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On the Delay Performance of In-network Aggregation in Lossy Wireless Sensor Networks

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***Abstract*—In this paper, we study the implication of wireless broadcast for data aggregation in lossy wireless sensor networks. Each sensor node generates information by sensing its physical environment and transmits the information to a special node called the sink, via multi-hop communications. The goal of the network system is to compute a function at the sink from the information gathered by spatially distributed sensor nodes. In the course of collecting information, in-network computation at intermediate forwarding nodes can substantially increase network efficiency by reducing the number of transmissions. On the other hand, it also intensifies the number of the information contained in a single packet and makes the system vulnerable to packet loss. Instead of retransmitting lost packets, which incurs additional delay, we develop a wireless system architecture that exploits the diversity of the wireless medium for reliable operations. To elaborate, we show that for a class of aggregation functions, wireless broadcasting is an effective strategy to improve delay performance while satisfying reliability constraints. We provide scaling law results on the performance improvement of our solution over unicast architecture with retransmissions. Interestingly, the improvement depends on the transmission range as well as the reliability constraints.**

I. Introduction

Wireless sensor networks consist of a large number of sensor nodes with limited resources of energy, transmission power, network bandwidth, and computation power. Each sensor node monitors physical environment in its neighborhood, collects data, and processes information. In many applications, the goal of the wireless sensor networks is to compute a global function of the information gathered by spatially distributed sensors at a special node called the *sink*. Due to the limited resource of sensor nodes, they often connect each other in an ad hoc fashion and relay others' information via multi-hop communications. Hence, the information gathered at each sensor can be forwarded to the sink.

Distributed *in-network computation* (or *aggregation*) [1] can improve the communication efficiency of the system. It allows for an intermediate node to participate in the computation of the global function: a sensor node can collect information from a subset of sensors and aggregate it by performing computations with the partial information. Compared with previous end-to-end information delivery paradigms, in which intermediate nodes simply relay the received information without change, distributed in-network computation can result in significant performance improvements in energy consumption, memory usage, bandwidth, and delay.

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In this paper, we focus on the delay performance of in-network aggregation in lossy wireless networks. Given a noisy wireless channel, distributed computations at intermediate nodes have a fundamental challenge in maintaining the overall reliability of the function computation [1]–[4]. Since the information contained in a single packet is highly intensified after several in-network computations, a packet loss can significantly impact the computation result, and thus a higher level of protection is required for each packet transmission. A packet can be protected by Error Correcting Code (*EEC*) [5] or can be restored by retransmitting the lost packet. In either case, additional delay is unavoidable. In many applications, it is important to compute the global function in a *timely* and *reliable* manner, and thus limiting the amount of additional delay is important.

To this end, we develop a new network architecture for in-network computation for a class of functions called *Minimal Masking* functions. We focus on the delay performance of the function computation subject to reliability constraints in lossy wireless environments. The delay is expressed in terms of the number of transmissions before a packet is successfully transmitted. We show that aggregation with wireless broadcast can improve the delay performance while satisfying the constraints. Our scaling law result clarifies the relationship between delay performance, reliability, and transmission range. We also provide a distributed algorithm and evaluate its performance through simulations.

In-network aggregation also has been studied in many other aspects [1]. Giridhar and Kumar investigated the maximum achievable computation rate for a class of functions [6]. Energy efficiency in lossy environments has been considered in [3], [4], [7]. Time and energy complexity of distributed computation has been provided in [8], [9]. Our work can be differentiated from the previous work in that i) we focus on the delay performance of in-network computation, ii) we consider reliability constraints in lossy wireless networks, and iii) we investigate the effect of wireless broadcast on the delay performance.

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The paper is organized as follows. We first describe the system model in Section II. We provide scaling law results of asymptotic delay bounds under different reliability constraints and transmission ranges in Section III. In Section IV, we develop a distributed algorithm to implement in-network aggregation with broadcast and simulate to evaluate its performance. Finally, we conclude in Section V.

II. System Model

We consider a sensor network $G(V,E)$ having a set V of sensor nodes and a set E of links, in which n sensor nodes are deployed. The network goal is to compute a global function with information obtained at each sensor node. We assume that the function should be calculated at a special node, called the sink, which is located at the center of the network. Each sensor node not only generates its own data but also relays others data to the sink via multi-hop wireless communications. The wireless channel is assumed to be lossy. A packet loss can be restored by retransmitting the lost packet, which however results in additional delay. Since many applications have both reliability and delay constraints, we focus on the relationship between the reliability and the delay performance and show how they can improve when in-network aggregation is appropriately employed in the wireless system.

