}_i(t), & \text{if } B_i(t)+r_i(t) \geq (1-\epsilon)\hat{r}_i(t), \\ B_i(t)+r_i(t), & \text{otherwise.} \end{cases} \tag{21}$$

- *Routing*: routing in each time slot t is determined by solving the following optimization problem.

$$\max_{\vec{w}_{ij},\vec{x}^s} \quad \sum_s U_s\big(x^s(t)\big)$$
$$\text{subject to} \quad w_{ij}^d(t) \geq 0,\ \forall d,\ \forall (i,j)\in\mathcal{L},$$
$$\sum_{j:(i,j)\in\mathcal{L}} w_{ij}^d(t) - \sum_{j:(i,j)\in\mathcal{L}} w_{ji}^d(t) - \sum_{f_s=i,d_s=d} x^s(t) \geq 0,$$
$$\forall d,\ \text{and for}\ i\neq d$$
$$\sum_{j:(i,j)\in\mathcal{L}} \sum_d w_{ij}^d(t) \leq \mu(e_i(t)), \quad \forall i\in\mathcal{N}. \tag{22}$$

We denote the solution to Eqn. (22) by $\vec{x}_{lb}(t) = [x_{lb}^s(t)]$. Note that the difference from Scheme-UB is the energy allocation, which is now based on the estimated average replenishment rate. Let $J^{lb}(T) \triangleq \mathbb{E}[\sum_s U_s\left(\frac{1}{T}\sum_{t=1}^T x_{lb}^s(t)\right)]$, where the expectation is taken over the sample space of the replenishment process. Also, let x_{lbc}^s denote the solution to Eqn. (22) when $e_i(t) = (1-\epsilon)^2\bar{r}_i$ for all $i\in\mathcal{N}$ and $J_c^{lb} \triangleq \sum_s U_s\left(x_{lbc}^s\right)$. Then we can obtain the following proposition.

Proposition 4*:* When T tends to infinity, $J^*(T)$ is lower bounded by $J^{lb}(T)$, and we have that $\liminf_{T\to\infty} J^*(T) \geq \liminf_{T\to\infty} J^{lb}(T) \geq J_c^{lb}$.

We refer to Appendix B for the proof.

Recall that both J_c^{ub} and J_c^{lb} are function of ϵ. Next we show via the following proposition that the lower bound J_c^{lb} can be arbitrarily close to the upper bound J_c^{ub} by setting ϵ sufficiently small.

Proposition 5*:* For any $\delta > 0$, there exists $\epsilon > 0$, such that $|J_c^{ub} - J_c^{lb}| < \delta$.

Proof: We define the ratio of two transmission rates:

$$k \triangleq \min_{i\in\mathcal{N}} \frac{\mu((1-\epsilon)^2\bar{r}_i)}{\mu((1+\epsilon)\bar{r}_i)}. \tag{23}$$

Since $\mu(\cdot)$ is an increasing concave function, we have from Jensen's inequality that

$$\mu((1-\epsilon)^2\bar{r}_i) = \mu((1-\epsilon)^2\bar{r}_i + (2\epsilon-\epsilon^2)\times 0) \geq (1-\epsilon)^2\mu(\bar{r}_i).$$

and similarly we have $\mu((1+\epsilon)\bar{r}_i) \leq (1+\epsilon)\mu(\bar{r}_i)$ for all $i\in\mathcal{N}$. Hence, from the definition of k, it follows that

$$(1-\epsilon)^3 < \frac{(1-\epsilon)^2}{(1+\epsilon)} \leq k \leq 1. \tag{24}$$

Let $(\vec{w}_{ij}^{d*}, \vec{x}^{s*})$ denote an optimal solution to Eqn. (19). Clearly, we have that $J_c^{ub} = \sum_s U_s\big(x^{s*}(t)\big)$. Then we consider another vector $(k\vec{w}_{ij}^{d*}, k\vec{x}^{s*})$. Since $(\vec{w}_{ij}^{d*}, \vec{x}^{s*})$ is an optimal solution to Eqn. (19), it satisfies all the constraints of Eqn. (19). From the first and the second constraints of Eqn. (19), we can easily show that the constant-mulitplied vector $(k\vec{w}_{ij}^{d*}, k\vec{x}^{s*})$ satisfies the first two constraints of Eqn. (22). Also from the third constraint of Eqn. (19) and the definition of k, we have that

$$\sum_j \sum_d k w_{ij}^{d*}(t) \leq k\mu(\bar{r}_i(1+\epsilon)) \leq \mu((1-\epsilon)^2\bar{r}_i)).$$