We are interested in a class of functions called *Minimal Masking*. In this class, the function result is a vector such that it masks out all the function arguments and each of its components is obtained from an argument. For instance, '$\max$' is a minimal masking function, in which the returned function value is the largest argument that masks out all the others. A minimal masking function has the following properties:

- *Symmetric*: A function f is symmetric if $f(\vec{x},\vec{y}) = f(\vec{y},\vec{x})$.
- *Decomposable*: A function f is decomposable if $f(\vec{x},\vec{y}) = f(f(\vec{x}), f(\vec{y}))$.
- *Componentwise Transitive*: A function f is componentwise transitive if $[f(\vec{x},\vec{y})]_i = [f(\vec{x})]_i$ and $[f(\vec{y},\vec{z})]_i = [f(\vec{y})]_i$ imply that $[f(\vec{x},\vec{z})]_i = [f(\vec{x})]_i$, where $[\cdot]_i$ denote the i-th element of the vector.

Many sensor network services can be realized through minimal masking functions such as finding $\max$ or $\min$, ranging (i.e., $[\min,\max]$), and alerting of events. The properties of the minimal masking functions promote in-network aggregation. Specifically, an intermediate node can collect information from other sensor nodes, and instead of directly relaying the received packets, it processes and aggregates them into a unit of information, i.e., a packet. It then forwards the computed value to the sink or to the next hop. Appropriate use of in-network aggregation can significantly reduce the amount of traffic generated over the network [3], [4], [6].

We assume that the information generated at each sensor node is exact without error. The message passing computation model [9] is assumed, i.e., all the information has to be explicitly transmitted and silence periods (including listening to others' activities) cannot be used to convey information. Hence, if a sensor node does not transmit a packet, its information cannot contribute to the global function computation. Time is slotted (each slot is equal to a sampling period) and all sensor nodes are assumed to be synchronized. We assume that routing is fixed. We first consider a tree topology rooted at the sink, which is a popular structure in wireless sensor networks because information heads for the sink. We also modify the topology later to incorporate wireless broadcast. As in the case above, in trying to obtain scaling laws, we assume perfect scheduling in TDMA network systems. The wireless channel between each pair of transmitting and receiving nodes is assumed to be independent and modeled as a binary channel with non-zero packet loss probability. Let p denote the highest loss probability among the links in the network.

At time slot t, each sensor node ν_i generates information β_i by sensing its physical environments. Our objective is to calculate a *minimal masking* function value $f(\beta_1,\beta_2,\ldots,\beta_n)$ at the sink in a timely and reliable manner. Let β^* denote the correct function value that has to be reported, i.e., $\beta^* := f(\beta_1,\ldots,\beta_n)$. The information β_j is said to be *critical* if the function result without β_j is different from β^*, i.e, $f(\beta_1,\ldots,\beta_{j-1},\beta_{j+1},\ldots,\beta_n) \neq \beta^*$. For instance, let $f(\cdot) = \min\{\cdot\}$ and $\beta_1 = 5$, $\beta_2 = 2$, and $\beta_3 = 9$. Then β_2 is the critical information because $f(\beta_1,\beta_3) = 5 \neq 2 = f(\beta_1,\beta_2,\beta_3)$. Note that if the information is represented by a vector with m components, there can be at most m critical information because of the componentwise transitive property of the minimal masking function. In the sequel, we assume that $m = 1$. This can be easily extended to $m > 1$. As long as m is a constant, our scaling law results do not change. Let $\hat{\beta}$ denote the critical information. Since $\beta^* = \hat{\beta}$ for $m = 1$, we use β^* and $\hat{\beta}$ interchangeably in the remainder of the paper.

III. Asymptotic Analysis of the Delay Bound

Let P_s denote the minimum probability that the sink computes the function correctly. We study the asymptotic delay performance of the sensor system for the following reliability constraint:

$$1 - P_s = O\left(\frac{1}{c(n)}\right), \quad (1)$$

i.e., there exists $n_0, c_0 > 0$ such that for all $n > n_0$, $1 - P_s \leq \frac{c_0}{c(n)}$, where $c(n)$ is an increasing function of n with $c(n) \to \infty$ as $n \to \infty$, indicating how fast reliability is achieved as the number of nodes n increases.

A. Aggregation with unicast

We first consider a point-to-point communication system with a tree topology [9], where a node has a parent and multiple children (except the root node and leaf nodes). Each node obtains information in two ways: from its own sensor and from its children. Once a node collects information from all its children, it aggregates the information including its own into a single packet using the minimal masking function and transmits the packet to its parent over a point-to-point communication link (unicast). The procedure repeats from leaf nodes to the root. We call this network architecture as aggregation with unicast and denote it by $\mathbb{U}$.

Since routes follow the tree structure rooted at the sink, each node ν has a unique parent $\mu(\nu)$. Let $r_u(n) \geq 0$ denote the maximum number of retransmissions allowed at each link. The worst-case probability $P_s(\nu)$ of a successful transmission over link (ν, $\mu(\nu)$) can be obtained as

$$P_s(\nu) = 1 - p^{1+r_u(n)}. \tag{2}$$

We define the depth $d(\nu)$ as the number of hops between node ν and the sink. Let $d^*(n)$ denote the maximum depth over all sensor nodes, i.e., $d^*(n) := \max_{\nu \in V} d(\nu)$. Then, P_s, the worst-case probability of success, is given by the success probability that the critical information arrives at the sink through the longest path. Letting $\hat{\nu}$ denote the node that generates the critical information $\hat{\beta}$, we have