Hence, the vector $(k\vec{w}_{ij}^{d*}, k\vec{x}^{s*})$ also satisfies the third constraint of Eqn. (22) when $e_i(t) = (1-\epsilon)^2\bar{r}_i$ for all $i \in \mathcal{N}$. Since J_c^{lb} is the achievable maximum utility of Eqn. (22) when $e_i(t) = (1-\epsilon)^2\bar{r}_i$ for all $i \in \mathcal{N}$, we have that

$$\begin{aligned} J_c^{lb} &\geq \sum_s U_s\big(kx^{s*}(t)\big) \\ &\geq k\sum_s U_s\big(x^{s*}(t)\big) \qquad (25) \\ &= kJ_c^{ub} \\ &\geq (1-\epsilon)^3 J_c^{ub}, \qquad (26) \end{aligned}$$

where Eqn. (25) holds because $U_s(\cdot)$ is an increasing concave function and $k \leq 1$, and Eqn. (26) comes from Eqn. (24).

Therefore, we have $(1-\epsilon)^3 J_c^{ub} \leq J_c^{lb} \leq J_c^{ub}$, where the latter inequality directly comes from Propositions 3 and 4. Thus, for any $\delta > 0$, we can find $\epsilon > 0$, such that $|J_c^{ub} - J_c^{lb}| < \delta$. ■

Proposition 5 implies that if ϵ is chosen to be sufficiently small, the performance of Scheme-LB approaches the optimal performance, as T tends to infinity. Hence, Scheme-LB is asymptotically optimal.

C. Distributed Algorithm based on Duality

Note that Scheme-LB should solve a convex optimization problem, i.e., Eqn. (22), in each time slot t in a centralized manner. In this section, we extend our solution and develop a low-complexity distributed scheme that approximates Scheme-LB using the standard optimization technique of duality [15], [16].

From the dual counterpart to Eqn. (22), we can obtain the following solution, denoted by *DualNet*, which can be implemented in a distributed manner. Since the technique is quite standard, we omit details and refer interested readers to our technical report [18].

- At each time t, source s generates data at rate $x^s(t)$ by solving

$$\max_{0 \leq x^s(t) \leq x_{max}} \quad U_s(x^s(t)) - p_{f_s}^{d_s}(t)x^s(t), \qquad (27)$$

  where x_{max} is a constant for the maximum data rate and $p_i^d(t)$ is the associated Lagrange multiplier for each second constraint of Eqn. (22).
- Routing at each node i is determined by solving

$$\max_{0 \leq \sum_j \sum_d w_{ij}^d(t) \leq \mu(e_i(t))} \sum_j \sum_{d \neq i} w_{ij}^d(t)\big(p_i^d(t) - p_j^d(t)\big) \qquad (28)$$

- The Lagrange multipliers are updated as

$$p_i^d(t+1) = \Big[p_i^d(t) - h\Big(\sum_{j:(i,j)\in\mathcal{L}} w_{ij}^d(t) - \sum_{j:(i,j)\in\mathcal{L}} w_{ji}^d(t) - \sum_{f_s=i,d_s=d} x^s(t)\Big)\Big]^+, \qquad (29)$$

  where h is a small step size.

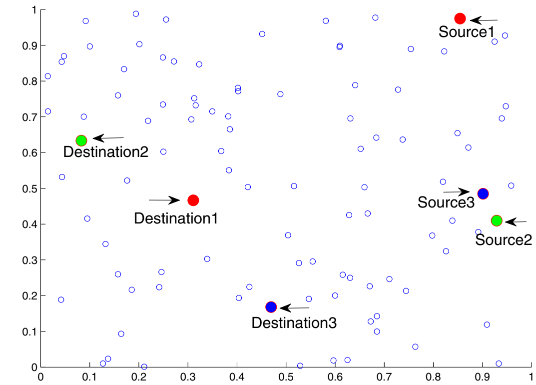

Fig. 1. A sensor network with 100 nodes

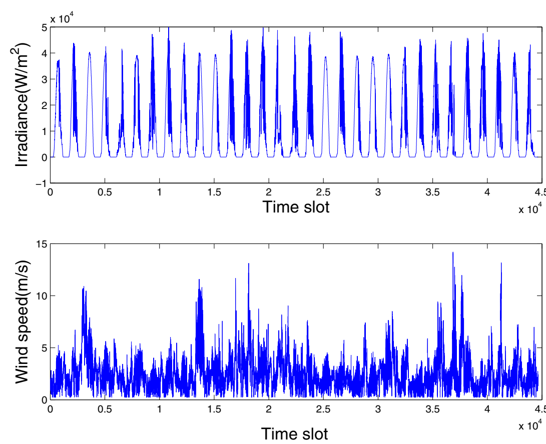

Fig. 2. Measurement for solar and wind energy

It is worthwhile pointing out that Eqn. (28) allocates energy for node i to transmit the data of commodity d to node j, where j and d are chosen for the largest $p_i^d(t) - p_j^d(t)$, which is similar to the well-known back-pressure scheme without interference constraint. Note that using the standard optimization technique, the performance of the dual solution gets closer to the optimal by increasing the number of iterations. Hence, the performance of *DualNet*, which performs a single iteration in each time slot, will improve if we embed multiple iterations in each time slot. Nevertheless, we show via simulations that *DualNet* with a single iteration still achieves good empirical performance that is close to the upper bound.

V. Numerical Evaluation

We evaluate our schemes through simulations. We consider a network with 100 nodes, which are randomly deployed in a 1×1 field, as shown in Fig. 1. We connect each pair of nodes within distance 0.2 by a link. We set three flows in the network, where the source and the destination for each flow are marked with the same color in the figure. We compare the performance of *DualNet* with a state-of-the-art scheme called *ESA* [13], which achieves asymptotic optimality under i.i.d.

energy replenishment profiles. We assume that the rate-power function follows $\mu(P) = \ln(1+10P)$ (bits/sec), and the utility function is given as $U_s(x_s) = \ln(1+x_s)$. We set the parameter ϵ to 10^{-4}.

We simulate the schemes with two different types of renewable energy, solar and wind. We adopt raw data collected at the National Renewable Energy Laboratory [17] for a period of one month (June 5th, 2011-July 5th, 2011) and set each time slot to one minute. Fig. 2 illustrates the two types of replenishment profiles during the month. The solar energy data set (Global 40-South LI-200) measures solar resource for collectors tilted 40 degrees from the horizontal and optimized for year-round performance. From the data, we can obtain the replenishment profile for the solar energy, assuming that each node is equipped with a solar panel of dimension $20mm \times 20mm$. For the wind resource, the data is measured using sensors placed 2 meters from the ground. The power can be calculated from the measured wind speed V as in [19]: $P_{wind} = 0.5 \times \rho \times A \times V^3$, where ρ denotes the air density set to $\rho = 1.23(\text{kg/m}^3)$, and A is the swept area of the wind turbine set to $A = 50mm \times 50mm$.