$$P_s = \min_{\hat{\nu}=\nu;\nu \in V} \prod_{k=1}^{d(\hat{\nu})} P_s(\nu_k) = \prod_{k=1}^{d^*(n)} P_s(\nu_k), \tag{3}$$

where $\nu_1 := \hat{\nu}$ and $\nu_{k+1} := \mu(\nu_k)$ for all $k > 1$. The last equality holds because in the worst-case, $\hat{\nu}$ has the largest depth $d^*(n)$. By substituting (2) into (3) and expanding it, we obtain the following two inequalities:

$$P_s \geq 1 - c_1 \cdot d^*(n) \cdot p^{1+r_u(n)},$$
$$P_s \leq 1 - c_1 \cdot d^*(n) \cdot p^{1+r_u(n)} + c_2 \cdot (d^*(n) \cdot p^{(1+r_u(n))})^2,$$

where c_1 and c_2 are some constants. Letting $S(n) := d^*(n) \cdot p^{1+r_u(n)}$, we have $c_1 S(n) - c_2 S(n)^2 \leq 1 - P_s \leq c_1 S(n)$. Note that we must have $S(n) \to 0$ as $n \to \infty$ for the reliability constraint (1) with $c(n) \to \infty$. Hence, we should have[1]

$$1 - P_s = \Theta(S(n)). \tag{4}$$

Again the reliability constraint of (1) requires that $S(n) \leq \frac{c_3}{c(n)}$ for some constant c_3, which can be rewritten as

$$1 + r_u(n) \geq c_4 \cdot \log(d^*(n) \cdot c(n)) + c_5, \tag{5}$$

where c_4 and c_5 are some constants. Hence, we have the following necessary and sufficient condition to satisfy the reliability constraint (1):

$$r_u(n) \geq \Theta(\log(d^*(n) \cdot c(n))). \tag{6}$$

We now consider the delay caused by retransmissions to achieve the given reliability constraints. Estimating the delay as the number of transmissions, the worst-case delay D_u^* can be presented as $D_u^* = \min_{r_u(n)}\{d^*(n) \cdot (1+r_u(n))\}$. From (6), we have $\min(1+r_u(n)) = \Theta(\log(d^*(n) \cdot c(n)))$, which implies that a packet should be transmitted at least $\Theta(\log(d^*(n) \cdot c(n)))$ times to satisfy the reliability constraints. Hence, we obtain the worst-case delay under aggregation with unicast as

$$D_u^* = \Theta(d^*(n) \cdot \log(d^*(n) \cdot c(n))). \tag{7}$$

[1] $f(n) = \Theta(g(n))$ means that there exists constants $c_1, c_2 > 0$ and n_0 such that for all $n \geq n_0$, $c_1 g(n) \leq f(n) \leq c_2 g(n)$.

B. Aggregation with wireless broadcast

In this section, we propose a new network architecture with wireless broadcast to improve the delay performance while achieving the same level of reliability. We explicitly exploit diversity from wireless broadcast. We first describe the system architecture and then analyze its delay performance.

We modify the tree structure in Section III-A by allowing nodes to broadcast a packet to *multiple parents*. To elaborate, each node (at depth d) is assumed to have at least $x(n)$ parents (at depth $d-1$), and transmits a packet through wireless broadcast channel to all parents (1+$r_b(n)$) times. At the root, we assume that the sink has $x(n)$ antennas and it can process signals from multiple antennas. In this architecture, we say that a node *successfully transmits* a packet if the broadcasted packet is successfully received by one of $x(n)$ parents. Note that each packet contains aggregated information abstracting all the information successfully collected by the transmitter. Due to the properties of the minimal marking function, it suffices that each node successfully transmit the aggregated information to *one of its parents* in order to ensure that the critical information $\hat{\beta}$ is successfully delivered to the sink. We call this architecture as aggregation with broadcast and denote it by $\mathbb{B}$.

The intuition can be better described using Fig. 1. Assuming that links are bidirectional, the dotted lines in the figure are links between two end nodes, and arrows indicate a transmission from a child to a parent. A failed transmission is marked by a cross. Fig. 1(a) illustrates that two transmissions from node 2 fail under $\mathbb{U}$. On the other hand, Fig. 1(b) shows that a single broadcast can transmit the packet to node 6 successfully. Hence, $\mathbb{U}$ requires four transmissions to deliver information B to the sink, whereas $\mathbb{B}$ needs two transmissions.

Note that the aggregation with broadcast $\mathbb{B}$ appears to be a little like *flooding*, but there are significant differences. While flooding is very ineffective because of broadcasting multiple duplicate packets, $\mathbb{B}$ reduces this inefficiency by in-network aggregation. Moreover, it takes advantage of the diversity of wireless broadcast, which is not exploited in flooding.

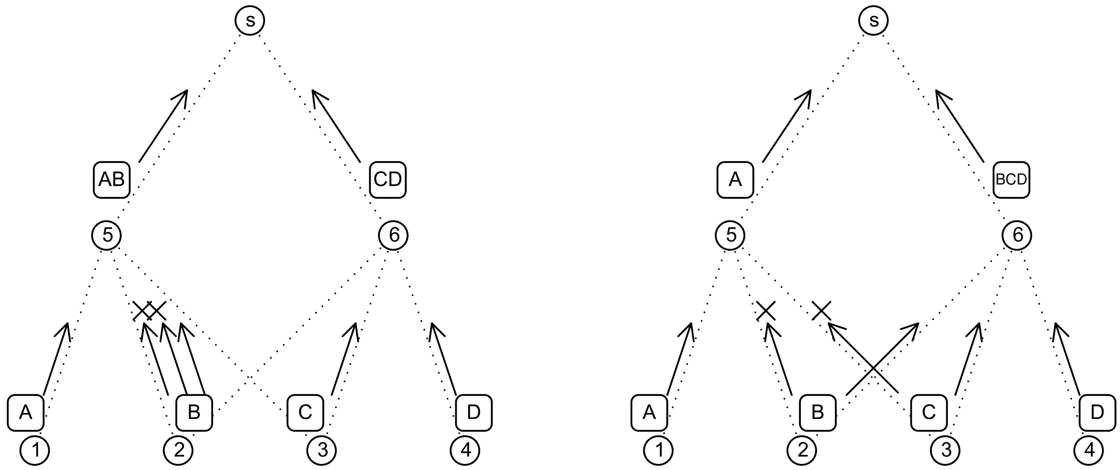
We now estimate the worst-case probability of successful function computation P_s under $\mathbb{B}$. Assuming independent packet losses over $x(n)$ links, a packet transmission from node ν is successful with probability

$$P_s(\nu) = 1 - p^{x(n) \cdot (1+r_b(n))}, \tag{8}$$

where $r_b(n)$ is the maximum number of retransmissions. Again, since the critical information $\hat{\beta}$ has to be delivered via at most $d^*(n)$ hops to reach the sink, the guaranteed probability P_s of a successful information delivery can be represented by

$$P_s \geq \min_{\hat{\nu}=\nu;\nu \in V} \prod_{k=1}^{d(\hat{\nu})} P_s(\nu_k) = \prod_{k=1}^{d^*(n)} P_s(\nu_k), \tag{9}$$

where $\nu_1 := \hat{\nu}$ and ν_{k+1} is one of parents of ν_k. Note that unlike $\mathbb{U}$, the first equality in (3) is changed to an inequality in (9) because the information $\hat{\beta}$ can take multiple paths to



(a) Aggregation with unicast ($\mathbb{U}$)

(b) Aggregation with wireless broadcast ($\mathbb{B}$)

Fig. 1. Transmissions over lossy wireless links. Each transmission is denoted by an arrow, and a failed transmission is denoted by a cross at the end of the arrow. Under aggregation with unicast, it needs four transmissions for information B to be successfully delivered to the sink, while it needs two transmissions under aggregation with broadcast.

the sink. From (8), we can obtain

$$1 - P_s \le d^*(n) \cdot p^{x(n)(1+r_b(n))}. \tag{10}$$

Then the following inequality suffices to satisfy the reliability constraint (1):

$$x(n) \cdot (1 + r_b(n)) \ge c_6 \cdot \log(d^*(n) \cdot c(n)) + c_7, \tag{11}$$

where c_6 and c_7 are some constants. Hence, we obtain a sufficient condition for the reliability as

$$x(n) \cdot (1 + r_b(n)) \ge \Theta(\log(d^*(n) \cdot c(n))). \tag{12}$$

Note that if each node broadcasts its packet to $\Theta(\log(d^*(n) \cdot c(n)))$ parents, the reliability constraint (1) can be satisfied with $r_b(n) = 0$. Since the delay bound D_b^* can be presented as $D_b^* = \min_{r_b(n)}\{d^*(n) \cdot (1 + r_b(n))\}$, we have

$$D_b^* \le \Theta\left(d^*(n) \cdot \max\{1, \tfrac{\log(d^*(n)\cdot c(n))}{x(n)}\}\right). \tag{13}$$

C. Performance in geometric networks

We now consider a popular scenario in which sensor nodes are randomly deployed in a geometric space, and evaluate the delay performance of aggregation schemes with unicast $\mathbb{U}$ and with broadcast $\mathbb{B}$. We derive the gain of $\mathbb{B}$ over $\mathbb{U}$ for geometric networks, where the reliability constraint and transmission range are a function of the number of nodes. We show that in general a higher gain can be achieved with stronger reliability requirements and larger transmission ranges.