Figs. 3 and 4 show the simulation results for the solar energy and the wind energy, respectively. The red dotted curve represents the upper bound J_c^{ub} that is obtained by solving Eqn. (19) for the given ϵ. It can be considered as the utility achieved by the infeasible scheme Scheme-UB. The blue dashed curve represents the utility achieved by *DualNet*. For both energy sources, the performance of *DualNet* approaches the upper bound as time increases. Also, an interesting observation in both results is that the performance achieved by *DualNet* has been once close to the upper bound when time is fairly small. This phenomenon occurs because the estimated average harvested energy at that time is greater than the actual (long-term) average. The results also show that *DualNet* outperforms *ESA*, and the performance differences are significant even after a long time period. This is because the Lyapunov optimization technique adopted by *ESA* requires an assumption that the replenishment energy in each time slot is either i.i.d. or Markovian. In contrast, our solution is developed under a mild assumption requiring only the existence of mean replenishment rate.

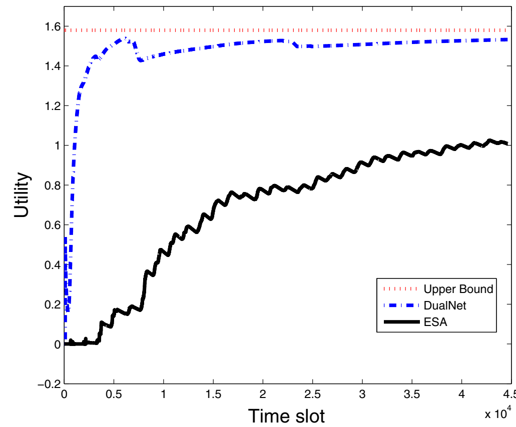

Fig. 3. Utility performance for solar energy

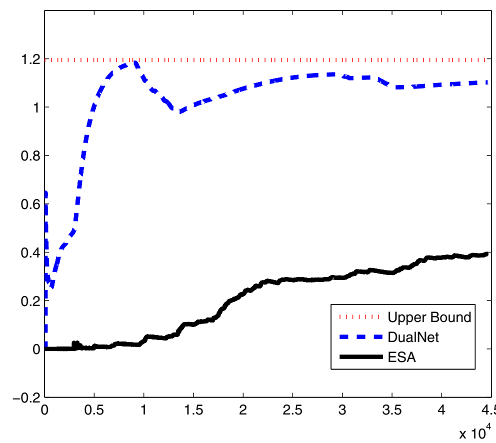

Fig. 4. Utility performance for wind energy

VI. Conclusion

In this paper, we study the joint problem of energy allocation and routing to maximize total user utility in a sensor network with energy replenishment. Under general replenishment profiles with finite mean value, we develop a low-complexity online solution that is asymptotically optimal. Characterizing the optimal performance by an upper bound achieved by an infeasible solution, we show that the long-term performance of our online solution approaches the upper bound. To the best knowledge of the authors, this is the first result that achieves asymptotic optimality in multi-hop networks with general energy replenishment profiles. Also, by removing time coupling properties between controls, our online solution achieves low complexity and can be approximated by a distributed algorithm. Through simulations based on traces from two different types of energy source, we evaluate our solutions and show that it outperforms a state-of-the-art scheme and achieves the performance close to the optimal.

Appendix A
Proof of Proposition 3

Proof: From the stationary property of the problem, we have $x_{ub}^s(t) = x_{ubc}^s$ for all t. Thus, we have that

$$\limsup_{T\to\infty} J^{ub}(T) = \limsup_{T\to\infty} \sum_s U_s\left(x_{ubc}^s\right) = J_c^{ub}.$$

We will prove that $\limsup_{T\to\infty} J^*(T) \leq J_c^{ub}$ by showing that the achievable maximum utility of Eqn. (19) is no smaller than that of a solution to Problem **A**.

We first consider the following problem, where the difference from Problem **A** is the last constraint.

$$\begin{aligned}
\max \quad & \sum_s U_s\big(\frac{1}{T}\sum_{t=1}^{T} x^s(t)\big) \\
\text{subject to} \quad & w_{ij}^d(t) \geq 0,\ \forall t,\ \forall d,\ \forall (i,j) \in \mathcal{L}, \\
& \sum_{t=1}^{T}\sum_j w_{ij}^d(t) - \sum_{t=1}^{T}\sum_j w_{ji}^d(t) - \sum_{t=1}^{T}\sum_{s:f_s=i,d_s=d} x^s(t) \geq 0, \\
& \forall d, \text{ and for } i \neq d, \\
& \sum_{t=1}^{T}\sum_{j:(i,j)\in\mathcal{L}}\sum_d w_{ij}^d(t) \leq T\mu(\bar{r}_i(1+\epsilon)),\ \ \forall i \in \mathcal{N}.
\end{aligned} \tag{30}$$