We first start with the gain for the previous (non-geometric) tree network. We define the maximum delay gain of $\mathbb{B}$ over $\mathbb{U}$ as $G^* := D_u^*/D_b^*$. From (7) and (13), we have[2]

$$\begin{aligned} G^* := \frac{D_u^*}{D_b^*} &= \Omega\left(\frac{d^*(n) \cdot \log(d^*(n) \cdot c(n))}{d^*(n) \cdot \max\{1, \frac{\log(d^*(n)\cdot c(n))}{x(n)}\}}\right) \\ &= \Omega\left(\frac{x(n)\log(d^*(n) \cdot c(n))}{x(n) + \log(d^*(n) \cdot c(n))}\right). \end{aligned} \tag{14}$$

Suppose that the network has depth $d^*(n) = \log n$ with the reliability constraint $c(n) = \log n$. From (7) and (13), we have $D_u^* = \Theta(\log n \cdot \log\log n)$ under $\mathbb{U}$, and $D_b^* \le \Theta(\log n \cdot \max\{1, \log\log n/x(n)\}) = \Theta(\log n)$ under $\mathbb{B}$ when $x(n) = \Theta(\log\log n)$. Hence, if each node can broadcast to $\Theta(\log\log n)$ parents, $\mathbb{B}$ outperforms $\mathbb{U}$ by $G^* = \Omega(\log\log n)$.

However, the achievability of $x(n) = \Theta(\log\log n)$ depends on the topology of the underlying network. In geometric networks, both $d^*(n)$ and $x(n)$ are related to the topological structure and we need to incorporate some topological notion into our analysis. To this end, we study the delay performance of aggregation schemes in random networks, where nodes are uniformly placed, subject to reliability constraints. In our analysis, we do not take into account edge effects, assuming that all nodes have the same order of parent nodes. Note that in sensor networks, most traffic heads for the sink. Hence, by carefully locating the sink, there would be few transmissions on the edge of the network. The assumption can be further supported by our development of a distributed algorithm in Section IV.

Given a network of n sensor nodes uniformly and independently distributed on a disk of radius 1, we assume that each node has an identical transmission range $t(n)$ and that the

[2] $f(n) = \Omega(g(n))$ means that there exists constants $n_0, c_0 > 0$ such that for all $n \ge n_0$, $f(n) \ge c_0 g(n)$.

sink is located at the center. We also assume that straight-line routing has been employed, thus achieving $d^*(n) = \frac{1}{t(n)}$.

Under aggregation with unicast $\mathbb{U}$, the delay bound directly comes from (7). By replacing $d^*(n)$ with $\frac{1}{t(n)}$, we have

$$D_u^* = \Theta\left(\frac{1}{t(n)} \log \frac{c(n)}{t(n)}\right). \quad (15)$$

On the other hand, under aggregation with broadcast $\mathbb{B}$, we have $x(n) \leq \Theta(nt(n)^2)$ because each node has $nt(n)^2$ neighboring nodes in its transmission range. We can achieve the equality by setting the parents of each node to the set of nodes within a sector of its transmission range (to the direction of the sink). Then, from (13) and $d^*(n) = \frac{1}{t(n)}$, we can obtain the delay bound as

$$D_b^* \leq \Theta\left(\max\left\{\frac{1}{t(n)}, \frac{1}{nt(n)^3} \log \frac{c(n)}{t(n)}\right\}\right). \quad (16)$$

From (15) and (16), we can present the gain G^* of $\mathbb{B}$ over $\mathbb{U}$ in geometric networks as

$$G := \frac{D_u^*}{D_b^*} = \Omega\left(\frac{nt(n)^2 \cdot \log \frac{c(n)}{t(n)}}{nt(n)^2 + \log \frac{c(n)}{t(n)}}\right). \quad (17)$$

As an example, we consider a random geometric network with minimal connectivity. It has been shown in [10] that $t(n)$ should be at least $\Theta(\sqrt{\frac{\log n}{n}})$ for the network to be asymptotically connected with high probability. Using $t(n) = \Theta(\sqrt{\frac{\log n}{n}})$, we have $d^*(n) = \Theta\left(\sqrt{\frac{n}{\log n}}\right)$. Suppose that $c(n) = \log n$, i.e., the reliability requirement enforces that $1 - P_s = O(\frac{1}{\log n})$. In this case, the delay bound of $\mathbb{U}$ can be written as $D_u^* = \Theta(\sqrt{n \log n})$ from (15). For $\mathbb{B}$, it suffices to satisfy $x(n)(1 + r_b(n)) = \Theta\left(\log\left(\frac{n \cdot c(n)}{\log n}\right)\right)$ to achieve the same level of reliability. Since each node can have $x(n) = \Theta(nt(n)^2) = \Theta(\log n)$ parents, the condition can be satisfied with $r_b(n) = 0$ when $c(n) = \log n$. This implies that there is no need of retransmission under $\mathbb{B}$. Hence, we can obtain the delay bound $D_b^* = O\left(\sqrt{\frac{n}{\log n}}\right)$ and the gain $G^* = \Omega(\log n)$. Note that $nt(n)^2 = \log n$ is the number of nodes in the transmission area of a node. This implies that $\mathbb{B}$ can potentially achieve a gain in delay as large as the diversity gain of wireless broadcast.

In general, from (17), the gain depends on both $t(n)$ and $c(n)$. We tabulate the gains for various network environments in Table I. The first column shows that the gain is dominated by the broadcasting areas in multi-hop networks with minimal connectivity. The last column shows that the gain is dominated by the reliability constraint in single-hop networks.