From Eqn. (7), it is clear that the last constraint in Problem **A**, i.e., $\sum_{j:(i,j)\in\mathcal{L}} \vec{w}_{ij} \in \Lambda_i$, is stricter than the last constraint of Eqn. (30), $\sum_t \sum_{j:(i,j)\in\mathcal{L}}\sum_d w_{ij}^d(t) \leq T\mu(\bar{r}_i(1+\epsilon))$, when T is sufficiently large. Hence, by letting $J^B(T) \triangleq \max \sum_s U_s\left(\frac{1}{T}\sum_{t=1}^{T} x^s(t)\right)$ denote the achievable maximum utility of (30), we have that

$$\limsup_{T\to\infty} J^*(T) \leq \limsup_{T\to\infty} J^B(T). \tag{31}$$

We also consider another strictly convex optimization problem with the same objective function and show that its solution is also the solution to Eqn. (30), which implies that both optimization problems have the same maximum utility.

$$\begin{aligned}
\max \quad & \sum_s U_s\big(\frac{1}{T}\sum_{t=1}^{T} x^s(t)\big) \\
\text{subject to} \quad & w_{ij}^d(t) \geq 0,\ \forall t,\ \forall d,\ \forall (i,j) \in \mathcal{L}, \\
& \sum_{j:(i,j)\in\mathcal{L}} w_{ij}^d(t) - \sum_{j:(i,j)\in\mathcal{L}} w_{ji}^d(t) - \sum_{f_s=i,d_s=d} x^s(t) \geq 0, \\
& \forall t,\ \forall d, \text{ and for } i \neq d, \\
& \sum_{j:(i,j)\in\mathcal{L}}\sum_d w_{ij}^d(t) \leq \mu(\bar{r}_i(1+\epsilon)),\ \forall t,\ \ \forall i \in \mathcal{N}.
\end{aligned} \tag{32}$$

Note that the difference from Eqn. (30) is the last two constraints, where now we do not have summation over time. The solution space of Eqn. (30) includes the solution space of Eqn. (32), since it can be easily shown that if $(\vec{w}_{ij}^d, \vec{x}^s)$ satisfies the constraints of Eqn. (32), it also satisfies the constraints of Eqn. (30). This implies that if the optimal solution to Eqn. (30) also satisfies all the constraints of Eqn. (32), then it is also the optimal solution to Eqn. (32).

Let $(\vec{w}_{ij}^{d'}, \vec{x}^{s'})$ denote the optimal solution to Eqn. (30). Also, we define two constants $w_{ij}^{d''} \triangleq \frac{1}{T}\sum_{t=1}^{T} w_{ij}^{d'}(t)$ and $x^{s''} \triangleq \frac{1}{T}\sum_{t=1}^{T} x^{s'}(t)$. We consider a time-invariant vector $(\vec{w}_{ij}^{d''}, \vec{x}^{s''})$, where $w_{ij}^{d''}(t) = w_{ij}^{d''}$ and $x^{s''}(t) = x^{s''}$ for all time slots. We will show that this time-invariant vector is a common optimal solution to both Eqn. (30) and Eqn. (32).