TABLE I

GAINS (D_u^*/D_b^*) OF $\mathbb{B}$ OVER $\mathbb{U}$ UNDER VARIOUS TRANSMISSION RANGES AND RELIABILITY CONSTRAINTS.

	$t(n) = \sqrt{\frac{\log n}{n}}$	$t(n) = \frac{1}{\sqrt{\log n}}$	$t(n) = 1$
$c(n) = \log n$	$\Omega(\log n)$	$\Omega(\log \log n)$	$\Omega(\log \log n)$
$c(n) = n$	$\Omega(\log n)$	$\Omega(\log n)$	$\Omega(\log n)$
$c(n) = \exp n$	$\Omega(\log n)$	$\Omega(\frac{n}{\log n})$	$\Omega(n)$

IV. DISTRIBUTED ALGORITHM

In this section, we develop a distributed algorithm for aggregation with broadcast using a tiered routing structure. Although the tiered structure has appeared in the literature for light-weight routing [11] and efficient sleep/wake scheduling [12], the purpose of our design is quite different. Unlike [11], [12], we assume that wireless links are lossy, and that the network has a specific goal of computing a minimal masking function. By exploiting the diversity of the wireless medium, we intend to improve the delay performance while satisfying reliability constraints.

We first describe our algorithm, and show that the algorithm achieves the delay performance of (16). To this end, we show that under the algorithm, each sensor node has at least $\Theta(nt(n)^2)$ parents and the maximum hop distance to the sink is at most $\Theta(\frac{1}{t(n)})$. We also evaluate its performance through simulations.

A. Algorithm with tiered structure

We assume that n wireless sensor nodes are uniformly deployed over a disk of radius 1. Each node has an identical transmission range of $t(n)$. We divide the networks into $\frac{1}{\delta t(n)}$ circular tiers as shown in Fig. 2, with $0 < \delta < 1$. Each tier has an identical width of $\delta t(n)$. Let T_i denote the set of nodes in the i-th tier, which is an area within distance of $(\delta t(n) \cdot (i-1), \delta t(n) \cdot i]$ from the sink. The sink is the only node in T_0.

The network is a time-slotted TDMA system. At the beginning of each time slot, each sensor node generates a packet with the sensed information. A time slot is further divided into mini-slots and in each mini-slot, a single packet can be transmitted. Let D_b denote the delay performance of the algorithm, which is estimated in the number of transmissions, i.e., mini-slots for the sink to compute the function.

Routing is simplified using the tiered structure; Every node μ in T_i is a parent of node ν in T_{i+1} if its distance is no greater than $t(n)$. Transmissions are scheduled from the outermost tier to the sink tier-by-tier one at a time, so that nodes in T_i can transmit only after all nodes in T_{i+1} finish their transmissions. Assuming the two-hop interference model, under which two links within two-hop distance cannot transmit simultaneously, multiple nodes within a tier can transmit simultaneously if the distance between any two of them is greater than $2t(n)$. We group nodes in each tier into mutually exclusive subsets such that all nodes in a subset can transmit simultaneously. Let $H(i,j)$ denote the j-th subset in T_i, and let h_i denote the total number of subsets in each tier T_i such that $\cup_{j=1}^{h_i} H_{i,j} = T_i$. Clearly, all nodes in T_i can finish a *single* transmission in h_i mini-slots. We have the total delay D_b to compute the function

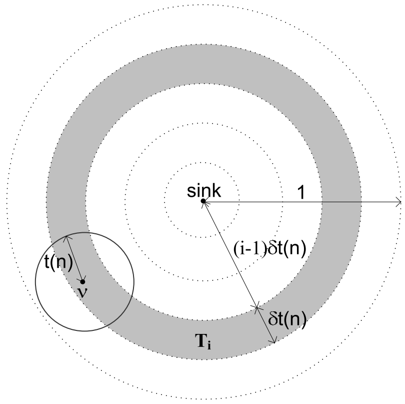


Fig. 2. Network with tiered structure.

as

$$D_b = \sum_{i=1}^{1/\delta t(n)} h_i(1 + r_b(n)). \tag{18}$$

The overall algorithm proceeds as follows: At the beginning of every time slot, each node originates a packet with sensed information. Nodes in $H(i,j)$ broadcast their packets $(1 + r_b(n))$ times in decreasing order of i and increasing order of j, such as $H(\frac{1}{\delta t(n)}, 1), H(\frac{1}{\delta t(n)}, 2), \ldots, H(\frac{1}{\delta t(n)}, h_{\frac{1}{\delta t(n)}}), H(\frac{1}{\delta t(n)} - 1, 1), H(\frac{1}{\delta t(n)} - 1, 2), \ldots, H(1, h_1)$. Note that with this ordering, nodes in T_i start their transmissions after all nodes in T_{i+1} finish transmissions. Then nodes in T_i who receive a packet from a node in T_{i+1} do aggregation using the minimal masking function, and update its packet if necessary. The detailed algorithm is as shown in Algorithm 1.

Algorithm 1 Distributed aggregation with wireless broadcast.

for $i = \frac{1}{\delta t(n)}$ to 1 **do**
  for $j = 1$ to h_i **do**
    Each node ν in $H(i,j)$ broadcasts its (aggregated) information $(1 + r_b(n))$ times.
    if node $\mu \in T_{i-1}$ receives the packet **then**
      Node μ do aggregation and update its information.
    end if
  end for
end for

Now we show that under Algorithm 1, each node has at least $\Theta(nt(n)^2)$ parents and the maximum hop distance from a node to the sink is $\Theta(\frac{1}{t(n)})$. Since two nodes transmitting simultaneously in $H(i,j)$ are separated by at least $2t(n)$, they have no common parent under the two-hop interference model. Then the minimum number of parents $x(n)$ can be bounded as follows. Suppose that node ν is located in T_i as shown in Fig. 3. The number of parents of node ν in T_{i-1} is no smaller than the number of nodes in the shaded area. For each node $\nu \in V$, there exists $\delta < \delta_\nu < 1$ such that the distance between ν and the shaded area is $\delta_\nu t(n)$. Let $\delta^* := \max_{\nu \in V} \delta_\nu$. Since nodes are uniformly distributed with density $\frac{n}{\pi}$, it can be easily shown that the number of nodes in the shaded area is bounded below by[3]

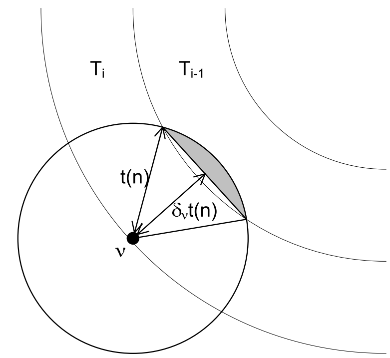


Fig. 3. Parents (in T_{i-1}) of node ν (in T_i) is located in the shaded area.

$$x(n) \geq \left(\frac{\cos^{-1}\delta^*}{\pi} - \delta^*\sqrt{1-\delta^{*2}}\right)\frac{nt(n)^2}{\pi} = \Theta(nt(n)^2). \tag{19}$$

Since each node has at least $\Theta(nt(n))^2$ parents, Algorithm 1 can achieve the required reliability (1) by satisfying (12).

Further, since each tier has the width $\delta t(n)$ and a packet is transmitted tier-by-tier, there are at most $\frac{1}{\delta t(n)}$ tiers and we have the maximum number of hops to the sink as

$$d^*(n) = \frac{1}{\delta t(n)} = \Theta\left(\frac{1}{t(n)}\right). \tag{20}$$

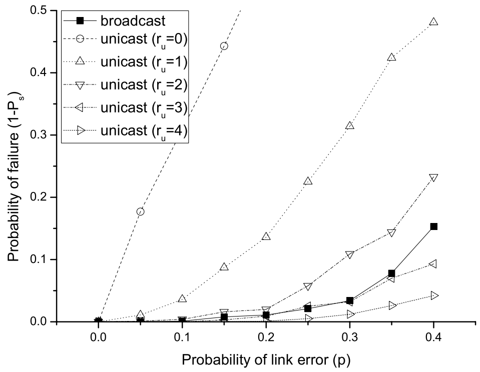
Hence, from (19) and (20), Algorithm 1 achieves the delay performance (16) and the gain (17).

B. Simulations

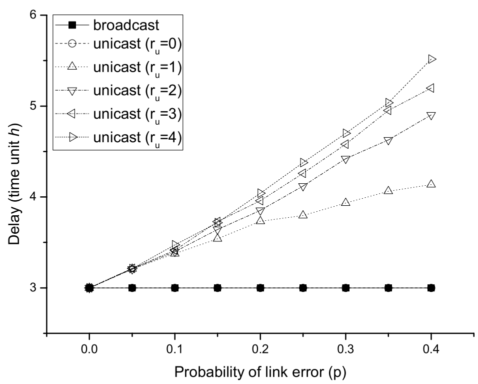
In this section, we simulate both Algorithm 1 and aggregation with unicast to compare their performance. We are interested in the successful probability of the function computation as well as the delay.

We consider a wireless sensor network with 100 nodes, which are randomly placed in a disk of radius 1. The transmission range of each node is set to 0.5. The tiered structure has the width 0.25 ($\delta = \frac{1}{2}$), and a parent-child relationship has been established between every pair of nodes if the two nodes are located in neighboring tiers and their distance is less than 0.5. In this settings, there are four tiers. We locate the sensor node that generates the critical information at the boundary of the network, i.e., in the 4-th tier. Since routing follows the tiered structure, both aggregations with unicast and broadcast takes at least four transmissions for the packet generated from the sensor node to arrive at the sink. We assume that for all tiers i, it takes the same number of mini-slots $\hat{h}$ for all nodes in T_i to finish a single transmission, and consider $\hat{h}$ as a time unit for the delay performance. For aggregation with unicast, we change the number of retransmissions r_u from 0 to 5, and for aggregation with broadcast we set $r_b = 0$. All links

[3] Although we implicitly assume that the shaded area is completely included in T_{i-1}, the same order results can be obtained when the shaded area stretches to inner tiers.