We first show that it is an optimal solution to Eqn. (30). Since $(\vec{w}_{ij}^{d'}, \vec{x}^{s'})$ is a solution to (30), it satisfies the constraints and we have that $\sum_{t=1}^{T}\sum_j w_{ij}^{d'}(t) - \sum_{t=1}^{T}\sum_j w_{ji}^{d'}(t) - \sum_{t=1}^{T}\sum_{s:f_s=i,d_s=d} x^{s'}(t) \geq 0$. Dividing by T, we obtain that

$$\sum_j w_{ij}^{d''}(t) - \sum_j w_{ji}^{d''}(t) - \sum_{s:f_s=i,d_s=d} x^{s''}(t) \geq 0, \tag{33}$$

for all $t \in [1,T]$, since $(w_{ij}^{d''}(t), x^{s''}(t))$ are equal over time. Hence, the inequality is also true when summing from $t=1$ to T. Hence, $(\vec{w}_{ij}^{d''}, \vec{x}^{s''})$ satisfies the second constraint of Eqn. (30). Similarly, since we have $\sum_{t=1}^{T}\sum_{j:(i,j)\in\mathcal{L}}\sum_d w_{ij}^{d'}(t) \leq T\mu(\bar{r}_i(1+\epsilon))$, dividing by T, we have that

$$\sum_{j:(i,j)\in\mathcal{L}}\sum_d w_{ij}^{d''}(t) \leq \mu(\bar{r}_i(1+\epsilon)). \tag{34}$$

By taking the summation from $t=1$ to T, it yields that $\sum_{t=1}^{T}\sum_{j:(i,j)\in\mathcal{L}}\sum_d w_{ij}^{d''}(t) \leq T\mu(\bar{r}_i(1+\epsilon))$. Therefore, $(\vec{w}_{ij}^{d''}, \vec{x}^{s''})$ satisfies all the constraints of Eqn. (30). Also, we have that

$$\sum_{t=1}^{T} x^{s'}(t) = T\cdot\frac{1}{T}\sum_{t=1}^{T} x^{s'}(t) = \sum_{t=1}^{T} x^{s''}(t).$$

This means that $(\vec{w}_{ij}^{d''}, \vec{x}^{s''})$ achieves the same utility value as the optimal solution $(\vec{w}_{ij}^{d'}, \vec{x}^{s'})$, which implies that it is another optimal solution to Eqn. (30).

We next show that $(\vec{w}_{ij}^{d''}, \vec{x}^{s''})$ is also an optimal solution to Eqn. (32). Note that from our earlier statement on the solution spaces of Eqn. (30) and Eqn. (32), it suffices to show that $(\vec{w}_{ij}^{d''}, \vec{x}^{s''})$ satisfies all the constraints of Eqn. (32), which has already been obtained from Eqn. (33) and Eqn. (34). Hence, $(\vec{w}_{ij}^{d''}, \vec{x}^{s''})$ is an optimal solution to Eqn. (32).

Let $J^C(T)$ denote the achievable optimal utility of Eqn. (32). Since both optimization problem Eqn. (30) and Eqn. (32) have an identical objective function and share at least a common maximizer, the achievable optimal utility should be equal, i.e.,

$$J^B(T) = J^C(T). \tag{35}$$

Further, from our development of the common solution, we can always find an optimal solution to Eqn. (32) that is time-invariant, and thus we can reduce the solution space to time-invariant vectors without affecting the achievable maximum utility. Next, we will prove that $J^C(T) = J_c^{ub}$, which is the achievable maximum utility of the optimal solution to Eqn. (19).

First, note that the time invariant solution $(\vec{w}_{ij}^{d''}, \vec{x}^{s''})$ to Eqn. (32) satisfies the constraints of Eqn. (19), since the constraints of both equations are the same. This implies that $J_c^{ub} \geq \sum_s U_s\big(x^{s''}\big) = \sum_s U_s\big(\frac{1}{T}\sum_{t=1}^{T} x^{s''}(t)\big) = J^C(T)$.