(a) Loss rate



(b) Delay

Fig. 4. Loss rate and delay of information deliverty. The loss rates of aggregation with unicast improves with the number of retransmissions r_u in Fig. 4(a). However, the increase of r_u leads to higher delay performance in Fig. 4(b).

are assumed to fail transmission with the same probability p. Changing p, we count the number of time units ($\hat{h}$) required for the sink to receive the critical information and measure the rate of failure, i.e., loss of the information. We run each simulation 1000 times and average the results.

Fig. 4 illustrates the loss rate of the critical information and the delay performance. Fig. 4(a) shows that aggregation with unicast can improve the loss rate with more retransmissions. However, it also increases the delay as shown in Fig. 4(b). If we have the delay bound of $6\hat{h}$, which is the minimum achievable delay plus $2\hat{h}$, then it is observed in Fig. 5 that the retransmission strategy cannot improve the loss rate beyond a certain threshold, and that aggregation with broadcast achieves better performance.

In the next experiment, we show that the performance of aggregation with broadcast improves with a higher node

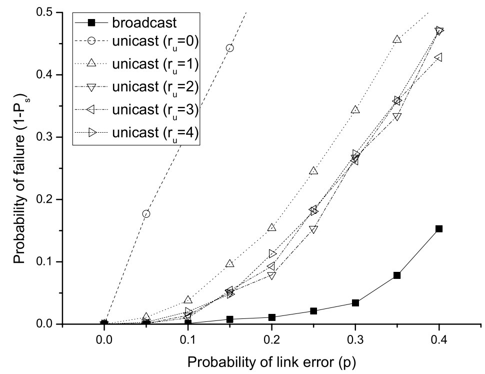


Fig. 5. Loss rate taking into account delay. When accounting for information delivery within delay bound, broadcasting without retransmissions shows the better performance than unicast with retransmissions. The delay bound is set to the minimum plus $2\hat{h}$.

density. We also consider a sensor networks with randomly placed nodes in a unit disk. Transmission range is fixed to 0.3, and the width of tier is set to 0.15 ($\delta = \frac{1}{2}$) resulting in 7 tiers. Links between parents and children are established as before. The sensor node that initiates the critical information is placed in the outermost tier. Hence, it takes at least $7\hat{h}$ for the information to arrive at the sink. For aggregation with unicast, we set $r_u = 2$. The delay bound is set to $9\hat{h}$ (the minimum plus $2\hat{h}$). For aggregation with broadcast, we set $r_b = 0$, which implies that that all successful information delivery is within $7\hat{h}$ (the minimum). The simulation results are shown in Fig. 6. Under aggregation with broadcast, the more nodes are deployed, the less probabilities of failure for the information delivery are achieved. This implies that under wireless network circumstances with severely noisy channels, broadcast strategy with a number of sensor nodes will be desirable, in particular when a high delay performance is requested.

V. Conclusion

In a wireless sensor network, in-network aggregation can significantly improve efficiency when the goal of the network is to compute a global function. However, since a loss of an aggregated packet is more harmful than an unaggregated packet, a higher level of protection is required for reliable operations in lossy wireless environments. In this paper, we use wireless broadcast as a means of protecting the aggregate information for a class of functions. Exploiting the diversity of wireless medium, broadcasting spreads information spatially, and the properties of the function enable distributed in-network computation with the spread information. We show that aggregation with broadcast can improve delay performance while satisfying the same level of reliability. The gain can be presented as a function of reliability constraints and transmission range. We also develop a distributed algorithm for aggregation with broadcast. Simulation results show that aggregation with broadcast outperforms aggregation with unicast, especially, in severely lossy network environments.

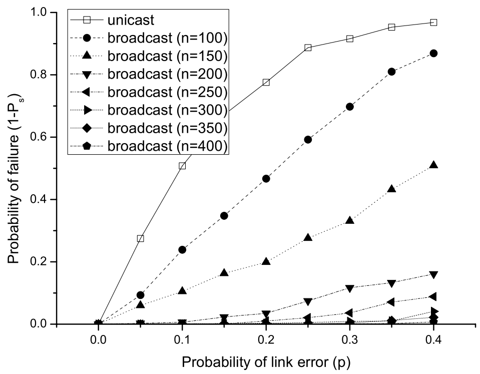


Fig. 6. Loss rate for aggregation with broadcast. Delay is taken into account with the bound as the minimum plus $2\hat{h}$. As the number of nodes increases, aggregation with broadcast can further exploit the diversity resulting in the decrease of the loss rate. Aggregation with unicast is set to $r_u = 2$.

There are many interesting open questions to consider. Aggregation functions besides the minimal masking functions should be considered. Although we focus on the delay performance, other performance metrics such as time complexity and achievable sampling rate are also of importance. It would be interesting to study the relationship between these metrics with aggregation functions and network topologies.

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