On the other hand, let $(\vec{w}_{ij}^{d*}, \vec{x}^{s*})$ represent one solution to Eqn. (19). Thus, we have $J_c^{ub} = \sum_s U_s\big(x^{s*}\big)$. Consider the time-invariant vector $(\vec{w}_{ij}^{d*}, \vec{x}^{s*})$, where $w_{ij}^{d*}(t) = w_{ij}^{d*}$ and $x^{s*}(t) = x^{s*}$ for all time slots. Note that $(\vec{w}_{ij}^{d*}, \vec{x}^{s*})$ satisfies all the constraints of Eqn. (32) and thus leads to a suboptimal value, i.e., $J^C(T) \geq \sum_s U_s\big(\frac{1}{T}\sum_{t=1}^{T} x^{s*}(t)\big) = \sum_s U_s\big(x^{s*}\big) = J_c^{ub}$. Thus, we have proved that

$$J^C(T) = J_c^{ub}. \tag{36}$$

Therefore, we have that from Eqns. (31), (35) and (36),

$$\limsup_{T\to\infty} J^*(T) \le \limsup_{T\to\infty} J^B(T) = \limsup_{T\to\infty} J^C(T) = J_c^{ub}.$$

■

Appendix B
Proof of Proposition 4

Proof: Since Scheme-LB is a feasible scheme, we have $J^*(T) \ge J^{lb}(T)$ by definition.

The energy allocation component of Scheme-LB is exactly the same as Scheme-LBONE for the single node case, thus all the results in Section III.B also hold. Let A_i denote the event $e_i(t) \ge (1-\epsilon)^2\bar{r}_i$. From Eqn. (12), we have that $\lim_{t\to\infty} P(A_i) = 1$ for each i. Given a finite number of nodes in the network, we can obtain that

$$\lim_{t\to\infty} P(e_i(t) \ge (1-\epsilon)^2\bar{r}_i,\ \forall i \in \mathcal{N}) = 1, \tag{37}$$

which immediately implies (as in Eqn. (13))

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} P(e_i(t) \ge (1-\epsilon)^2\bar{r}_i,\ \forall i \in \mathcal{N}) = 1. \tag{38}$$

Then we can obtain that

$$\begin{aligned}
J^{lb}(T) &= \mathbb{E}\Big[\sum_s U_s(\frac{1}{T}\sum_{t=1}^{T} x_{lb}^s(t))\Big] \\
&\ge \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\Big[\sum_s U_s(x_{lb}^s(t))\Big] \\
&\ge \frac{1}{T}\sum_{t=1}^{T}\Big\{\mathbb{E}\Big[\sum_s U_s(x_{lb}^s(t))|\ e_i(t) \ge (1-\epsilon)^2\bar{r}_i, \forall i \in \mathcal{N}\Big] \\
&\qquad \cdot P(e_i(t) \ge (1-\epsilon)^2\bar{r}_i, \forall i \in \mathcal{N}) \\
&\qquad + \mathbb{E}\Big[\sum_s U_s(x_{lb}^s(t))|\ e_i(t) < (1-\epsilon)^2\bar{r}_i, \text{for some } i \in \mathcal{N}\Big] \\
&\qquad \cdot P(e_i(t) < (1-\epsilon)^2\bar{r}_i, \text{for some } i \in \mathcal{N})\Big\} \\
&\ge \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\Big[\sum_s U_s(x_{lb}^s(t))|\ e_i(t) \ge (1-\epsilon)^2\bar{r}_i, \forall i \in \mathcal{N}\Big] \\
&\qquad \cdot P(e_i(t) \ge (1-\epsilon)^2\bar{r}_i, \forall i \in \mathcal{N}) \\
&\ge \frac{1}{T}\sum_{t=1}^{T} J_c^{lb} \cdot P(e_i(t) \ge (1-\epsilon)^2\bar{r}_i, \forall i \in \mathcal{N}),
\end{aligned} \tag{39}$$

where the first inequality holds due to Jensen's Inequality as well as the concavity of $U(\cdot)$, the second inequality holds because of $\mathbb{E}[X] = \mathbb{E}[X|A]P(A) + \mathbb{E}[X|A^c]P(A^c)$ and the last inequality holds since J_c^{lb} is achieved when $e_i(t) = (1-\epsilon)^2\bar{r}_i, \forall i \in \mathcal{N}$. Taking liminf on the both sides and from (38), we can obtain that

$$\liminf_{T\to\infty} J^*(T) \ge \liminf_{T\to\infty} J^{lb}(T) \ge J_c^{lb}.$$

■

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